Today's agenda (part I):

Electric potential energy.

You must be able to use electric potential energy in work-energy calculations.

Electric potential.

You must be able to calculate the electric potential for a point charge, and use the electric potential in work-energy calculations.

Electric potential and electric potential energy of a system of charges.

You must be able to calculate both electric potential and electric potential energy for a system of charged particles (point charges today, charge distributions next lecture).

The electron volt.

You must be able to use the electron volt as an alternative unit of energy.

Potential energy vs. electric potential

Two new quantities:

- (electric) potential energy U
- electric potential V

Do not mix them up!

(electric) potential energy ≠ electric potential

(Electric) potential energy

Work done by electric (Coulomb) force:

$$\left[W_E\right]_{i\to f} = \int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell}$$

- **independent** of path -> force is **conservative**

Define **potential energy U:** (unit J or Nm)

$$U_f - U_i = -\left[W_E\right]_{i\to f} = -\int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell}$$

equivalently: $\vec{F}_E = -\dfrac{\partial U}{\partial \vec{r}}$

- potential energy defined w.r.t. (initial) **reference state**

Potential energy of two point charges

- two point charges q_1 and q_2, initially at infinite distance
- moved to distance r_{12}

$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell} = -\int_{\infty}^{r_{12}} \frac{k\ q_1 q_2}{r^2}\,dr = \frac{k\ q_1 q_2}{r_{12}}$$

potential energy of two point charges

$$\boxed{U(r_{12}) = k\ \frac{q_1 q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}}$$

works for positive and negative charges

potential energy is zero in **reference state** when particles are infinitely far apart

You **must** use this convention if you want to use the equation for potential energy of point charges! If you use the above equation, you are "automatically" using this convention.

Potential energy of point charge in electric field

- force on point (test) charge: $\vec{F} = q\vec{E}$

$$U_f - U_i = -\int_{r_i}^{r_f} \vec{F} \cdot d\vec{\ell} = -q\int_{r_i}^{r_f} \vec{E} \cdot d\vec{\ell}$$

potential energy of point charge in electric field:

$$\boxed{U_f - U_i = -q\int_{i}^{f} \vec{E} \cdot d\vec{\ell}}$$

Example: calculate the electric potential energy of two protons separated by a typical proton-proton intranuclear distance of $2x10^{-15}$ m.

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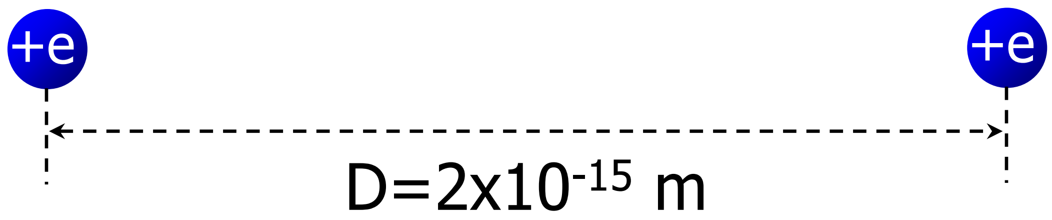

$$U = k\,\frac{q_1 q_2}{r_{12}} = k\,\frac{(+e)(+e)}{D} = 9\times10^{9}\,\frac{\left(+1.6\times10^{-19}\right)\left(+1.6\times10^{-19}\right)}{\left(2\times10^{-15}\right)}$$

$$U = +1.15\times10^{-13}\ \text{J}$$

What is the meaning of the + sign in the result?

Example: calculate the electric potential energy of a hydrogen atom (electron-proton distance is 5.29×10^{-11} m).

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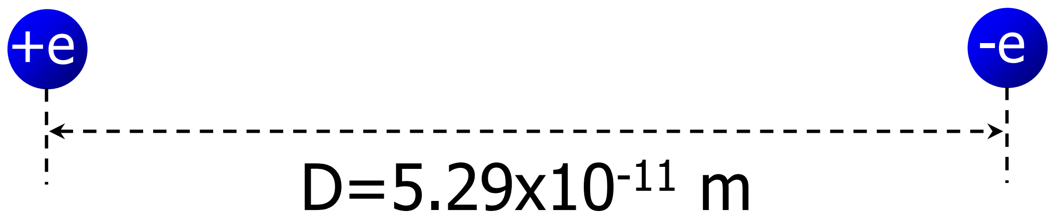

$$U = k\,\frac{q_1 q_2}{r_{12}} = k\,\frac{(+e)(-e)}{D} = 9\times10^{9}\,\frac{\left(+1.6\times10^{-19}\right)\left(-1.6\times10^{-19}\right)}{\left(5.29\times10^{-11}\right)}$$

$$U = -4.36\times10^{-18}\ \text{J}$$

What is the meaning of the - sign in the result?
Is that a small energy?

Today's agenda:

Electric potential energy (continued).

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Work-energy problems

- system of charged particle has electric potential energy
- if charges move, kinetic and potential energies change

Energy conservation law:

$$E_f - E_i = (K_f + U_f) - (K_i + U_i) = \left[W_{other}\right]_{i \to f}$$

This is from Mechanics.

Important: Distinguish conservative work and external work

$$\Delta U = U_f - U_i = -\left[W_{\text{conservative}}\right]_{i\to f}$$

change in potential energy is defined as the negative of the work done by the conservative force which is associated with the potential energy (today, **the electric force**).

If an external force moves an object "against" the conservative force,* and the object's kinetic energy remains constant, then

$$\left[W_{\text{external}}\right]_{i\to f} = -\left[W_{\text{conservative}}\right]_{i\to f}$$

Always ask yourself which work you are calculating!

*for example, if you "slowly" lift a book, or "slowly" push two negatively charged balloons together

Example: two isolated protons are constrained to be a distance $D = 2 \times 10^{-10}$ meters apart (a typical atom-atom distance in a solid). If the protons are released from rest, what maximum speed do they achieve, and how far apart are they when they reach this maximum speed?

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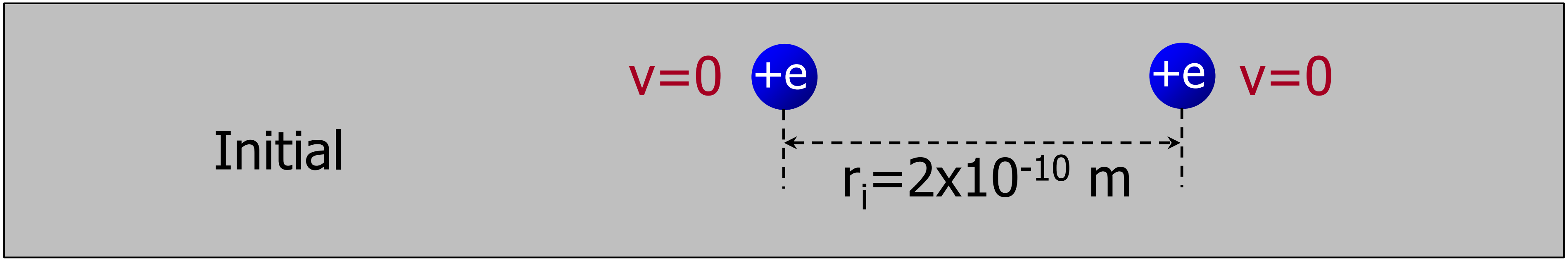
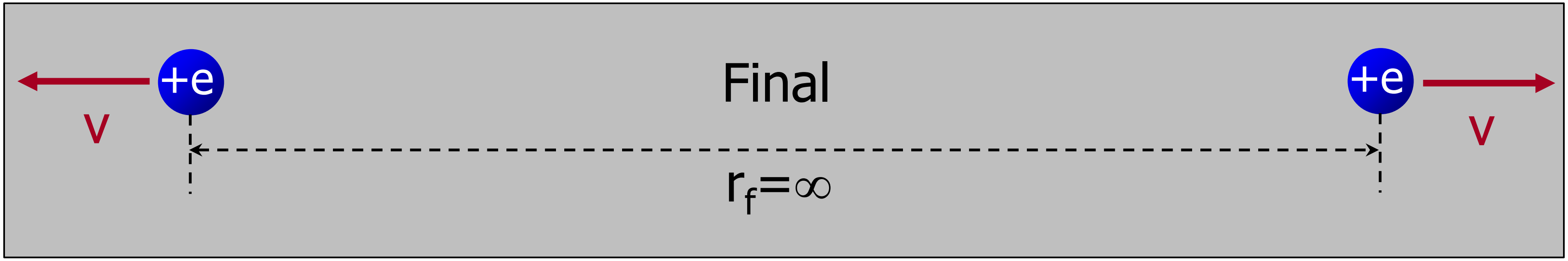
We need to do some thinking first.

What is the proton’s potential energy when they reach their maximum speed?

