PHYS-201(e)
General Physics: Electromagnetism

Prof. P. Scarlino

Mock Exam

20.12.2024

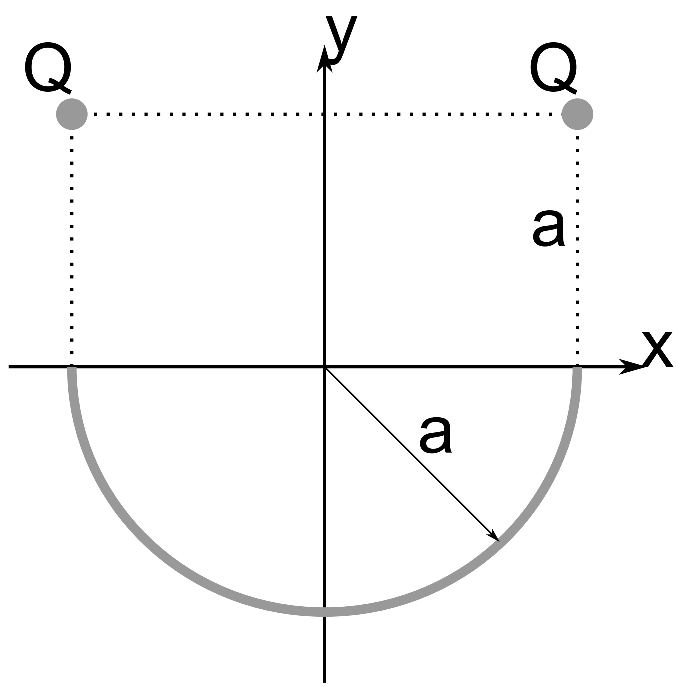
Name: ____________________ Sciper: ____________________

- **P**ut your name and Sciper number on every paper sheet (like the sticker in the first page).
- **Please give at least a brief explanation of the calculations you are undertaking. Any answer requires a justification, even a brief one.**
- The exam takes place from 9:15 to 12:15 a.m.
- Please put your valid ID on the table.
- There are six problems. Each problem provides different points as indicated. Check that you have received all problems.
- One page of handwritten notes is allowed as well as a calculator without equation solver. The items will be checked in the course of the written exam.
- If you want to leave before the end of the written exam, please raise your hand and wait until an assistant arrives at your place.
- Please respect your colleagues and do not leave between 11:45 and 12:15 am. Use remaining time to check your solutions.
- Please remain seated while the papers are collected at the end of the exam.
- The copy must be written in blue or black pen or fountain pen, using indelible ink; pencil, in particular, is not allowed, except for graphs.
- Number each page and indicate the total number of double sheets on the first page at the end of the exam.
- For a graph to be considered correct, it must have a legend, the axes must be named and, if it makes sense, scaled.
- Useful constants: $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\mathrm{H\,m^{-1}}$, $c = 3 \times 10^{8}\mathrm{m\,s^{-1}}$.

Problem 1:

(6 pts), 3 questions

Consider the charge distribution shown in the figure below. A semicircle of radius a centered at the origin (indicated by the gray curved line) carries a uniform positive charge per unit length λ. Two positive point charges Q are located at points $(-a, a)$ and (a, a) (indicated by the two gray dots).

