Exercise sheet 6: Screened electric field, energy in capacitor, currents and Ohm's law

16/10/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

Exercise 1.
(Extraction of a dielectric slab/Category I)
We consider a parallel plate capacitor with a polystyrene slab ($\epsilon_r = 2.5$) between the plates. Its capacitance is $C = 10$ nF. The dielectric slab is extracted from the capacitor. During the process, the capacitor stays connected to a voltage generator applying a potential difference $\Delta\phi = 100$ V between the plates. Calculate:

a) the variation of the charge on one of the plates,

b) the variation of energy stored in the capacitor,

c) the amount of work needed to take the dielectric slab out.

Exercise 2.
(Current through shell-like metal/Category I)
Two long coaxial cylinders consisting of thin perfect conductors with radii a and b are separated by a metal of conductivity σ (Fig. 1). The two perfect conductors are maintained at a potential difference $\Delta\phi$. What current I flows from one conductor to the other one in a length L? Derive the formula for I as a function of L, σ, a, b, and $\Delta\phi$.

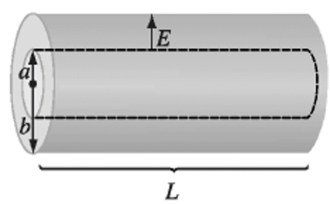
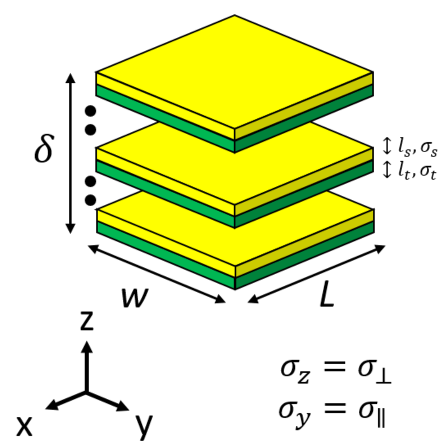


Figure 1: Two long coaxial cylinders consisting of thin perfect conductors with radii a and b are separated by a metal of conductivity σ.

Exercise 3.
(Laminated conductor - From Purcell/Category II/
expected time 35 mins to solve after training)

A laminated conductor is made by depositing, alternately, layers of silver with thickness l_s and conductivity σ_s, and layers of tin with thickness l_t and conductivity σ_t. Assume there are equal layers of silver and tin. The composite material shows different electrical conductivities depending on whether the current is applied in the plane of the layers ($\sigma_\parallel$) or perpendicular to them ($\sigma_\perp$). The individual conductivities are scalar (isotropic).



a) Find the expression of $1/\sigma_\perp$ as a function of σ_s, σ_t, l_s and l_t.
Consider $\sigma_s = 6.3 \times 10^7$ $(\Omega\,\text{m})^{-1}$ and $\sigma_t = 8.7 \times 10^6$ $(\Omega\,\text{m})^{-1}$. Calculate $\sigma_\perp$ in the case of $l_s = l_t$. Hint: Assume that the top-most surface is an equipotential

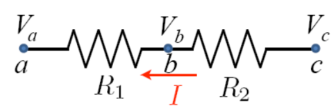
surface and the bottom-most surface is also an equipotential surface but at a different potential value. Evaluate the spatial variation of the electric field in the individual layers.

b) Find the expression of $\sigma_{\parallel}$ as a function of σ_s, σ_t, l_s and l_t.
Calculate $\sigma_{\parallel}$ in the case of $l_s = l_t$ considering the parameters of (a). Hint:
In a real experiment the two surfaces parallel to the x, z-plane would be covered by an ultra-thin conductor sheet which are in electrical contact with each layer and represent two equipotential surfaces at the two ends of all layers. Hence each layer experiences the identical potential difference in y-direction.

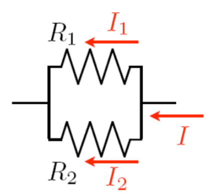
c) The anisotropy ratio $r_A = \sigma_{\parallel}/\sigma_{\perp}$ can be written as: $\frac{\sigma_{\parallel}}{\sigma_{\perp}} = 1 + \frac{l_s/l_t}{(1+l_s/l_t)^2}\left(\frac{(1+\sigma_s/\sigma_t)^2}{\sigma_s/\sigma_t} - 4\right)$.
At fixed conductivities, how to maximize or minimize the above ratio r_A? You can study the variations of the above function of $x = l_s/l_t$.

d) *Extra Question:* Find the formula for the anisotropy ratio (as given in part (c))

Exercise 4.
(Parallel and series resistors/Category I)



a) Consider the two resistors in series represented in the sketch. By expressing the voltage drop $V_c - V_a$ as a function of the current I flowing through both resistors in series, show that $R_{\text{tot}} = R_1 + R_2$. Calculate R_{tot} for $R_1 = 5\ \Omega$ and $R_2 = 30\ \Omega$. Resistors are illustrated by zig-zag symbols. Remember that the connecting wires between resistors are assumed to be perfect conductors through which charges move according to Newton's first law.



b) Consider the two resistors in parallel represented in the sketch. By expressing the current $I = I_1 + I_2$ as a function of the voltage drop across the resistors, show that $1/R_{\text{tot}} = 1/R_1 + 1/R_2$. Calculate R_{tot} for $R_1 = 5\ \Omega$ and $R_2 = 30\ \Omega$. Remember that the connecting wires between resistors are assumed to be perfect conductors and hence are equipotential lines or surfaces.

c) Consider the parallel resistors again. Assume I splits up in I_1 and I_2 in such a manner that the quantity $P = R_1 I_1^2 + R_2 I_2^2$ is minimized (the quantity P is the dissipated power due to charge scattering/Ohm's law). Express I_1 and I_2 as function of R_1, R_2 and I. Compare with I_1 and I_2 found based on the solution of (b).

d) Calculate the dissipated power P for (a) and (b) in case (i) a current $I = 1$ A or (ii) a potential difference (voltage) $\Delta V = 1$ V is applied.