Exercise sheet 11: Displacement current, magnetic energy

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27/11/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

Exercise 1.
(Magnetic field from displacement current/Category I)

Suppose that a circular parallel–plate capacitor has a radius of $R_0 = 3.0$ cm and a plate separation of $d = 5.0$ mm. A sinusoidal potential difference $V = V_0 \sin(2\pi f t)$ is applied across the plates, where $V_0 = 150$ V and $f = 60$ Hz. Ignore fringing fields of E. Hint: This exercise is meant to show that fields from the term dE/dt in a capacitor are small at 60 Hz, i.e., when AC currents/electric fields vary slowly.

a) In the region between the plates, show that the magnitude of the induced magnetic field is given by $B = B_0(R)\cos(2\pi f t)$, where R is the radial distance from the capacitor's central axis.

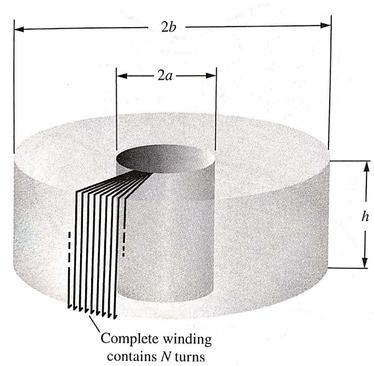
b) Determine the expression for the amplitude $B_0(R)$ of this time-dependent (sinusoidal) field when $R \leq R_0$ and when $R > R_0$.

c) Plot $B_0(R)$ in tesla for the range $0 \leq R \leq 10$ cm.

d) Now assume that a wire connected to the capacitor carries exactly the amount of absolute charge current I as given by the maximum displacement current I_{D} flowing through the whole cross-section of the capacitor in its gap. The wire is assumed to be infinitely long and has a diameter of 2 mm. Calculate the magnetic field generated by I just at its surface in the limit of an infinitely long wire. Compare this value with the field B in the gap of the capacitor at $R = 1$ mm. How much weaker is this field generated by the displacement current in the capacitor compared to the field of the charge-based current?

Exercise 2.
[Self-inductance and magnetic energy in toroidal solenoid /Category II (After training for solution: 25 min)]

Consider a toroidal solenoid (coil) with the geometrical parameters presented in the figure. Complete winding contains N turns all around the core.



Toroidal coil of rectangular cross section. Only a few turns are shown.

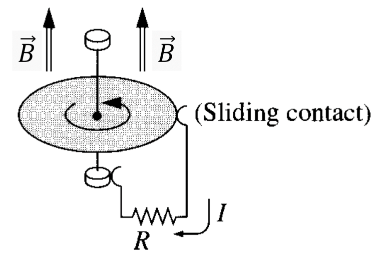
a) Calculate the self-inductance, L, of the system.

b) Obtain the magnetic energy density, u, stored in the system.

c) Calculate the total energy, U, stored in the system.

d) Evaluate L for a toroidal solenoid with dimensions of $2a = 10$ mm, $2b = 15$ mm and $h = 5$ mm with $N = 100$ turns of wire. How much magnetic energy is stored in the system if one applies a current of $I = 3$ A to the solenoid. Suppose that we store the same amount of energy on a circular plate capacitor with plate diameter of $R = 2b = 15$ mm and distance between plates $d = h = 5$ mm filled with a dielectric with dielectric constant of 5. How much voltage, V, needs to be applied?

Exercise 3.
(Faraday's disk / Category II (After training time needed: 20-25 min))



Faraday's disc was the first electromagnetic generator. The working principle is as follows: A conductor disc (metal) of radius a rotates with a constant angular velocity ω about a vertical axis, through a uniform field $\vec{B}$, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the edge of the disk (see sketch).

a) Find the expression for the current in the resistor. Hint: When the disk rotates the charges in the conductor inherit a velocity vector.

b) Now let's consider to construct a bicycle dynamo using Faraday's disk. Instead of the resistor, we power a lightbulb with an electrical power consumption $P_{\text{el.}} = 4$ W and resistance $R = 1\ \Omega$. Due to size restrictions, we choose a disk radius of $a = 2$ cm. Using permanent magnets, we provide $B = 0.2$ T. Calculate the frequency of rotation f (turns per second, $f = \omega/2\pi$) needed to supply 4 W to the bulb. Is the required frequency realistic for the application as a bicycle dynamo?

Exercise 4.

(Bicycle dynamo / Category II (After training time needed: 15-20 min))

Consider a commercial bicycle dynamo as sketched below. Instead of spinning a coil in a magnetic field (compare with Exercise 3), a permanent magnet is spinning with respect to a fixed coil (solenoid). The part labeled "Soft iron core" allows one to guide the magnetic field lines of the rotating magnet through the "fixed coil". As a consequence one can assume a magnetic field inside the coil which amounts to $B(t) = B_0 \sin(\omega t)$ with $B_0 = 0.2$ T. The coil has $N = 1000$ turns and all the turns have a radius $a = 2$ cm.

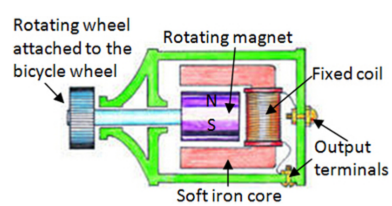


Figure 1: Bicycle Dynamo.

a) Calculate the frequency of rotation f (turns per second, $f = \omega/2\pi$) needed to apply a time-averaged electric power $P_{\text{avg}} = 4$ W to the bulb with $R = 1\ \Omega$. (Here we neglect any self-inductance of the coil. Time-averaged electric power means that one averages the time-dependent dissipated power $P(t)$ over one period T. The time-averaged power P_{avg} is found to be $emf_0^2/2R$ where emf_0 is the amplitude of the time-dependent electromotive force.)

b) Plausibility check (optional): A cyclist drives with a velocity $v = 20$ km/h. The bicycle wheel's radius is $r_{\text{W}} = 31$ cm. The radius of the dynamo wheel is $r_{\text{D}} = 0.5$ cm. Calculate the frequency f that is relevant for induction and compare to the value of a).