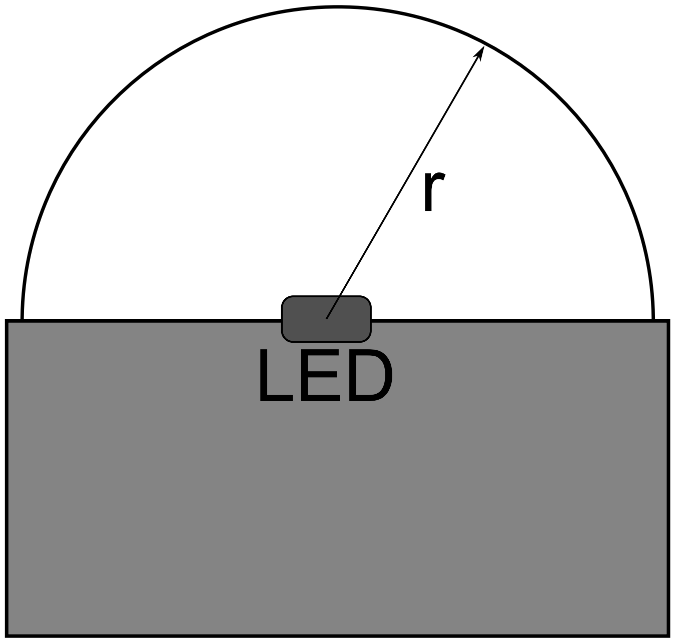


1. Find the expression of the electric field at the origin <u>due to the charged semicircle only</u>. Express your answer in unit vector notation.
2. Find the expression of the electric field at the origin <u>due to the point charges only</u>. Express your answer in unit vector notation.
3. Find the value of Q for which the total electric field at the origin is zero.

Problem 2:

(5 pts), 4 questions

A string of Christmas lights consists of LEDs covered by transparent hemispherical domes, as shown in the figure below. The dome has a radius r of 7.50 mm. The LEDs convert electrical energy into light radiation energy at a rate of 0.096 W. The LEDs radiate uniformly into the hemispherical dome.



1. Find the wavenumber k and angular frequency ω of the light knowing that the LEDs emit red light with a wavelength of 650 nm. Give the analytical expressions and numerical answers.

2. Find the intensity of the radiation at the surface of the hemispherical dome. Give the analytical expression and numerical answer.

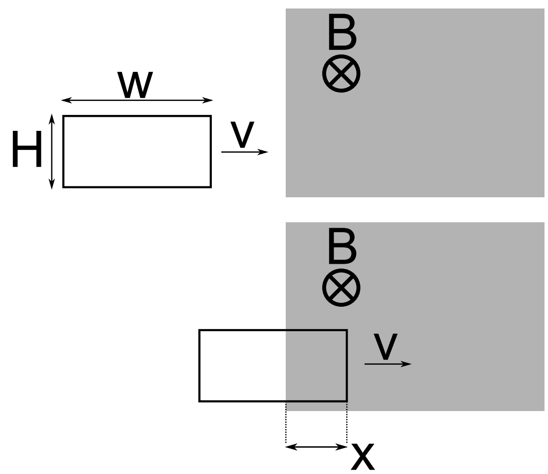
3. Find the amplitudes of the electric and magnetic fields at the surface of the hemispherical dome. Give the analytical expressions and numerical answers.

4. Find the average pressure that the light exerts on a perfectly absorbing particle of dust resting on the hemispherical dome. Give the analytical expression and numerical answer.

Problem 3:

(5 pts), 3 questions

A rectangular loop of wire of width w, height H and resistance R travels at a constant speed v into a uniform magnetic field B (gray region). The plane of the rectangular loop is perpendicular to the magnetic field, which is directed into the page. A schematic of the problem is shown below.



1. Find the magnitude of the induced current in the rectangular loop while it is entering the region with uniform magnetic field (as in the bottom panel of the figure above). Draw the direction of the induced current in the loop on a schematic (briefly justify qualitatively your answer). Clearly indicate the region where there is the uniform magnetic field on your drawing.