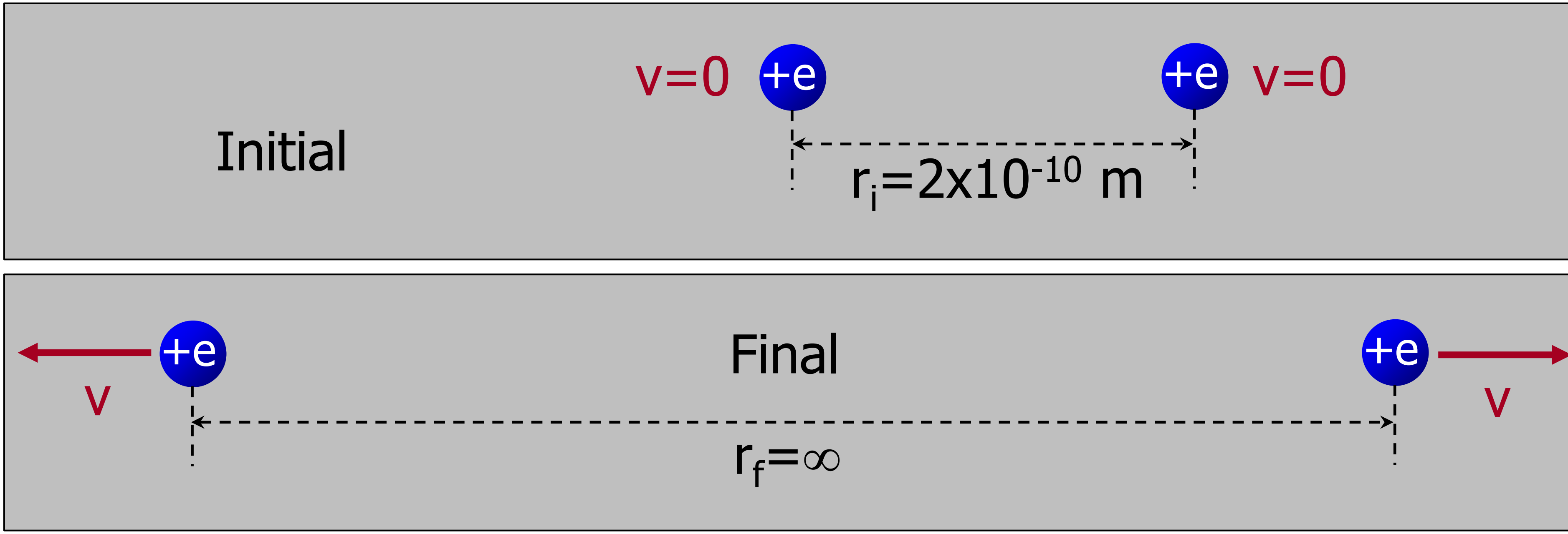
How far apart are the protons when they reach their maximum speed?

Example: two isolated protons are constrained to be a distance $D = 2\text{x}10^{-10}$ meters apart (a typical atom-atom distance in a solid). If the protons are released from rest, what maximum speed do they achieve, and how far apart are they when they reach this maximum speed?





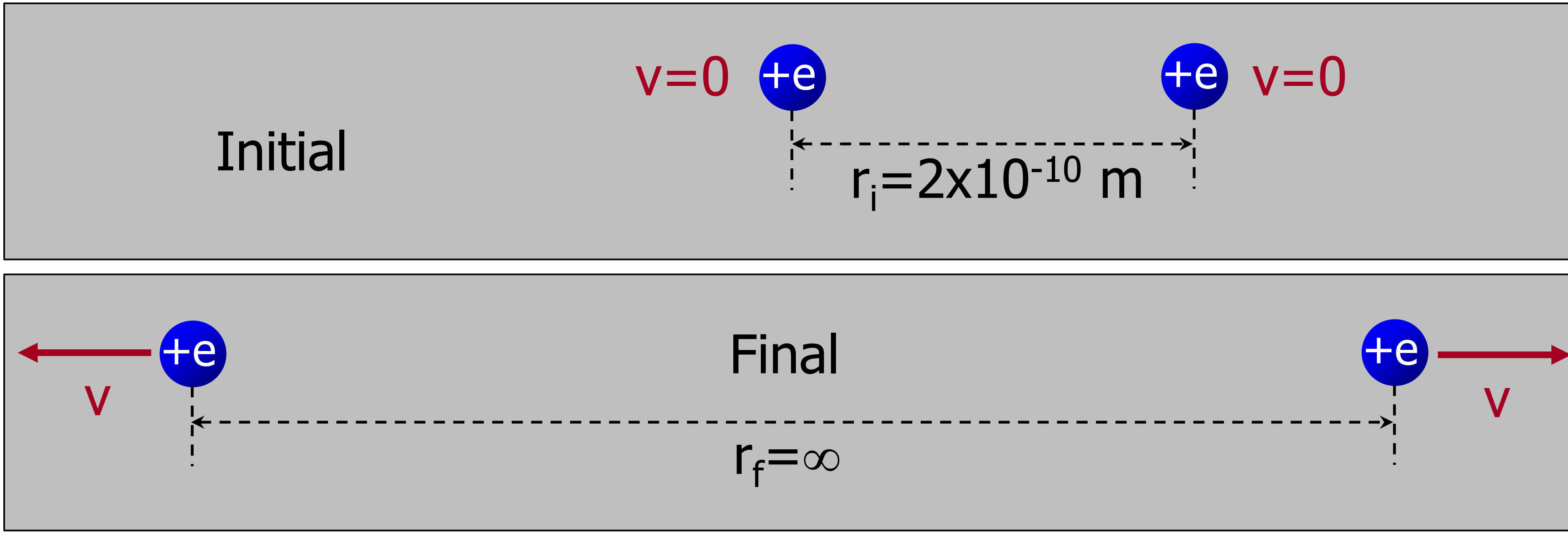
Why are the two speeds the same? There is a conservation of momentum problem buried in here!



$$E_f - E_i = [W_{other}]_{i\to f}$$

$$K_f + \cancelto{0}{U_f} - (\cancelto{0}{K_i} + U_i) = [\cancelto{0}{W}_{other}]_{i\to f}$$

$$K_f = U_i$$



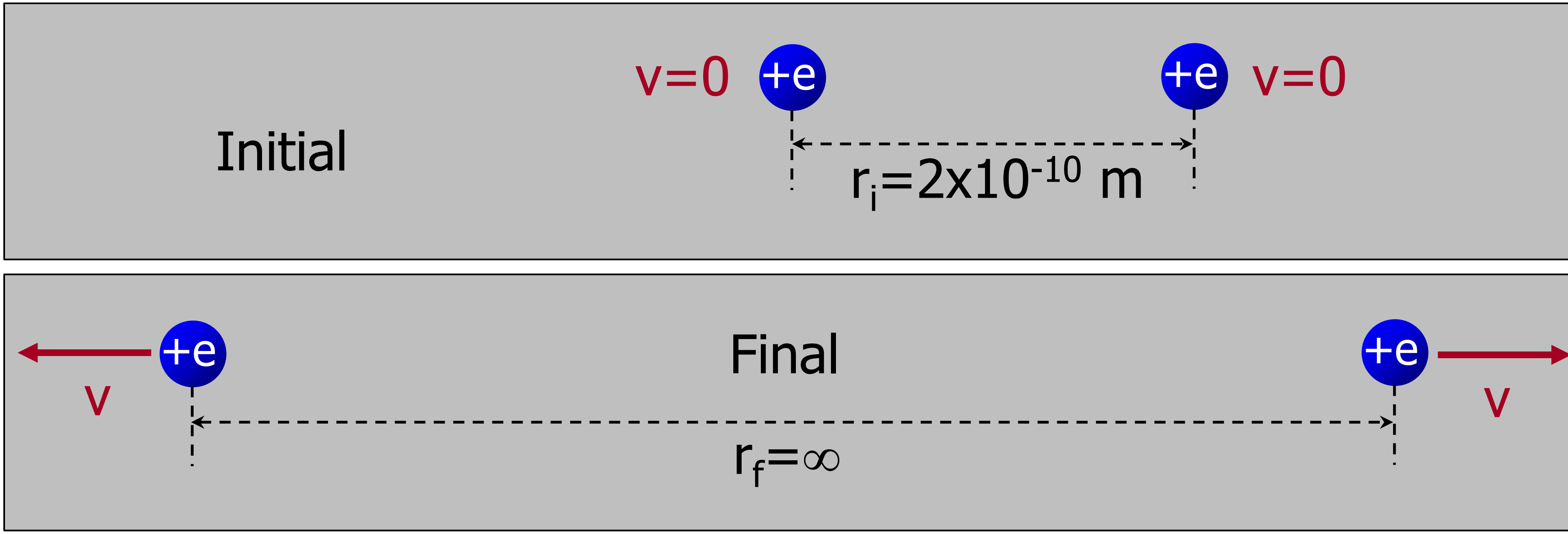
$$K_f = U_i$$

How many objects are moving in the final state? Two.

