

Exercise sheet 7: Currents in circuits with capacitors

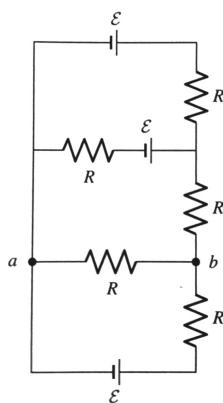
30/10/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

Exercise 1.

(Kirchhoff's laws - From Purcell/Category I)

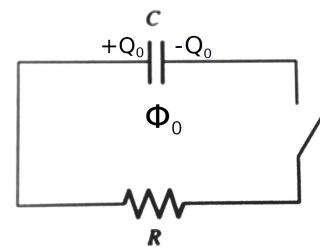
What is the potential difference between point a and point b in this circuit? Solve the circuit using Kirchhoff's laws assuming identical batteries for each EMF $\mathcal{E}$ and identical resistors R .



Exercise 2.

(Discharge of a capacitor/category II (after training about 25 min for solution))

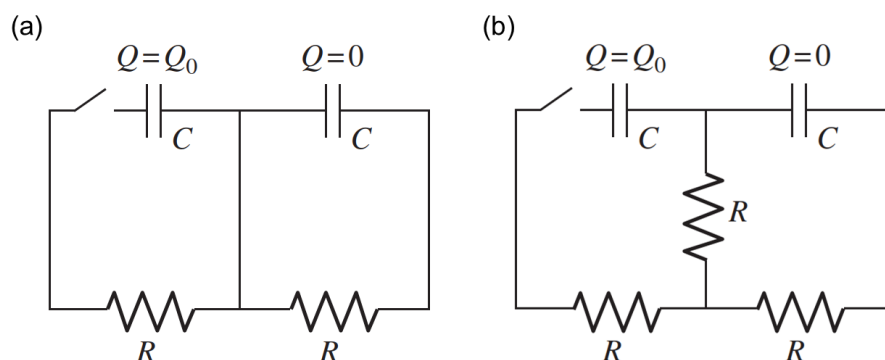
Consider the circuit shown in the sketch. The capacitor with capacitance C is initially charged at a potential difference $\phi_0 = \phi(t = 0)$. At $t = 0$, the switch is closed. The capacitor discharges as a function of time.



- In which direction does the current flow at $t > 0$? Draw a sketch.
- Express the potential difference $\phi(t)$ as a function of the charge $Q_c(t)$ on the capacitor and its capacitance C .
- Express the potential difference $\phi(t)$ as a function of the current $I(t)$ and the resistance R .
- Use your own words to explain why $I(t) = -\frac{dQ_c(t)}{dt}$.
- From the equations you have found so far, write the differential equation dictating the evolution of $Q_c(t)$ as a function of time, with R and C as parameters. Solve this differential equation. Don't forget to consider the initial conditions.
- How much charge is left in the capacitor at $t = \tau = RC$? τ is the characteristic time of the RC circuit.
- What was the energy initially stored in the capacitor? What is the energy left after a very long time, say $t \gg RC$? Where did it go?

Exercise 3.**(A discharge with two capacitors - From Purcell/Category II)**

- a) The circuit in (a) contains two identical capacitors and two identical resistors. Initially, the left capacitor has charge Q_0 (with the left plate positive), and the right capacitor is uncharged. If the switch is closed at $t = 0$, find the charges on the capacitors as functions of time. Hint: The loop equations should be simple ones.
- b) Answer the same question for the circuit in (b), in which one more identical resistor has been added. What is the maximum (or minimum) charge that the right capacitor achieves? Sketch the time dependencies of the charges Q_1 and Q_2 on the capacitors C_1 (left) and C_2 (right), respectively, and the currents I_1 and I_2 flowing in the two loops. Hint: Find the solutions of the differential equations in that take the sum and difference of the loop equations and solve for the sum and difference of the charges. Then compute each charge individually.

**Exercise 4.****(Advanced Question: Cubic network - From Purcell/Category III)**

A cube has identical resistors R along each edge. Find the equivalent resistance between two nodes that correspond to:

- diagonally opposite corners of the cube;
- diagonally opposite corners of a face;
- adjacent corners.

You do not need to solve a number of simultaneous equations; instead use symmetry arguments. Hint: Identify nodes that are on the same potential in each different scenario. For the further analysis you can assume that such nodes are interconnected by a perfectly conducting wire.

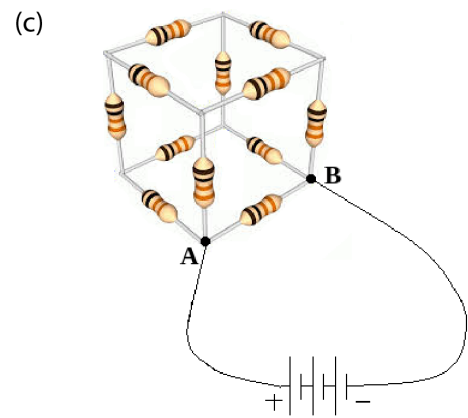
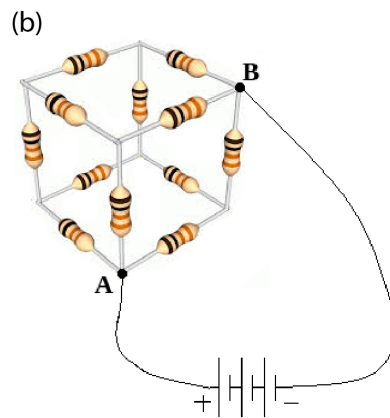
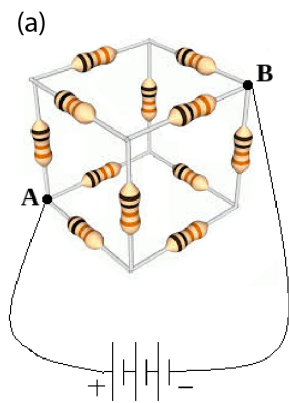


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