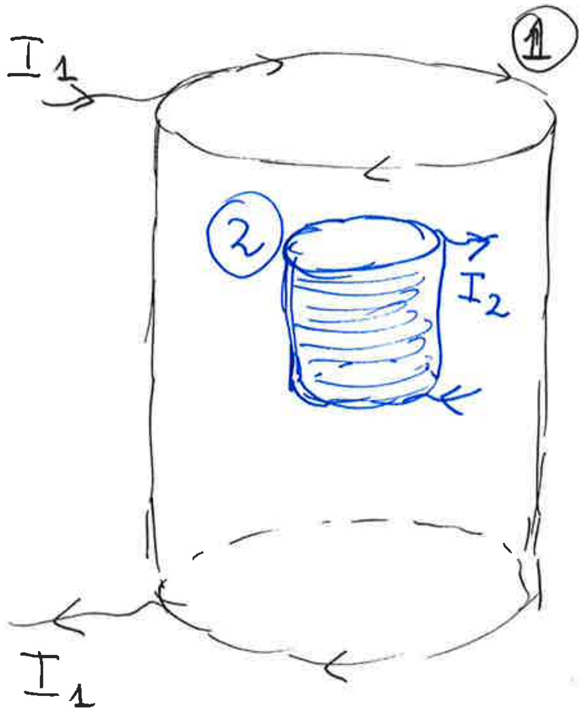
In this problem there are 2 coils, arranged as in the drawing.



The problem asks to calculate the mutual inductance.

But now we can choose to calculate M_{12} or M_{21} (both of them $= M$).

If we try to calculate M_{12}:

$$\mathcal{E}_1 = -\frac{d}{dt}\left(M_{12} I_2\right)$$

But the flux of B_2 = magnetic field generated by I_2 through coil 2 is not uniform!

$\Rightarrow$ M_{12} is very difficult to calculate!

If instead we consider a current I_1 in the coil 1 it generates a uniform magnetic field all inside coil 1 $\Rightarrow$ It is easy to calculate the flux of B_1 through coil 2

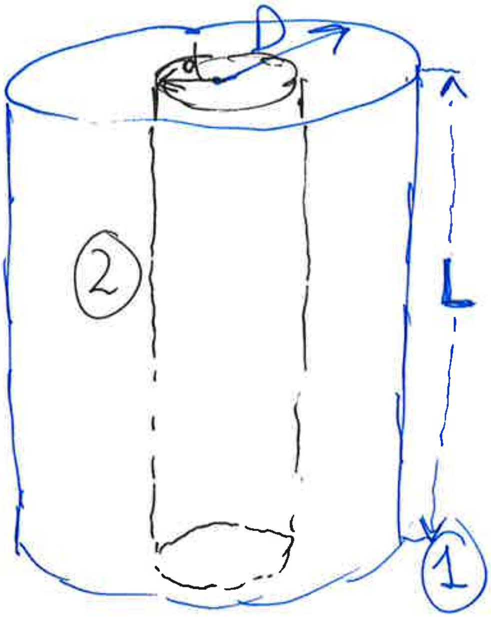
$$\Phi_2 = B_1 A_2 N_2$$

Therefore, M_{21} is much easier to calculate!

A_i, N_i (with $i = 1, 2$) are respectively the area and the number of loops of coil i

About the problem in slide 10.23

Two solenoids of diameter d and D inside each other



$$M_{21} = -\frac{\mathcal{E}_2}{\partial I_1/\partial t}$$

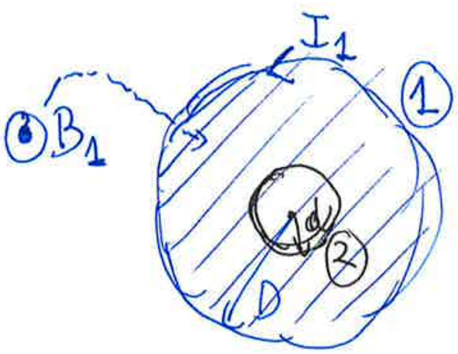
$$\mathcal{E}_2 = -\frac{d}{dt}\Phi_2 = -\frac{d}{dt}\left[A_2 N_2 B_1\right] =$$

$$= -\frac{d}{dt}\left[A_2 N_2\left(\mu_0 \frac{N_1}{L} I_1\right)\right] =$$

$$= -A_2 N_2 \mu_0 \frac{N_1}{L}\frac{dI_1}{dt}$$

If you now try to calculate $M_{12} = -\dfrac{\mathcal{E}_1}{\partial I_2/\partial t}$ you will need to consider the flux Φ_1 due to the magnetic field B_2 generated by coil (2) through the loops of coil (1)

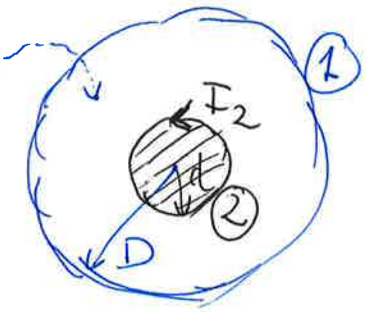
B_1 through coil (2)



$$\Phi_2 = \boxed{A_2} N_2 B_1$$

B_2 through coil (1)

In between there is not B_2 field



$$\Phi_1 = N_1 \oint \vec{B_2}\cdot d\vec{A_1} =$$

$$= \cancel{\int_D^d} 0 + \int_d^0 = B_2 \boxed{A_2} N_1$$