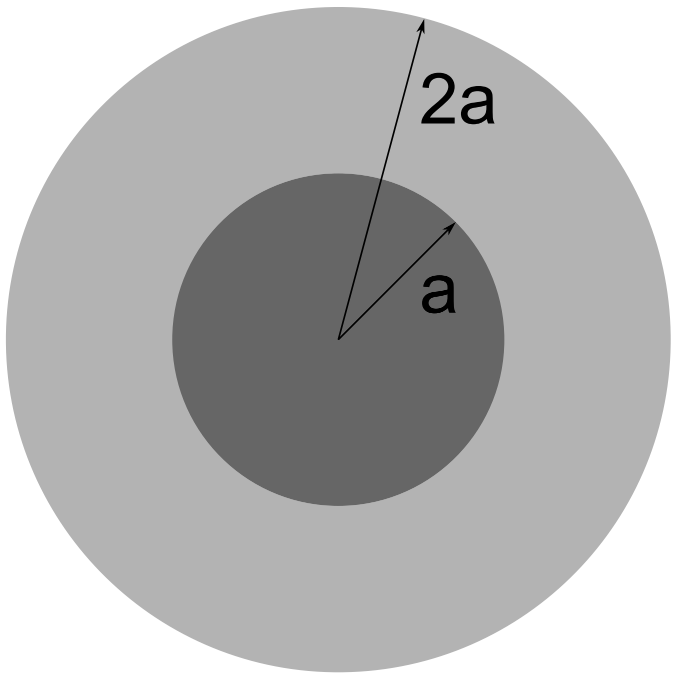
2. Find the magnitude of the total magnetic force exerted on the rectangular loop while it is entering the region with uniform magnetic field (as in the bottom panel of the figure above). Draw the direction of <u>all</u> the forces exerted on the loop on a schematic. Clearly indicate the region where there is the uniform magnetic field on your drawing.

3. Once the loop is entirely in the region of uniform magnetic field, what is the magnitude of the induced current ?

Problem 4:

(11 pts), 6 questions

A solid insulating plastic sphere of radius a (dark gray) carries a net positive charge $3Q$ uniformly distributed throughout its interior. The insulating sphere is coated with a metallic layer of inner radius a and outer radius $2a$ (light grey). The conducting metallic layer carries a total net charge of $-2Q$. The system is represented below. Assume that the system is at equilibrium.



1. Find the volume charge density ρ associated with the charge uniformly distributed in the plastic sphere in terms of the variables introduced above.

2. Apply Gauss law to derive the magnitude of the electric field in the different regions:

   (a) $r < a$

   (b) $a < r < 2a$

   (c) $r > 2a$

   For each region, draw the Gaussian surface you are using on a schematic, and indicate on the surface the direction of any vectors which appear in the analytical expression of Gauss law. Express your answer in terms of a, Q, r and ε_0.

3. Draw the electric field profile as a function of r in all different regions. Comment on the continuity of the electric field.

4. Find the electric potential as a function of r in all regions. Use $V(\infty) = 0$ as the reference point.

    (a) $r > 2a$

    (b) $a < r < 2a$

    (c) $r < a$

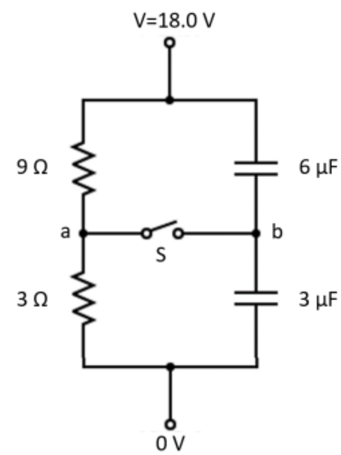
5. Draw the electric potential as a function of r in all regions. Comment on the continuity of the electric potential.

6. Find the net charge q_i residing on the inner surface of the metallic layer, and the net charge q_0 residing on the outer surface of the metallic layer.

Problem 5:

(8 pts), 3 questions

Consider the electrical circuit in the figure below.