How many K_f terms are there? Two.

How many **pairs** of charged particles in the initial state? One.

How many U_i terms are there? One.



$$K_f = U_i$$

$$2\left(\frac{1}{2}m_p v^2\right) = k\frac{(+e)(+e)}{r_i}$$

$$v = \sqrt{\frac{ke^2}{m_p r_i}} = \sqrt{\frac{\left(9\times10^{9}\right)\left(1.6\times10^{-19}\right)^2}{\left(1.67\times10^{-27}\right)\left(2\times10^{-10}\right)}} = 2.63\times10^{4}\,\frac{m}{s}$$

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Electric Potential

We have defined electric field by force it exerts on test charge

$$\vec{F} = q\vec{E}$$

Now: define **electric potential V** via potential energy of test charge

$$U = qV$$

$$\boxed{U_f - U_i = q\,(V_f - V_i)}$$

source charges create electric potential V,
test charge q feels the potential,
this produces potential energy U

unit of electric potential: Nm/C = V (Volt)

Electric Potential of a point charge:

$$V(r) = \frac{U(r)}{q_1} = \frac{1}{q_1}\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}}$$

q_1 is the test charge, q_2 is the source charge

so that the electric potential of a point charge q is

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ .}$$

Only valid for a point charge!

Relation between electric potential and electric field

$$V_f - V_i = \frac{U_f - U_i}{q} = \frac{-\int_{r_i}^{r_f} \vec{F}_E \cdot d\vec{\ell}}{q} = -\int_{r_i}^{r_f} \frac{\vec{F}_E}{q} \cdot d\vec{\ell} = -\int_{r_i}^{r_f} \vec{E} \cdot d\vec{\ell}\,.$$

$$\boxed{\mathrm{V_f} - \mathrm{V_i} = -\int_{\mathrm{i}}^{\mathrm{f}} \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\ell}}$$

Two conceptual examples.

Example: a proton is released in a region in space where there is an electric potential. Describe the subsequent motion of the proton.

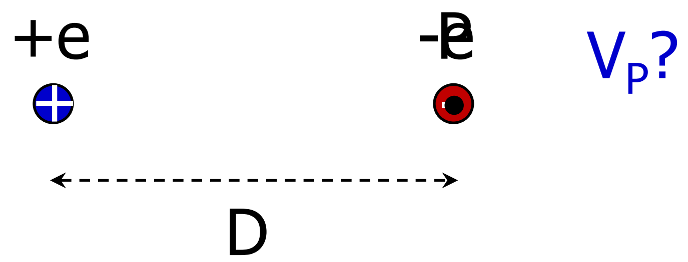
The proton will move towards the region of lower potential. As it moves, its potential energy will decrease, and its kinetic energy and speed will increase.

Example: a electron is released in a region in space where there is an electric potential. Describe the subsequent motion of the electron.

The electron will move towards the region of higher potential. As it moves, its potential energy will decrease, and its kinetic energy and speed will increase.

Protons fall down, electrons fall up.

What is the potential due to the proton in the hydrogen atom at the electron's position (5.29×10^{-11} m away from the proton)?



$$V_p = \frac{kq}{r} = \frac{k(+e)}{D} = \frac{(9 \times 10^{9})(+1.6 \times 10^{-9})}{(5.29 \times 10^{-11})} = 27.2\ \mathrm{V}$$

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Electric Potential Energy of a System of Charges

Electric potential energy comes from the interaction between pairs of charged particles, so you have to add the potential energies of **each pair** of charged particles in the system.

(Could be a pain to calculate!)

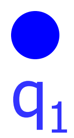
Electric Potential of a System of Charges

The potential due to a particle depends only on the charge of that particle and where it is relative to some reference point.

The electric potential of a system of charges is simply the sum of the potential of each charge. (Much easier to calculate!)

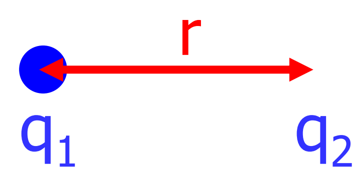
Example: electric potential energy of three charged particles

A single charged particle has no electrical potential energy. To find the electric potential energy for a system of two charges, we bring a second charge in from an infinite distance away:



$$U = 0$$

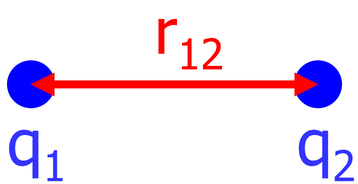
before



$$U = k\frac{q_1 q_2}{r}$$

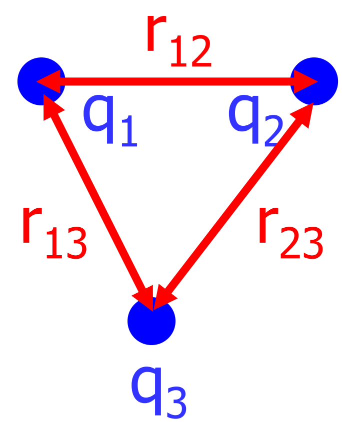
after

To find the electric potential energy for a system of three charges, we bring a third charge in from an infinite distance away:



before

$$U = k\frac{q_1 q_2}{r_{12}}$$



after

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

We have to add the potential energies of each pair of charged particles.

Electric Potential of a Charge Distribution

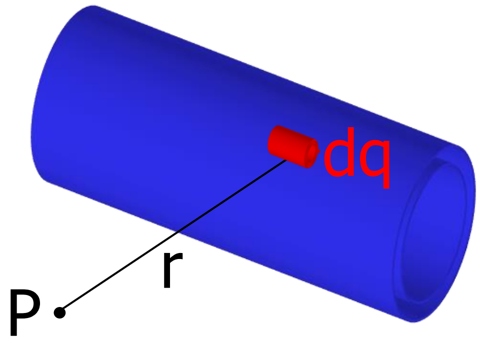
Collection of charges: $$V_P = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}.$$

P is the point at which V is to be calculated, and r_i is the distance of the i^{th} charge from P.

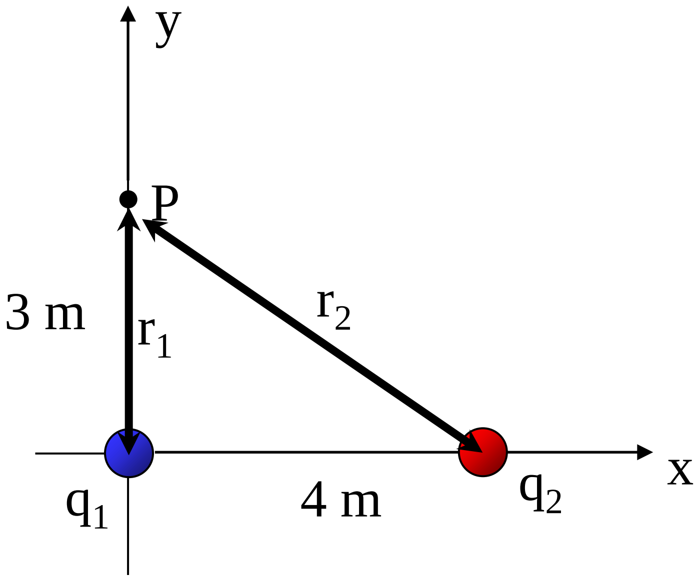
Charge distribution:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}\ .}$$

Potential at point P.



Example: a 1 μC point charge is located at the origin and a -4 μC point charge 4 meters along the +x axis. Calculate the electric potential at a point P, 3 meters along the +y axis.

