Exercise sheet 2: Electric Fields

18/09/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). Problems addressing pure mathematical aspects are extra and not rated. For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions).The aim here is to show how in an exam context one should approach the problem making use of symmetry analysis and explaining the development of the entire solution step-by-step. The exact problem setting cannot be repeated in an exam however.

Exercise 1.
(Electric fields from opposite charges put on a straight line)(Category I)

We consider two charges q of opposite sign, but with the same magnitude, to reside on the x axis. The separation between them is d.

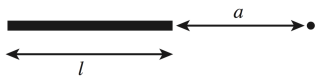
a) Find the electric field, $\vec{E}$ (magnitude and direction) a distance z above the midpoint between them, as a function of z and d.

b) Which dependence of $\vec{E}$ on z is predicted for $z \gg d$? Compare with the formula describing the electric field of a dipole moment that you might search for in a textbook. What do you conclude?

Hint: Consider the decomposition of vectors along x- and y- axis as done in the Exercise 2 of Exercise sheet 1.

Exercise 2.
(Charged Rod) (Category I)

We consider a rod of small radius such that it can be approximated as a line with length l, carrying a uniformly distributed charge Q.



a) Define the linear charge density λ.

b) Find the electric field at a distance a from the end of the rod, along its axis (see the sketch). The electric field can be calculated with the following equation:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_\Gamma \frac{\lambda(\vec{r}\prime)dl}{|\vec{r}-\vec{r}\prime|^3}(\vec{r}-\vec{r}\prime) \tag{1}$$

c) How does the calculated field behave for $a \gg l$ and for $a \ll l$? Hint: consider the electric field in the given limiting cases up to leading order.

Exercise 3.
(Charged semi-circle/taken from an exam) (Category II)
An ultra-thin rod is bent to a semi-circle and contains distributed charges as sketched (see figure). The charges $+Q$ and $-Q$ are each homogeneously distributed in the indicated segments of the semi-circle. Calculate the formula for the electric field vector $\vec{E}$ in point P as a function of $|Q|$ and the radius r. (For the calculation assume the rod to be a line mathematically)