

Exercise sheet 8: Magnetic fields, Ampère-Laplace (Biot-Savart) law, Ampère's law

6/11/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

Exercise 1.

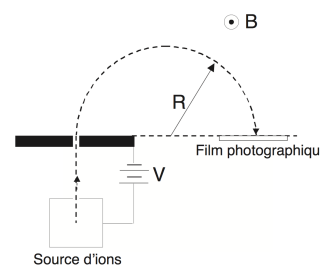
(Thomson's discovery/From Griffiths/Category I)

In 1897, J.J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of "cathode rays". He selected particles of a specific speed via the following "trick": He passed the beam through uniform crossed electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively. They were mutually perpendicular and both of them were perpendicular to the beam. He adjusted the magnitude of the electric field E until he got zero deflection for a fixed magnetic field B_0 . What was the speed of the particles in terms of E and B_0 which had a straight trajectory?

Exercise 2.

(Mass Spectrometer/Category I)

We consider the mass spectrometer represented in the sketch. Two isotopes of lithium, with an atomic masses of 6 au and 7 au, are ionized (Li^+) and accelerated by a potential difference of 900 V. The accelerated ions enter a uniform magnetic field $B = 0.04$ T, which deflects the beam. After moving along a semi-circle, the ions hit a photographic film. Two spots separated by a distance x appear on the film. Calculate x . Neglect friction, relativistic effects and gravitational force. Useful unit conversion: $1 \text{ au} = 1.66 \times 10^{-27} \text{ kg}$.



Exercise 3.

(Wire loop/Category II/After training: 15 min)

Using Ampère-Laplace law (Biot-Savart law), calculate the magnetic field $\vec{B}$ along the axis of a circular loop of radius R carrying a current i .

Exercise 4.

(Ribbon/Category I (by following the hint)/After training: 10 min)

The sketch (see Fig. 1) shows the cross section of a very thin ribbon of width w , carrying a uniform current i going into the plane of the sheet. The length of the ribbon is assumed to be infinite. Hint: Use the superposition principle for the solution if you consider the ribbon to be composed of a linear array of infinitely long wires. Their field was calculated in the lecture.

1. Find the norm and direction of the magnetic field $\vec{B}$ at a point P in the ribbon's plane at a distance d . Please make a drawing.
2. Draw the norm of the magnetic field as a function of d , with d going from 0 to ∞ , and comment on the behaviour of $\vec{B}$ in both limits.

Exercise 5.

(Current in sheet/Griffiths/Category I)

Find the magnetic field of a uniform surface current $\vec{K} = K\hat{x}$ flowing over the whole $x-y$ -plane (see Fig. 2), i.e. an infinitely long and wide sheet of current $\vec{K}$. Hint: Analyze first the expected symmetry and relevant components of the magnetic field.

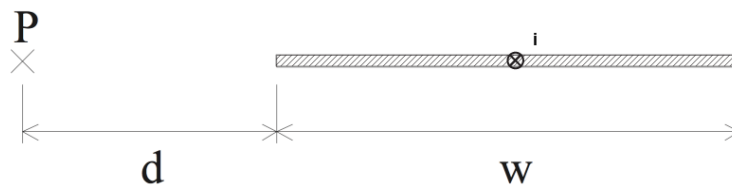


Figure 1: Schematic of the cross-section of a very thin ribbon of width w , carrying a uniform current i going into the plane of the sheet.

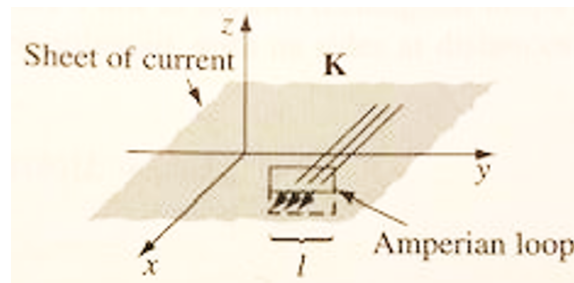


Figure 2: Sheet of current and possible Amperian loop to analyze the magnetic field B after symmetry analysis.