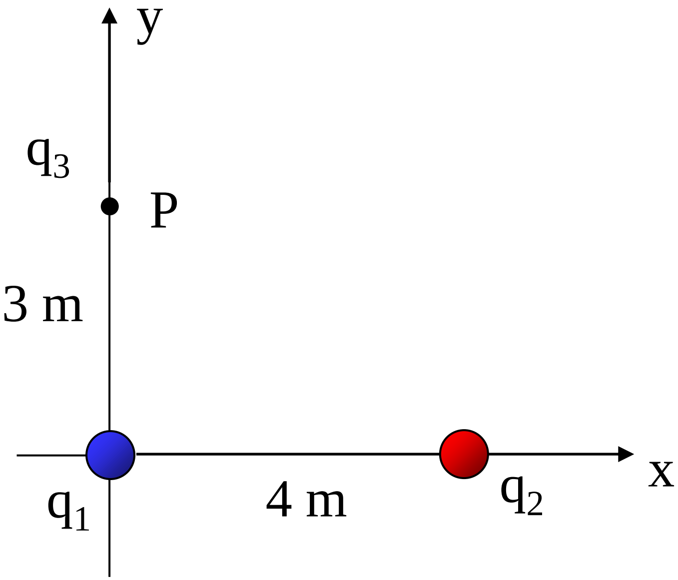


$$V_P = k\sum_i \frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 9\times10^9\left(\frac{1\times10^{-6}}{3} + \frac{-4\times10^{-6}}{5}\right)$$

$$= -4.2\times10^3\ \mathrm{V}$$

Example: how much work is required to bring a +3 μC point charge from infinity to point P? (And what assumption must we make?)



$$W_{external} = \Delta E = \cancelto{0}{\Delta K} + \Delta U$$

$$W_{external} = \Delta U = q_3 \Delta V$$

$$W_{external} = q_3\left(V_P - \cancelto{0}{V_\infty}\right)$$

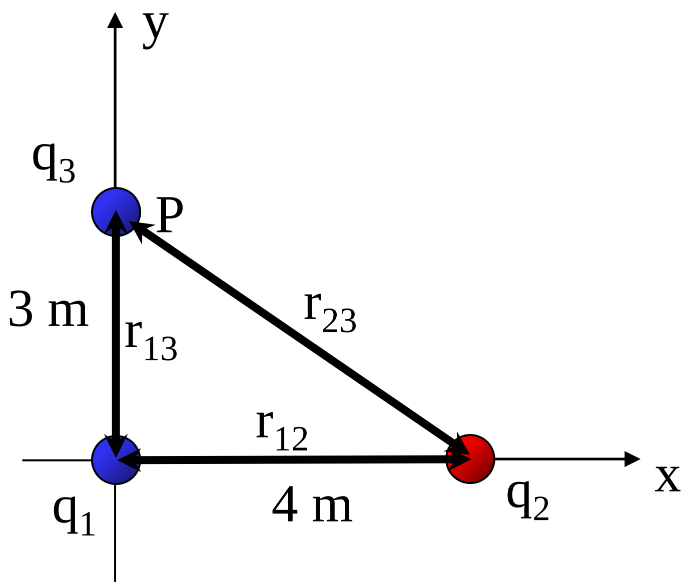
$$W_{external} = 3\times10^{-6}\left(-4.2\times10^{3}\right)$$

$$W_{external} = -1.26\times10^{-3}\,J$$

The work done by the external force was negative, so the work done by the electric field was positive. The electric field "pulled" q_3 in (keep in mind $|q_2|$ is 4 times $|q_1|$).

Positive work would have to be done by an external force to remove q_3 from P.

Example: find the total potential energy of the system of three charges.



$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$U = 9\times10^{9}\left(\frac{\left(1\times10^{-6}\right)\left(-4\times10^{-6}\right)}{4} + \frac{\left(1\times10^{-6}\right)\left(3\times10^{-6}\right)}{3} + \frac{\left(-4\times10^{-6}\right)\left(3\times10^{-6}\right)}{5}\right)$$

$$U = -2.16\times10^{-2}\ \text{J}$$

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The Electron Volt

An electron volt (eV) is the energy acquired by a particle of charge e when it moves through a potential difference of 1 volt.

$$\Delta U = q\Delta V$$

$$1\text{ eV} = \left(1.6\times10^{-19}\text{C}\right)\left(1\text{ V}\right)$$

$$\boxed{1\text{ eV} = 1.6\times10^{-19}\text{J}}$$

This is a very small amount of energy on a macroscopic scale, but electrons in atoms typically have a few eV (10's to 1000's) of energy.

Example: on slide 9 we found that the potential energy of the hydrogen atom is about -4.36x10^{-18} joules. How many electron volts is that?

$$U = -4.36\times10^{-18}\,\mathrm{J} = \left(-4.36\times10^{-18}\,\mathrm{J}\right)\left(\frac{1\ \mathrm{eV}}{1.6\times10^{-19}\,\mathrm{J}}\right) \approx -27.2\ \mathrm{eV}$$

"Hold it! I learned in Chemistry (or high school physics) that the ground-state energy of the hydrogen atom is -13.6 eV. Did we make here a physics mistake?"

The ground-state energy of the hydrogen atom includes the positive kinetic energy of the electron, which happens to have a magnitude of half the potential energy.

Add KE+PE to get ground state energy.

Today’s agenda (part II):

Electric potential of a charge distribution.

You must be able to calculate the electric potential for a charge distribution.

Equipotentials.

You must be able to sketch and interpret equipotential plots.

Potential gradient.

You must be able to calculate the electric field if you are given the electric potential.

Potentials and fields near conductors.

You must be able to use what you have learned about electric fields, Gauss’ law, and electric potential to understand and apply several useful facts about conductors in electrostatic equilibrium.

Electric potential V of charge distributions

- last lecture: potentials of **point charges**
- now: potentials of **extended charged objects** (line charges, sheet charges, spheres, cylinders, etc)

Two strategies:

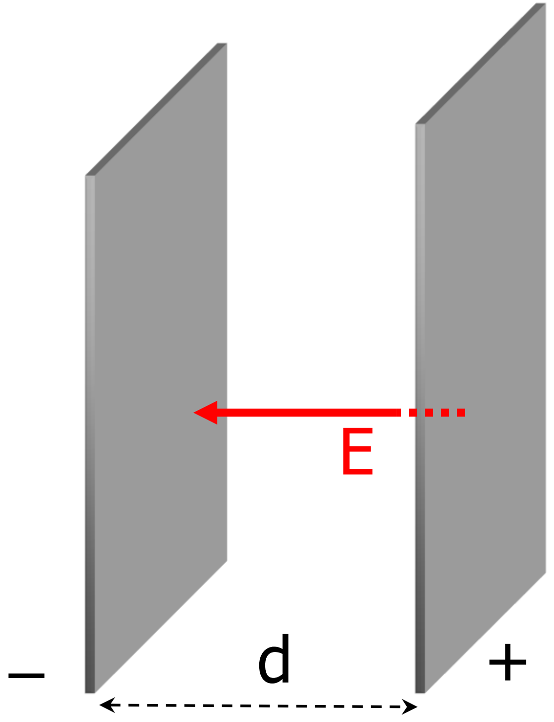
- decompose distribution into charge elements, integrate their contributions to V

- first find the electric field of the distribution (for example via Gauss' law), then integrate $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{\ell}$

Example 1: electric potential between two parallel charged plates.

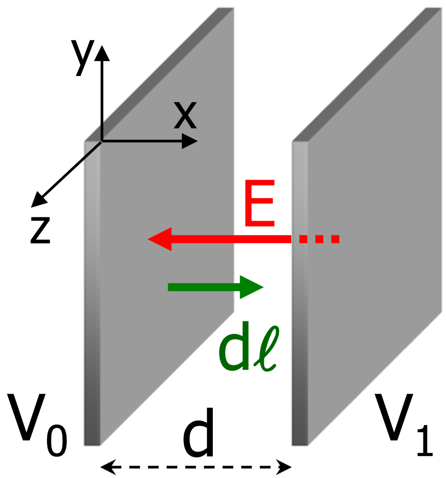
- plates carry surface charge density σ
- plates separated by distance d
- plates are large compared to d

$E = |\sigma| / \varepsilon_0$, perpendicular to plates



$$\Delta V = V_1 - V_0 = -\int_{\text{plate }0}^{\text{plate }1} \vec{E} \cdot d\vec{\ell}$$

$d\vec{\ell}$ is $d\vec{x}$



$$\Delta V = -\int_0^d \left(-E\ dx\right) = E\int_0^d dx = Ed$$

V is **higher** at the **positive** plate

$$E = \frac{|\Delta V|}{d},\quad \text{or}\ \boxed{|\Delta V| = Ed}$$

holds for constant field only

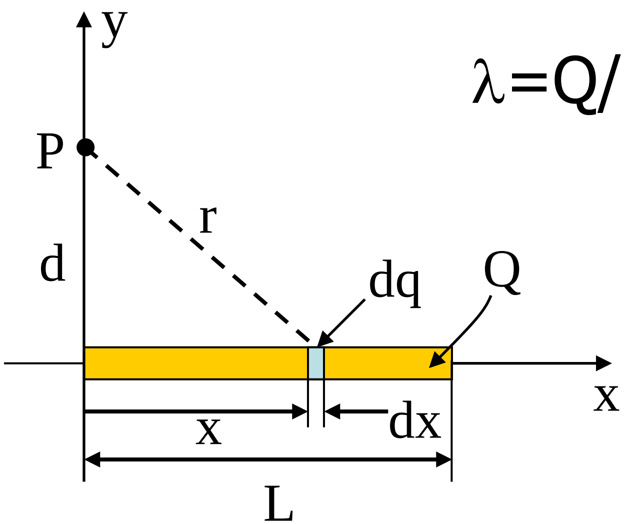
Example 2: A thin rod of length L located along the x-axis has a total charge Q uniformly distributed along the rod. Find the electric potential at a point P along the y-axis a distance d from the origin.

