Exercise sheet 10: Induction, inductances, LC oscillator

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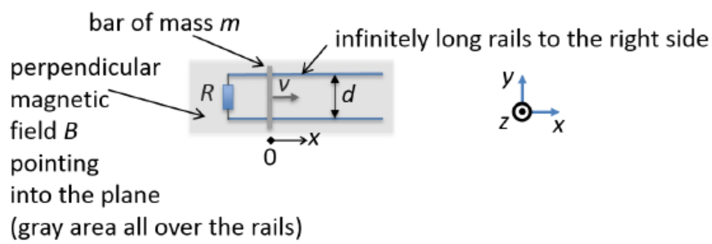
20/11/2024

We indicate the challenges of the problems by categories I ("warming-up"), II ("exam-level"), III ("advanced"). For your orientation: problems attributed to category II have been or could have been considered for an exam (assuming a specific duration for finding the solution; see comments in the solutions). The exact problem setting cannot be repeated in an exam however.

Exercise 1.

(Induction and motion as considered in General Physics I /Category II)

A metal bar of mass m slides with velocity v and without friction on two parallel conducting rails (see figure). The rails are semi-infinite and separated by a distance d. At one end of the rails a resistor with resistance R electrically connects the rails. Assume that the resistances of bar and rails are negligible. A uniform field B is perpendicular to the plane and points along the $-\vec{z}$-direction.



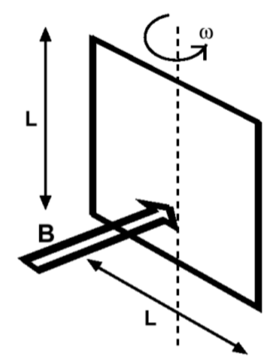
a) Calculate the current I flowing in the circuit. How does it depend on v, B, R and geometrical parameters (neglect the self-inductance)? Indicate the direction of the current when velocity v is pointing to the right.

b) Assume now that at a given position $x = 0$ and time $t = 0$, the bar has an initial velocity v_0 parallel to the rails, pointing away from the resistor. No external force is applied to the bar for $t > 0$. Calculate the formula for the time-dependent velocity $v(t)$ of the bar.

Exercise 2.

(Rotating square loop / Category II
(after training time needed: 20-25 min))

Consider a square loop rotating around a vertical axis with constant angular velocity ω. The square loop is made of a wire of resistance R. The loop is placed in a uniform magnetic field $\vec{B}$ that is perpendicular to the axis of the rotation, see figure.



a) Calculate the induced current I in the loop. Evaluate the current flow direction for the sketched graph.

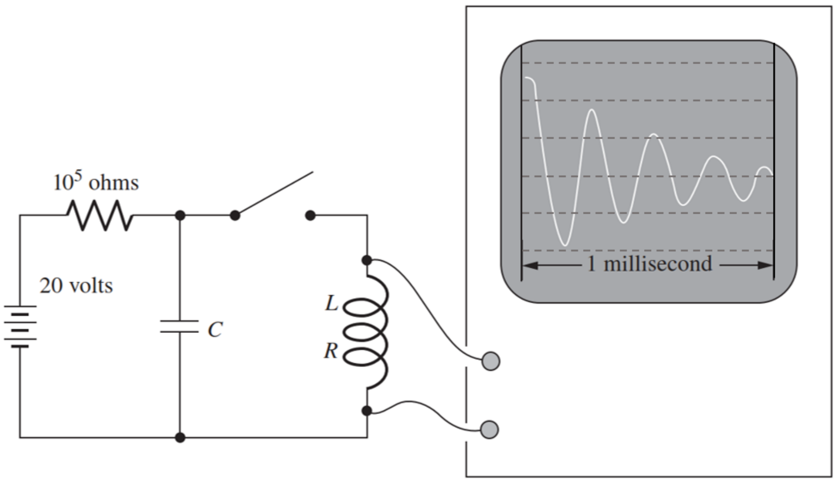
b) Calculate the torque τ necessary to keep the loop turning.

c) Show that the time-dependent mechanical power $P_{mech}(t) = \tau(t)\omega$ (General Physics I) is equal to the electrical power dissipated in the wire.

Exercise 3.

(Decaying signal/Category I)

The coil in the circuit of the figure below is shown to have an inductance of 0.01 H. When the switch is closed, the oscilloscope sweep is triggered. The 10^5 ohm resistor is large enough (as you will discover) so that it can be treated as essentially infinite for part (a) and (b) of this problem.

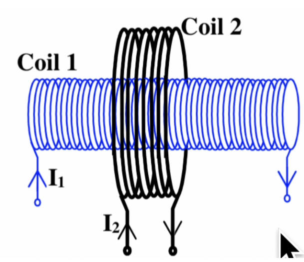


a) Determine, as well as you can, the value of the capacitance C neglecting first any resistance in the loop formed by the coil and capacitor.

b) Now estimate the value of the resistance R of the coil. We assume that only this value R is responsible for the observed decay of the oscillating signal. Hint: Consider R to be in series with L.

c) What is the magnitude of the voltage across the oscilloscope input, a“long” time after the switch has been closed, say several seconds after?

Exercise 4.
(Self-inductance versus mutual inductance/Category II/
(After training for solution: 30 min))



The current I_1 through a long solenoid (coil 1) with n_1 turns per meter and radius R_1 is changing with time (given by dI_1/dt). A further circular coil 2 surrounds coil 1 (see figure). For the calculation of the magnetic field in coil 1 assume the solenoid to be infinitely long.

a) First disregard coil 2. Calculate the strength of the electric field induced by dI_1/dt as a function of distance r from the central axis of the solenoid. Sketch the result as a function of r.

b) Use the previous approximation: (i) Calculate the maximum emf developed along the solenoid assuming the following parameters: $R_1 = 1$ cm, length $l_1 = 15$ cm, and an absolute number of turns $N_1 = 150$. Here, l_1 is the length of the considered segment. The current follows $I_1(t) = I_0 sin(\omega t)$ with $I_0 = 5$ A and $\omega = 2\pi 50$ Hz. (ii) Are the maximum I and maximum emf reached at the same time? (Hint: The field lines are encircled N_1 times by the wire of the solenoid)

c) Now consider coil 2 to be present. It has a total of N_2 turns. What is the mutual inductance M when one assumes that all the flux from the solenoid (coil 1) passes through the outer coil 2; how does it depend on the parameters n_1, N_2, and R_1? (Hint: No current flow in coil 2, ends of coil 2 are open. To calculate M one must consider that coil 2 encircles the flux of coil 1 N_2 times.)

d) Relate the mutual inductance M to the self-inductances L_1 and L_2 of the solenoid (coil 1) and the coil 2, respectively. Assume that coil 2 is wound very tightly onto the solenoid ($R_2 = R_1$), and it has the same length l. Still, the number of its turns is different. The approximation of the magnetic field of an infinitely long solenoid is considered to be valid in both cases.