Exercise sheet 5: Dipole moments, capacitors, dielectrics

9/10/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

For exercises on capacitors the video (link) is instructive.

Exercise 1.

(Potential of a spherical capacitor/Category II)

An initially neutral metallic ball (conductor) of radius $R_1 = 0.1$ m is charged up with a charge Q by connecting the ball to a potential of $\phi_0 = 600$ V with an ultrathin conductor wire (with respect to the electric ground (earth) which is the potential valid at infinity). After the charging, the connection is cut abruptly such that the charges stay on the disconnected ball. Two thin hemispheres (conductors), initially uncharged, of radii $R_2 = 0.11$ m, are taken from infinity and brought in a position such that they form a closed outer spherical shell with the same center as the ball, without touching it. Calculate the potential of the inner ball for the following cases:

a) the hemispheres have no connection to the ground (earth) or the ball,

b) the hemispheres are connected to the ground (earth) using a perfect conductor,

c) the hemispheres are isolated from the ground and from the other ball when they are moved. Then, once in position, they are connected to the ball using a perfect conductor.

Hints:

i) The ground (earth) is defined as a conductor of infinite capacity, such that it can provide/take any amount of charge without changing its potential which is set to zero. $\phi_{\text{ground}} = \phi_\infty = 0$. Something connected to the ground will always have zero potential.

ii) When a system is isolated, it means that no charge can leave it. Thus, its charge does not change. Its potential, however, can change.

Exercise 2.

(Two different dielectrics; category I)

We consider the plate capacitors represented in Fig. 1 which have large surface area S and are separated by a small distance d, brought to the potential difference $\Delta\phi$.

a. Express electric field $\mathbf{E}$, electric displacement $\mathbf{D}$, polarization $\mathbf{P}$, and surface charge density σ_{free} in those capacitors as a function of the distance d, the surface area S, the potential difference $\Delta\phi$, and the dielectric constants ϵ_1 and ϵ_2.

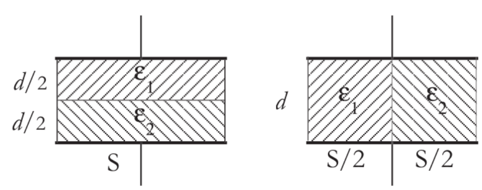

Figure 1: Type 1 (left) and type 2 (right) capacitors.

b. Calculate the absolute values of capacitances of the capacitors for $S = 400$ mm^2, $d = 10$ mm, $\epsilon_1 = 3.4$ (pyrex glass), $\epsilon_2 = 5$ (plexiglass). Refer to Fig. 1.

c. Using the expression obtained in a., sketch the electric field inside the capacitor for the type 1 capacitor. Consider the following cases: (i) $\epsilon_1 < \epsilon_2$, (ii) $\epsilon_1 = \epsilon_2$, and (iii) $\epsilon_1 > \epsilon_2$. What happens at the dielectric interface? Describe the physical origin of what you see.

Exercise 3.
(Insertion of a dielectric slab: isolated capacitor; category I)
A parallel plate capacitor is charged using a voltage generator. The generator is then disconnected, leaving the capacitor charged and isolated. A dielectric slab is then inserted between the conductive plates. Describe qualitatively how the charge Q_{free}, the potential difference $\Delta\phi$, the electric field E, and the capacitance C vary. The capacitance is defined as $C = \frac{Q_{\text{free}}}{\Delta\phi}$. Does the energy stored in the capacitor vary?

Exercise 4.
(Sphere with dielectric shell/Category II)
An insulating solid sphere with radius R is unevenly charged. The positive charge density is described by $\rho = Kr$ where K is a positive constant (with units of A.s/m^4) and $r \leq R$ is the distance (in units of m) from center of the sphere. This problem was discussed in exercise 4 of problem sheet 3. The electrical field outside the orange sphere (Fig. 2) is given in the solution of problem sheet 3. Now the same sphere is surrounded by a homogeneous dielectric shell (blue shell in Fig. 2) extending from $r = R$ to $r = 2R$. Consider an electric permittivity of $\varepsilon_r = 2$. Sketch $E(r)$ relative to the curve provided in in Ex. 4 on sheet 3. Hint: consult the solutions of exercise 4 on problem sheet 3.

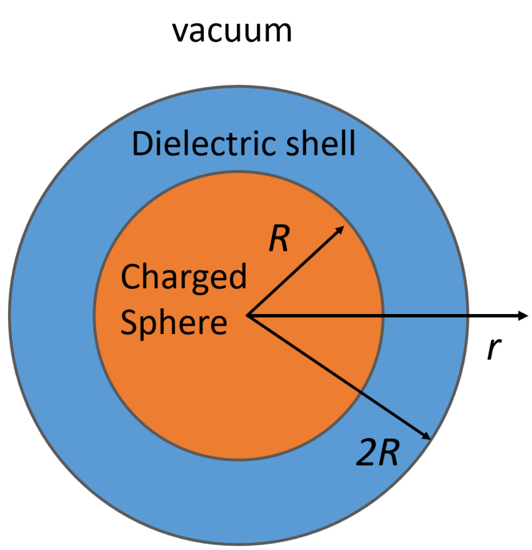

Figure 2: Sketch of the charged sphere with dielectric and vacuum surrounding.