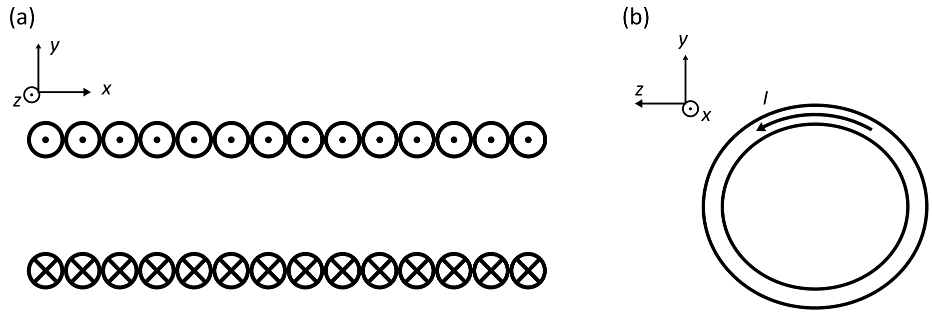


1. Calculate the potential difference between points a and b when the switch S is open. Write explicitly which of the two points is at a higher potential.
2. At some point the switch S is closed. Calculate the value of the potential in point b a very long time after the switch is closed.
3. Write the Kirchoff equations for the circuit at a generic time t after the switch S has been closed. Determine the characteristic time constant τ for the charge rebalancing between the two capacitors.

Problem 6:

(7 pts), 3 questions

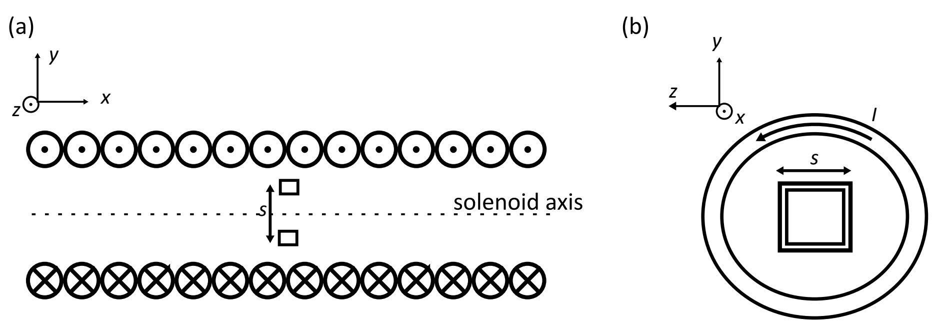
An infinitely long ideal solenoid has n turns per unit length, and it is carrying a current I. A schematic of the cross section of the side view and the front view of this solenoid are respectively shown in panels (a) and (b) below.



1. Use Ampere's law to derive the magnitude of the field produced inside this solenoid. Draw a schematic of the orientation of the magnetic field inside the solenoid, the integration path used in Ampere's law and indicate any vectors which appear in the analytical expression of Ampere's law.

2. The infinitely long ideal solenoid has $n = 400$ turns per meter. The current flowing through the solenoid is described by the function $I = 3t^2$, where t represents time in seconds and I is the current in amperes. A square coil with sides of length $s = 1.3$ cm is fixed inside the solenoid such that its axis coincides with that of the solenoid, as depicted in the figure below. At $t = 0.75$ seconds, the induced electromotive force (EMF) is 22.2 μV. A schematic of the cross section of the side view and the front view of the solenoid with the coil inside are respectively shown in panels (a) and (b) below.

   Find the number of turns of the square coil. Note that the thickness of the wires can be neglected throughout the problem. Give the analytical expression and numerical answer.



3. The current flowing through the solenoid is now constant in time and has a magnitude of $I_{\text{sol}} = 10$ mA. The square coil within the solenoid is now attached such that it can rotate around an axis aligned along the z direction and passing through its center. An external torque of $\vec{\tau}_{\text{ext}} = 5\,\text{N}\,\text{m}\hat{z}$ is applied on the coil.

   If the square coil axis is at a 45° angle relative to that of the solenoid, find the required current that must flow through the square coil and its direction to prevent any rotational movement of the square coil. Recall that the solenoid has 400 turns per meter, the square coil has sides of length $s = 1.3$ cm. For the number of turns in the square coil, use your answer to the question above. Give the analytical expression and numerical answer.