

Exercise sheet 1: Vector Algebra Tools

11/09/2024

By this problem sheet we intend to *stimulate preliminary considerations* about concepts related to vector algebra, which are relevant for the course. This exercise acts as a revision of the basic concepts.

Exercise 1.

Consider that a hockey puck with a mass of 0.5 kg slides on a friction-less horizontal surface of an ice rink. Two sticks strike the puck simultaneously as shown in figure 1. The first stick exerts a force $\vec{F}_1$ with a magnitude of 4.0 N, directed at an angle of -60° to the x axis. The second stick exerts a force $\vec{F}_2$ with a magnitude of 5.0 N at an angle of $+20^\circ$ to the x axis. Determine both the magnitude and the direction of the puck's acceleration.

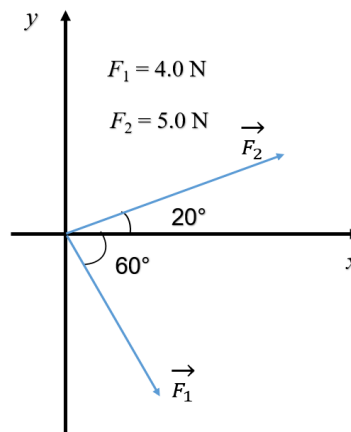


Figure 1: A hockey puck experiencing two forces

Exercise 2.

Consider that a car is on a friction-less inclined plane at an angle of θ as shown in Figure 2.

Part A. Draw a diagram showing the forces acting on the car and calculate the acceleration of the car. Discuss what happens if the inclination θ is 0° , 30° , 45° , and 90° .

Part B. How long does it take for the front bumper of the car to reach the bottom of the inclined plane, and what is the car's velocity when it reaches there.? Assume that the distance between the front bumper of the car to the bottom of the inclined plane is d .

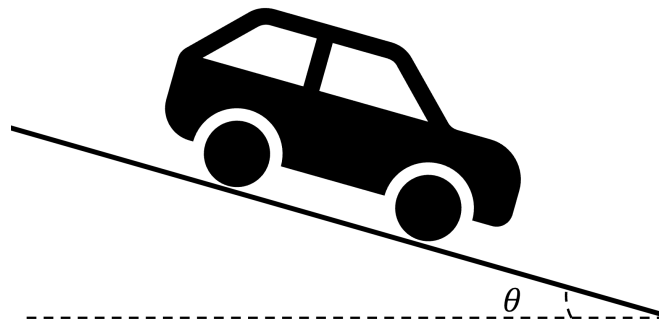
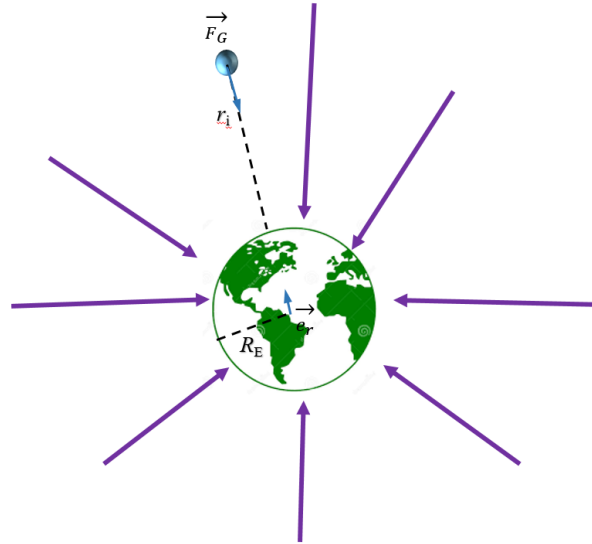


Figure 2: A car on a friction-less inclined plane

Exercise 3.

A space transportation vehicle releases a communication satellite with a mass $m = 500 \text{ kg}$ while in an orbit with a height $r_i = 300 \text{ km}$ above the surface of the Earth, as shown in Figure 3. The radius of the Earth is $R_E = 6370 \text{ km}$ and the mass M of the Earth is $5.97 \times 10^{24} \text{ kg}$. **Hint:** The gravitational force reads $\vec{F} = -\frac{GMm}{r^2}\vec{e}_r$ where $\vec{e}_r$ is

Figure 3: The satellite boosted from r_i to r_f

the unit vector pointing away from the center of the Earth and r is the distance from the center of the Earth, the satellite has a circular orbit.

Part A. How much is the kinetic energy of the satellite at this height?

Part B. What is the total mechanical energy at this height? **Hint:** Calculate the potential energy at a distance r ($r > R_E$) from the center of the earth. The mechanical energy is the sum of the kinetic and potential energy.

Part C. A rocket engine on the satellite is required to boost it into a circular geosynchronous orbit at $36\,000 \text{ km}$. How much energy does the engine have to provide?

1 Hints and Examples

We start with some examples on the basic vector algebra, which will help solve the problems given in the exercise. Consider figures 4 to 7 given below:

1. Consider the case of collinear forces: Finding the net force of the two forces shown in figure 4, $\vec{F}_1$ and $\vec{F}_2$.