Follow line charge recipe

1. Decompose line charge

$dq = \lambda dx$

$\lambda = Q/L$



2. Potential due to charge element

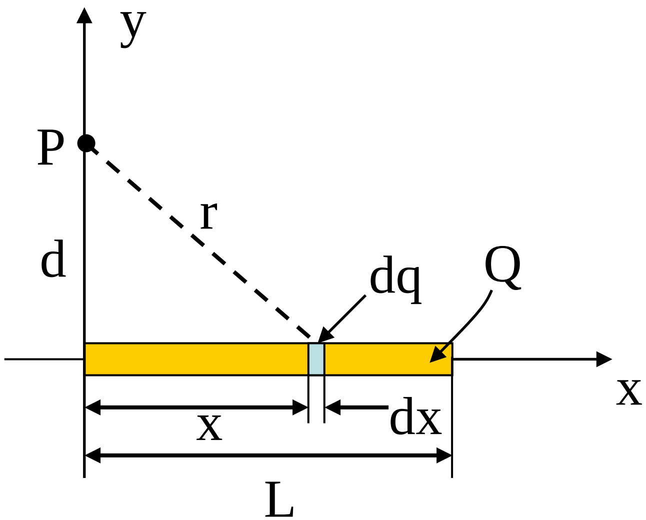
$$dV = k\frac{dq}{r} = k\frac{\lambda dx}{\sqrt{x^2 + d^2}} \quad *$$

***What are we assuming when we use this equation?**

3. Integrate over all charge elements

$$V = \int_{\text{rod}} dV$$

$$V = \int_0^L k \frac{\lambda dx}{\sqrt{x^2 + d^2}} = k \frac{Q}{L} \int_0^L \frac{dx}{\sqrt{x^2 + d^2}}$$



Use integral:

$$\int \frac{dx}{\sqrt{x^2 + d^2}} = \ln\left(x + \sqrt{x^2 + d^2}\right)$$

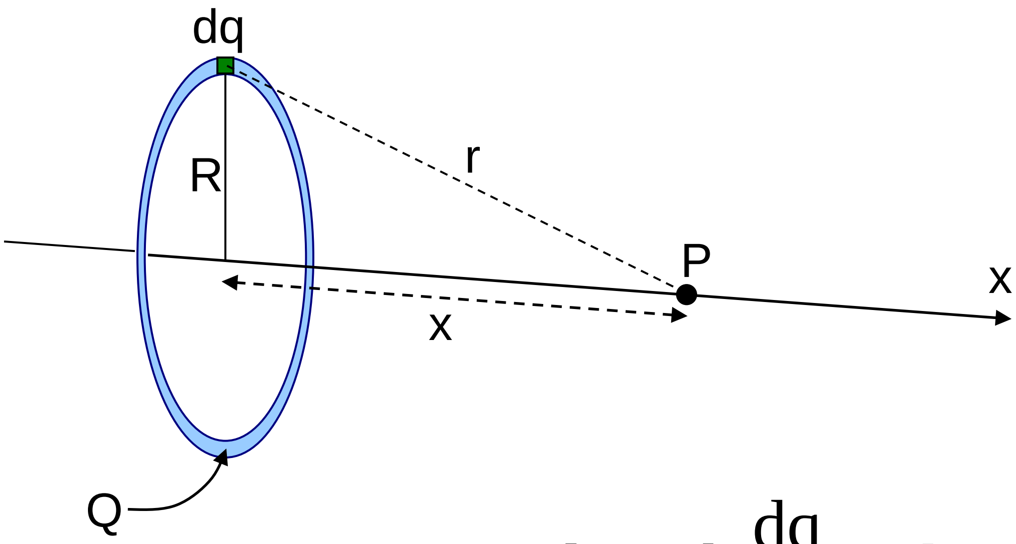
$$V = \frac{kQ}{L} \ln\left(\frac{L + \sqrt{L^2 + d^2}}{d}\right)$$

Include the sign of Q to get the correct sign for V.

note:
$\ln(a) - \ln(b) = \ln(a/b)$

What is the direction of V?

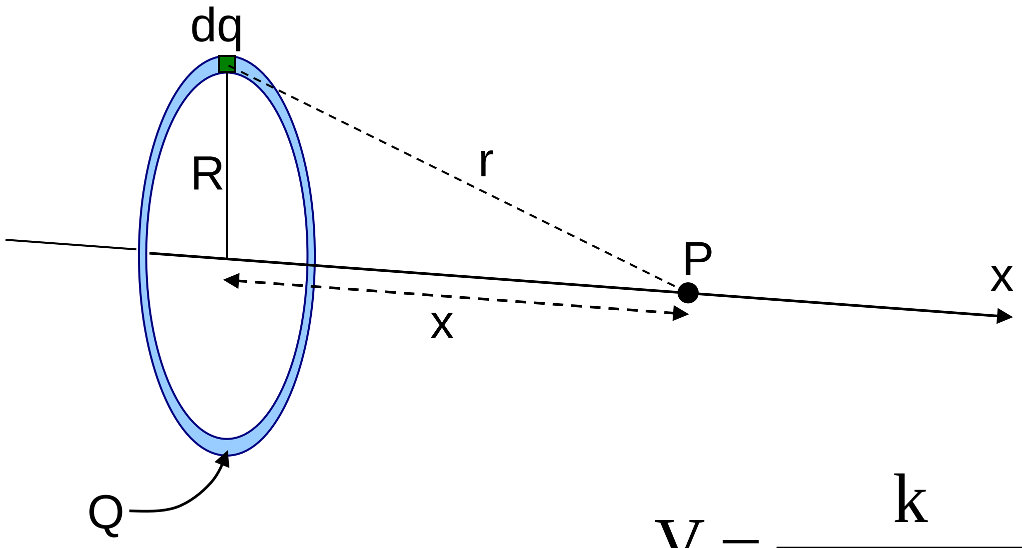
Example 3: Find the electric potential due to a uniformly charged ring of radius R and total charge Q at a point P on the axis of the ring.



Every dq of charge on the ring is the same distance from the point P.

$$dV = k\frac{dq}{r} = k\frac{dq}{\sqrt{x^2+R^2}}$$

$$V = \int_{ring} dV = \int_{ring} \frac{k\,dq}{\sqrt{x^2+R^2}} = \frac{k}{\sqrt{x^2+R^2}}\int_{ring} dq$$

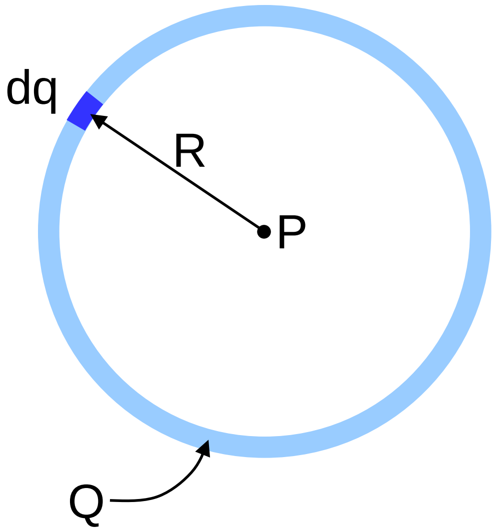


$$V = \frac{k}{\sqrt{x^2 + R^2}} \int_{\text{ring}} dq$$

$$V = \frac{kQ}{\sqrt{x^2 + R^2}}$$

Could you use this expression for V to calculate $\vec{E}$?

Example 4: Find the electric potential at the center of a uniformly charged ring of radius R and total charge Q.

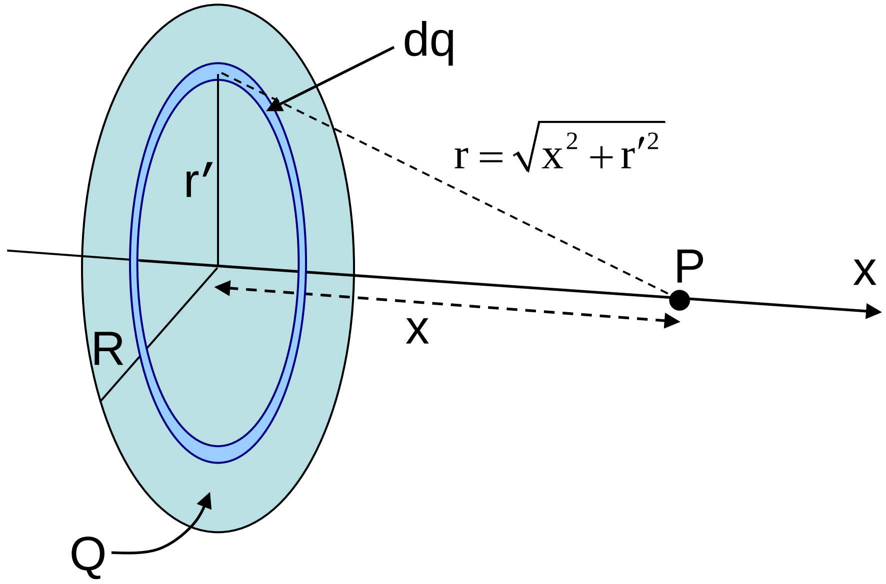


Every dq of charge on the ring is the same distance from the point P.

$$dV = k\frac{dq}{r} = k\frac{dq}{R}$$

$$V = \int_{ring} dV = \int_{ring} \frac{kdq}{R} = \frac{k}{R}\int_{ring} dq = \frac{kQ}{R}$$

Example 4: A disc of radius R has a uniform charge per unit area σ and total charge Q. Calculate V at a point P along the central axis of the disc at a distance x from its center.

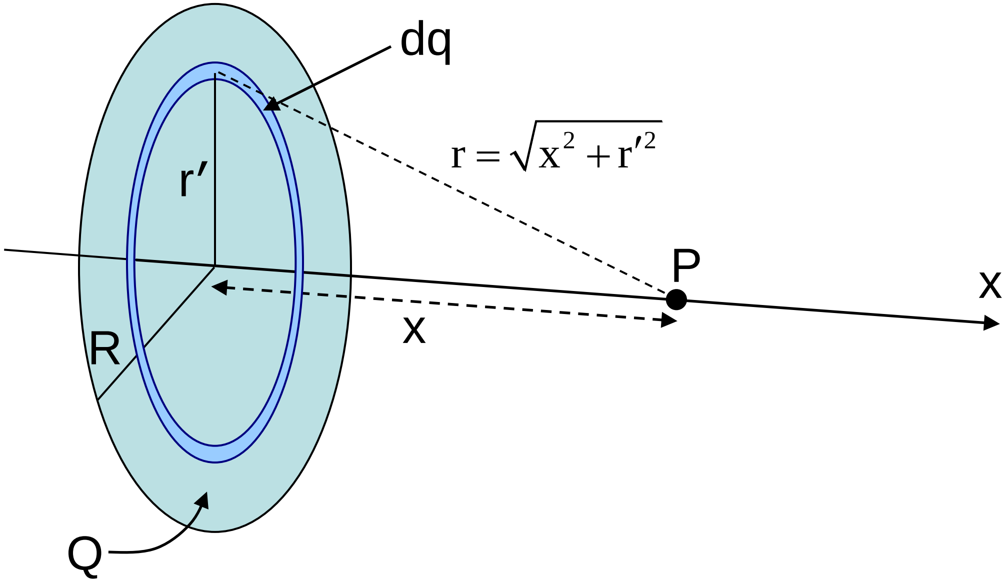


we already know V for a ring

$\rightarrow$ **decompose disk into rings**

- area of ring of radius r′ and thickness dr′ is $dA = 2\pi r' dr'$
- charge of ring is $dq = \sigma\, dA = \sigma(2\pi r' dr')$ with $\sigma = Q/\pi R^2$

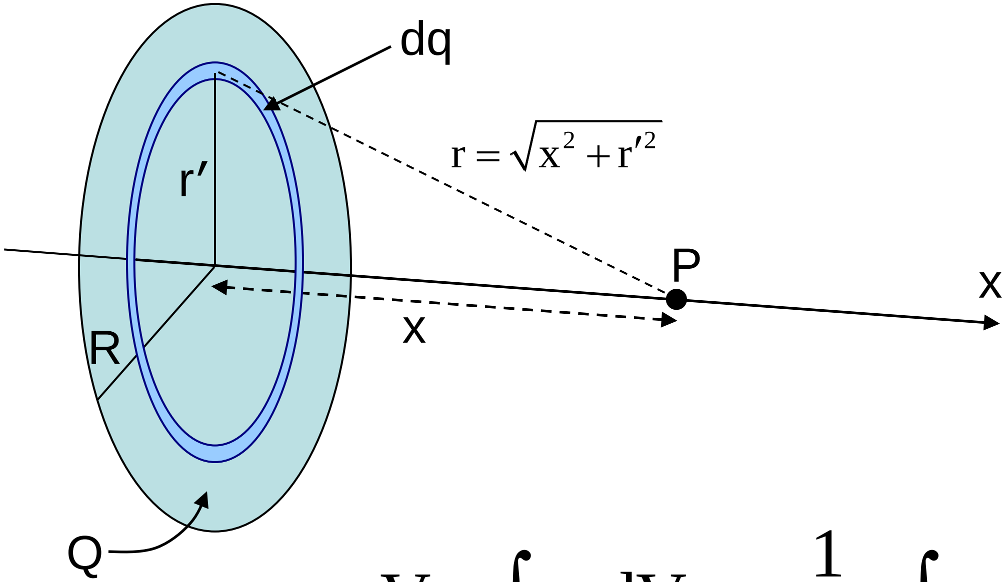
each ring is a distance $r = \sqrt{x^2 + r'^2}$ from point P



$$dV = k\frac{dq}{r}$$

$$= \frac{k\left[\sigma\left(2\pi r'\,dr'\right)\right]}{\sqrt{x^2 + r'^2}}$$

This is the (infinitesimal) potential
for an (infinitesimal) ring of radius r′.

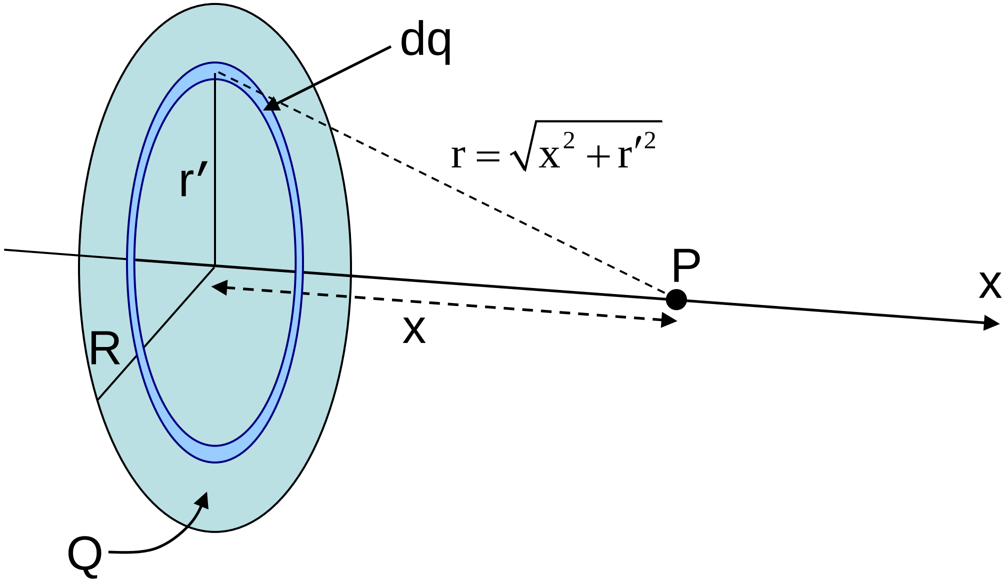


$k = 1/4\pi\varepsilon_0$

$$V = \int_{\text{ring}} dV = \frac{1}{4\pi\varepsilon_0}\int_{\text{ring}} \frac{\sigma 2\pi r' dr'}{\sqrt{x^2 + r'^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r' dr'}{\sqrt{x^2 + r'^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0}\sqrt{x^2 + r'^2}\,\Bigg|_0^R = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2 + R^2} - |x|\right) = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\sqrt{x^2 + R^2} - |x|\right)$$

$$\sigma = \frac{Q}{\pi R^2}$$



$$V = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\sqrt{x^2 + R^2} - |x|\right)$$