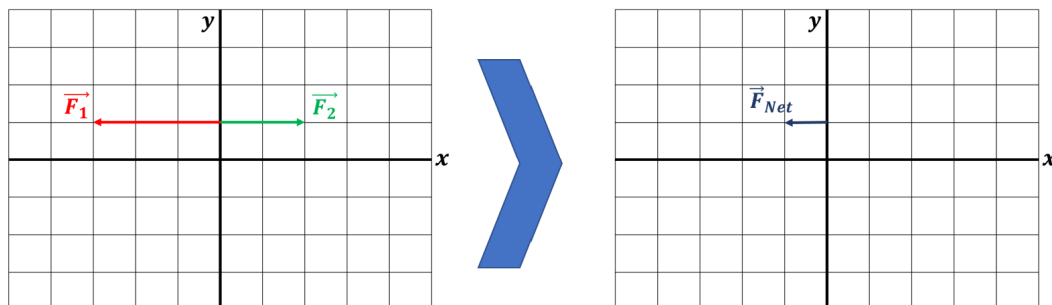


Figure 4: Net of two forces collinear with each other

2. Consider the case of non-collinear forces: Finding the net force of the two forces shown in figure 5, $\vec{F}_1$ and $\vec{F}_2$. This can be done in two ways: (i) Using the graphical *tip-to-tail* method, or (ii) by decomposing the vectors in their components on the x- and y-axis. Note that the choice of axis is important and relevant in Exercise 2 of the problem sheet.

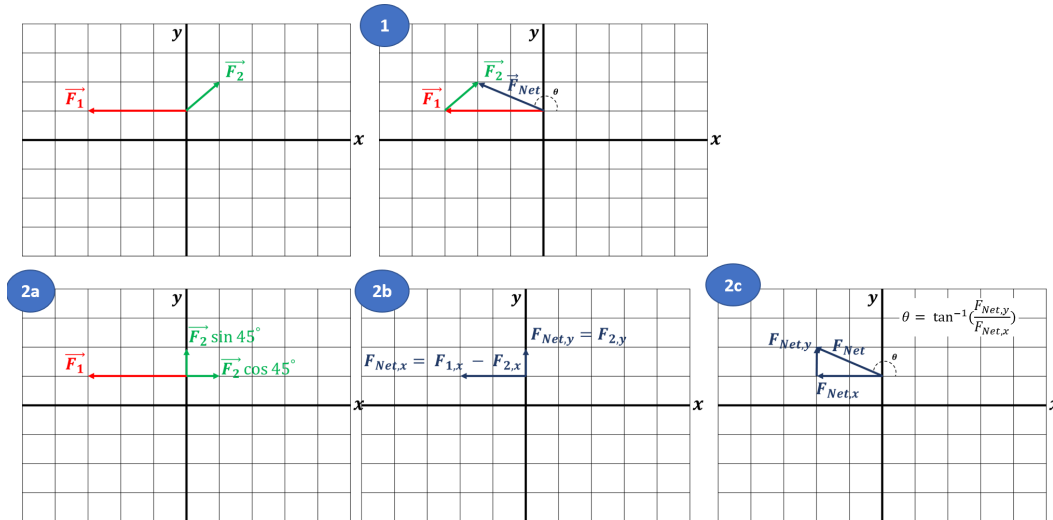


Figure 5: Net of two forces acting on a body. The first method marked (1) is the *tip-to-tail* method, and the second method marked (2a-c) is solving by resolving a vector on the axes.

3. Consider the case of more than two non-collinear forces: Finding the net force of the two forces shown in figure 6, $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. *Tip-to-tail* method is shown here.

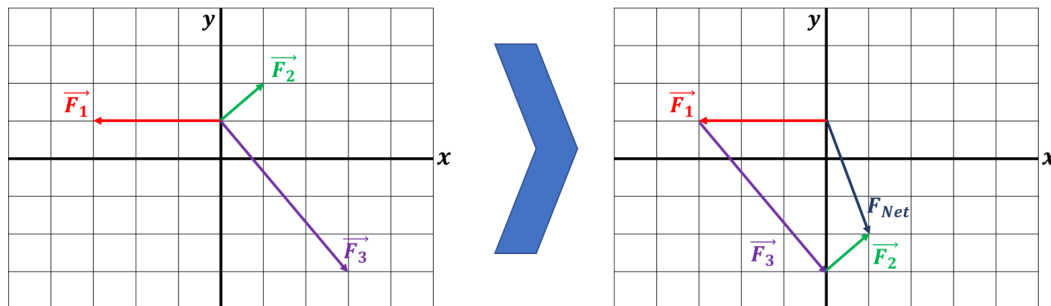


Figure 6: Net of three forces acting on a body

4. Consider taking components in a three-dimensional space as shown in figure 7. Note that this example helps in moving from Cartesian to Spherical coordinates.

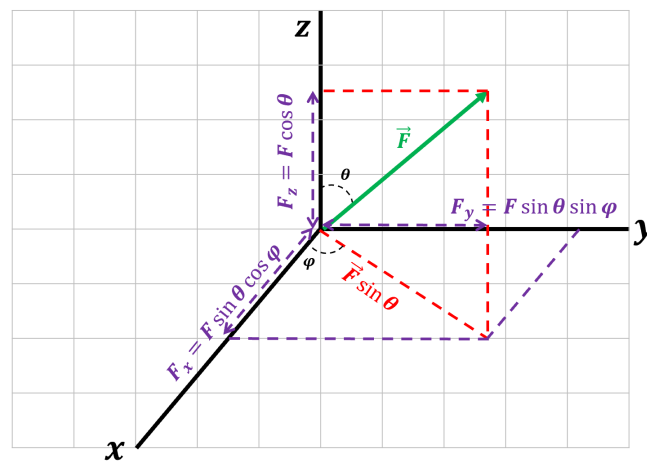


Figure 7: Example of a vector in three-dimensional space | Resolving the given vector in components projected on the x, y, z axes