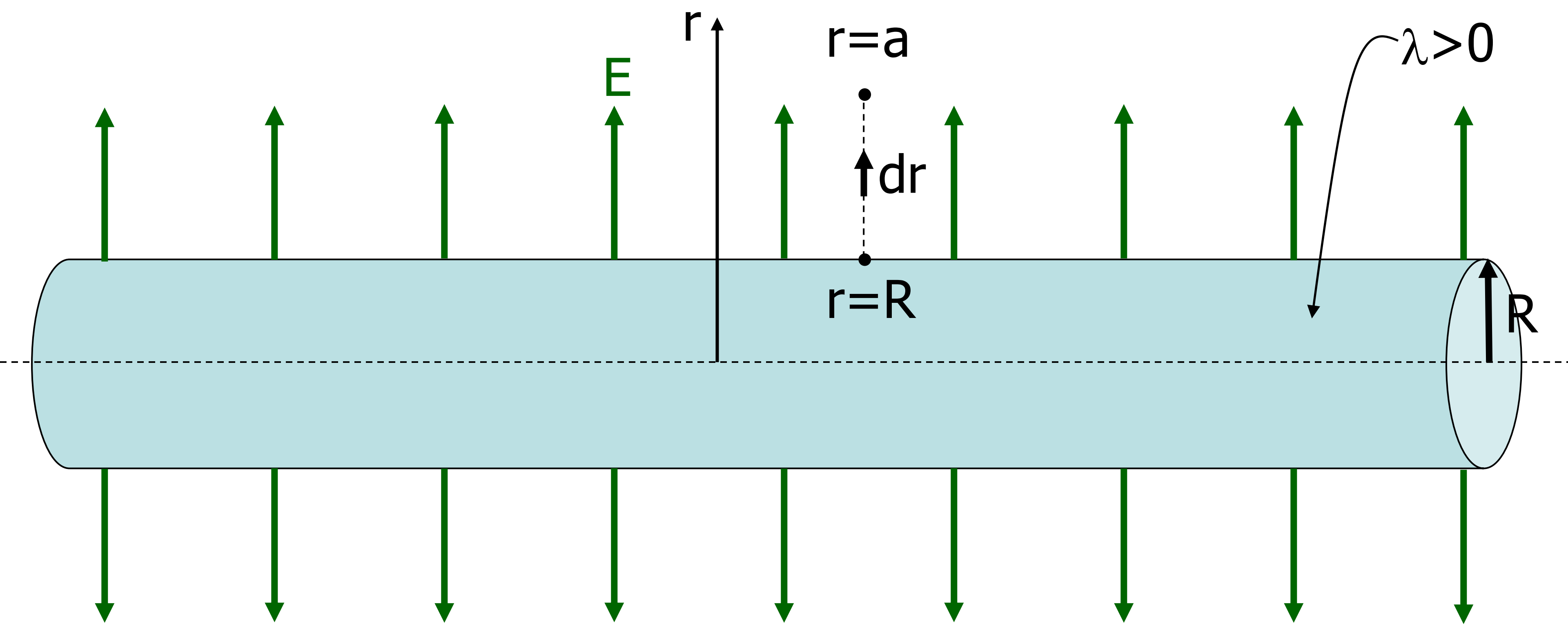
Could you use this expression for V to calculate $\vec{E}$?

Example 5: calculate the potential at a point outside a very long insulating cylinder of radius R and positive uniform linear charge density λ.

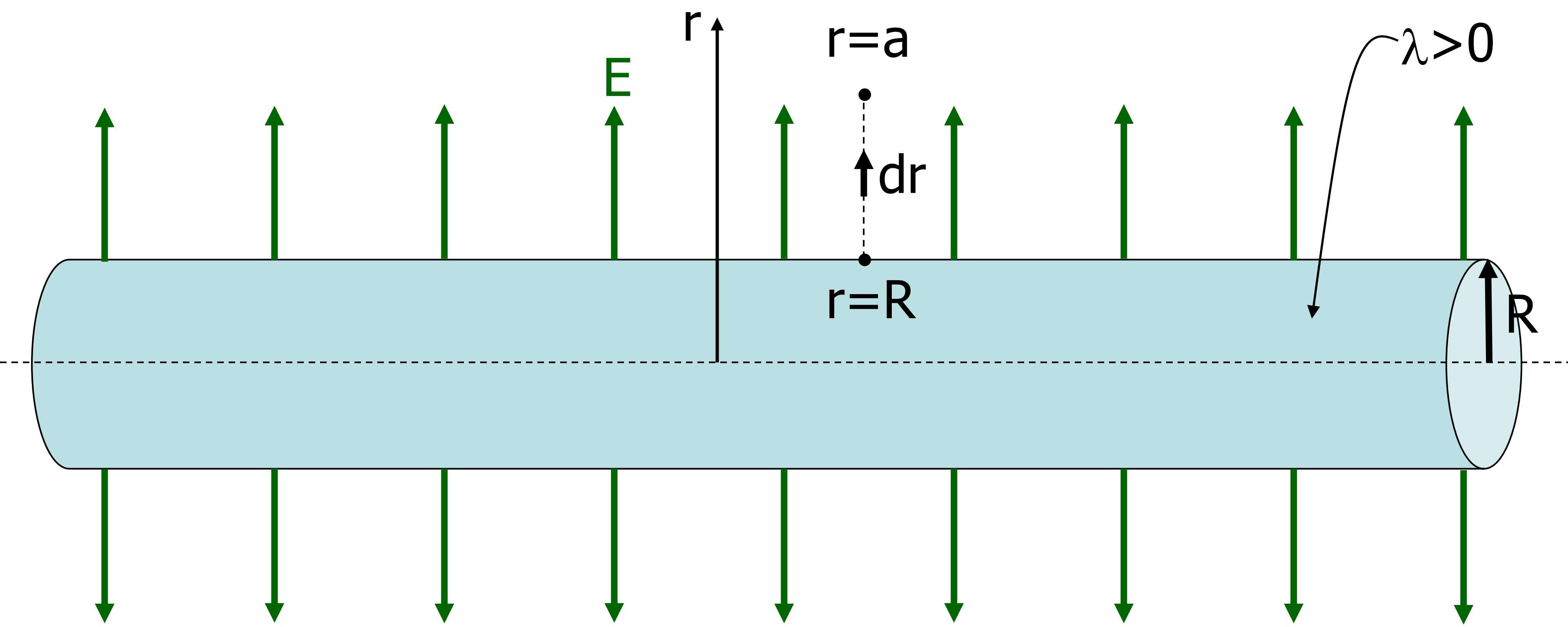
Which strategy to use?

- decomposition into charge elements and $dV = k\frac{dq}{r}$ **NO!**
  leads to complicated triple (volume) integral

- calculate $\vec{E}$ first, then use $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{\ell}$ **YES!**
  we already derived $\vec{E}$ using Gauss' law



Start with $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ and $\Delta V = -\int_i^f \vec{E} \cdot d\vec{\ell}$ to calculate $\Delta V_{R\to a}$.



$$V_a - V_R = V_{aR} = \Delta V_{R\to a} = -\int_R^a \vec{E}\cdot d\vec{\ell} = -\int_R^a \vec{E}(r)\cdot d\vec{r} = -\int_R^a (+E)\,dr$$

$$= -\int_R^a \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right) dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_R^a \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}(\ln r)\Big|_R^a$$

$$V_a - V_R = -\frac{\lambda}{2\pi\varepsilon_0}\left(\ln a - \ln R\right) = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{a}{R} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{a}$$

If we let a be an arbitrary distance r, then $V_r - V_R = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r}.$

If we take V=0 at r=R, then $V\left(r\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r}.$

Things to note:

$$V(r) = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R}{r}.$$

V is zero at the surface of the cylinder and decreases as you go further out. This makes sense! V decreases as you move away from positive charges.

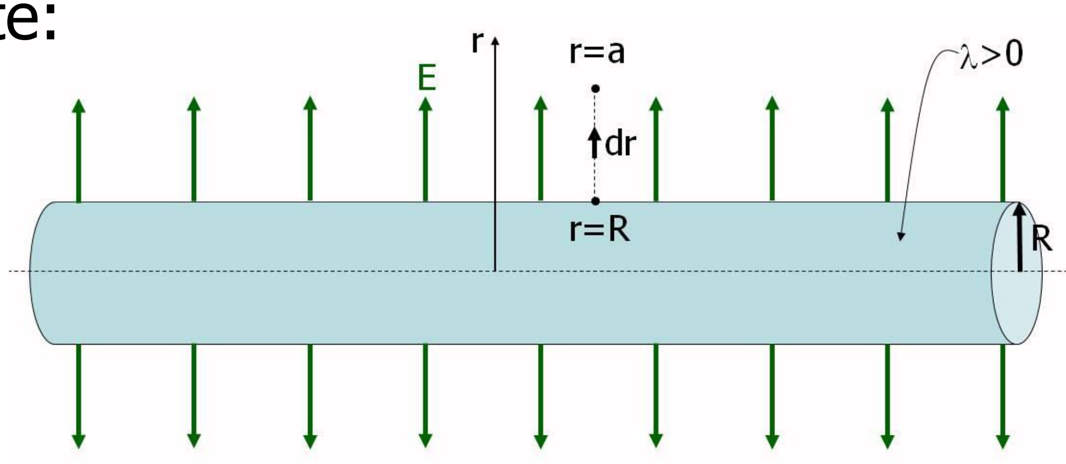
$$V_r - V_R = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R}{r}$$

If we tried to use V=0 at r=∞ then

$$V_r - V_\infty = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{\infty}{r} = \infty$$ (V is infinite at any finite r).

That's another reason why we can't start with $dV = k\frac{dq}{r}$.

Things to note:



$$V_r - V_R = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R}{r}$$

For $\lambda>0$ and $r>R$, $V_r - V_R < 0$.

By convention is $V_{ab} = V_a - V_b$.
Thus $V_{rR} = V_r - V_R$ is the potential difference between points r and R and for $r>R$, $V_{rR} < 0$.

Today's agenda:

Electric potential of a charge distribution.

You must be able to calculate the electric potential for a charge distribution.

Equipotentials.

You must be able to sketch and interpret equipotential plots.

Potential gradient.

You must be able to calculate the electric field if you are given the electric potential.

Potentials and fields near conductors.

You must be able to use what you have learned about electric fields, Gauss' law, and electric potential to understand and apply several useful facts about conductors in electrostatic equilibrium.

Equipotentials

Equipotentials are contour maps of the electric potential.

Equipotential lines:

- lines of constant electric potential V
- visualization tool complementing electric field lines

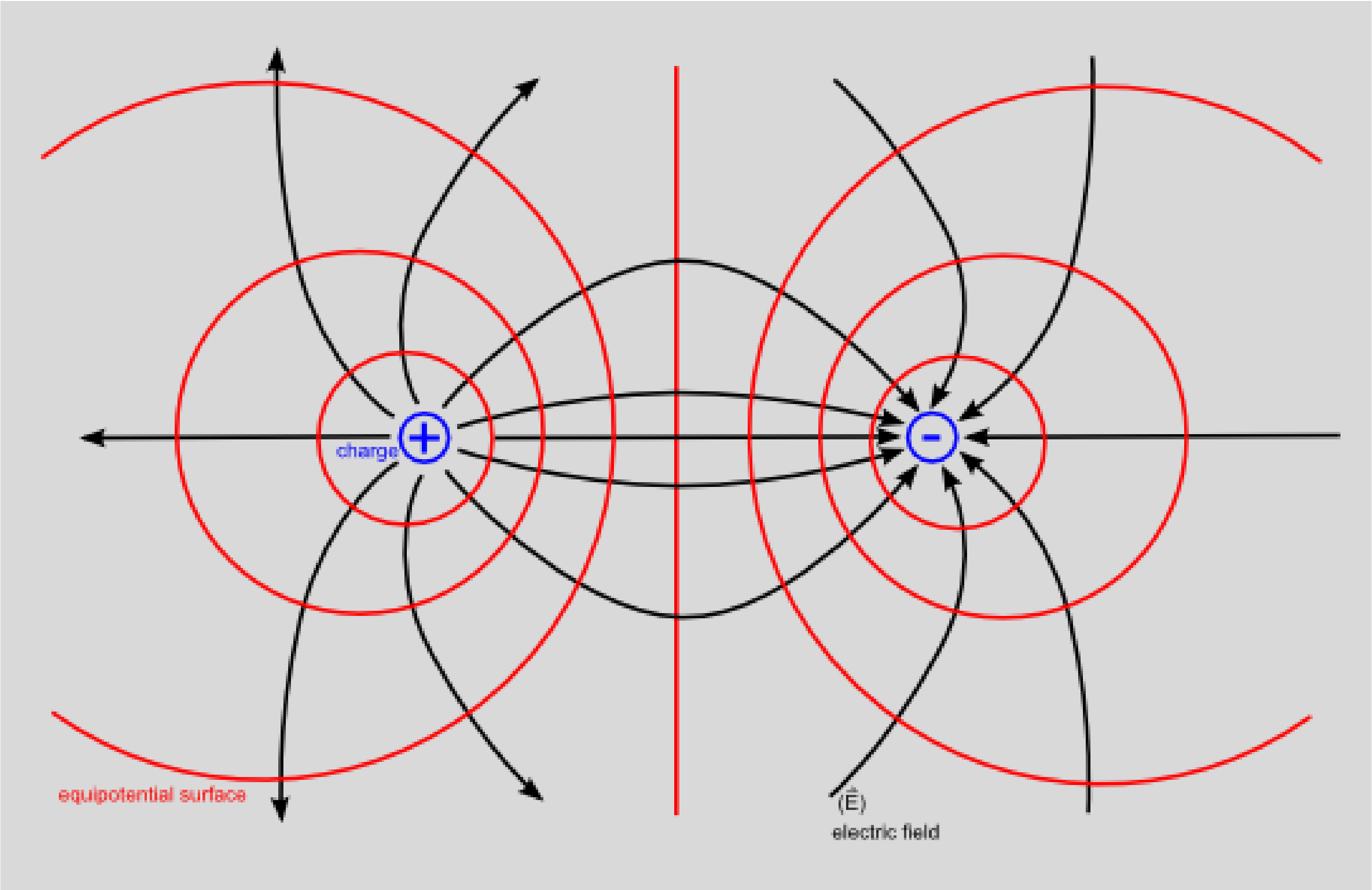
Electric field is perpendicular to equipotential lines. Why?

Otherwise work would be required to move a charge along an equipotential surface, and it would not be equipotential.

In static case (charges not moving), surface of conductor is an equipotential surface. Why?

Otherwise charge would flow and it wouldn't be a static case.

Here are electric field and equipotential lines for a dipole.



Equipotential lines are shown in red.

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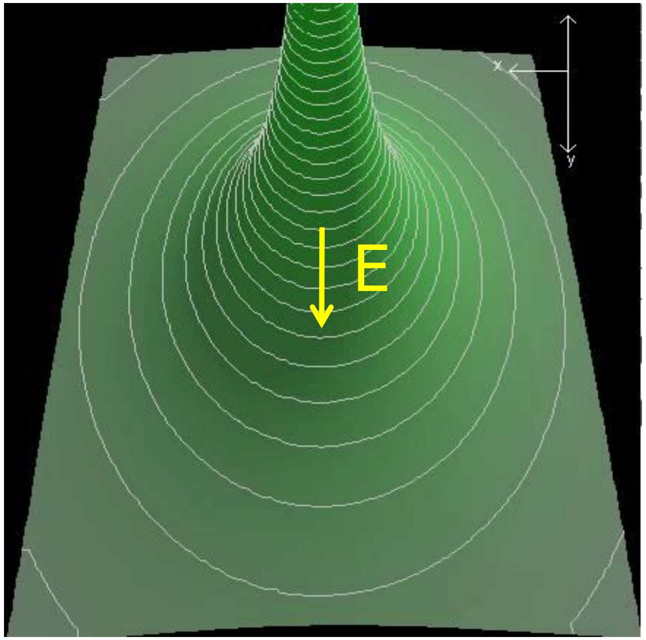
Potential Gradient
(Determining Electric Field from Potential)

Electric field vector points from + to -, this means from higher to lower potentials.

Remember: $$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}\ .$$

Inverse operation:

$$\vec{E} = -\frac{\partial V}{\partial \vec{r}}$$



E is perpendicular to the equipotentials

For spherically symmetric charge distribution:

$$E_r = -\frac{dV}{dr}$$

In one dimension:

$$E_x = -\frac{dV}{dx}$$

In three dimensions:

$$E_x = -\frac{\partial V}{\partial x}\ ,\quad E_y = -\frac{\partial V}{\partial y}\ ,\quad E_z = -\frac{\partial V}{\partial z}.$$

$$\text{or}\quad \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -\vec{\nabla} V$$

Example: In a region of space, the electric potential is V(x,y,z) = $Axy^2 + Bx^2 + Cx$, where A = 50 V/m^3, B = 100 V/m^2, and C = -400 V/m are constants. Find the electric field at the origin

$$E_x(0,0,0) = -\left.\frac{\partial V}{\partial x}\right|_{(0,0,0)} = -\left(Ay^2 + 2Bx + C\right)\Big|_{(0,0,0)} = -C$$

$$E_y(0,0,0) = -\left.\frac{\partial V}{\partial y}\right|_{(0,0,0)} = -(2Axy)\Big|_{(0,0,0)} = 0$$

$$E_z(0,0,0) = -\left.\frac{\partial V}{\partial z}\right|_{(0,0,0)} = 0$$

$$\vec{E}(0,0,0) = \left(400\frac{V}{m}\right)\hat{i}$$

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Potentials and Fields Near Conductors

When there is a net flow of charge inside a conductor, the physics is generally complex.

When there is no net flow of charge, or no flow at all (the electrostatic case), then a number of conclusions can be reached using Gauss' Law and the concepts of electric fields and potentials…

Eletrostatics of conductors

Electric field inside a conductor is zero.

Any net charge on the conductor lies on the outer surface.

Potential on the surface of a conductor, and everywhere inside, is the same.

Electric field just outside a conductor must be perpendicular to the surface.

Equipotential surfaces just outside the conductor must be parallel to the conductor's surface.