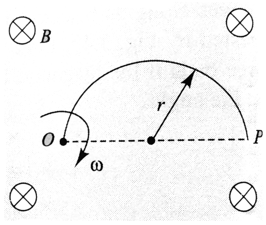
Quiz #2

Problem 1. A wire is in the form of a semicircle of radius r. One end is attached to an axis about which it rotates with an angular speed ω. The axis is normal to the plane of the semicircle. The wire is immersed in a uniform magnetic field B parallel to the axis. Find the induced emf between points O and P of the semicircle.



i) $\frac{B\omega a^2}{2}$ ii) $B\omega a^2$ iii) $2B\omega a^2$ iv) $\frac{B\omega a^2}{4}$

Problem 2. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{16} Hz and amplitude 48Vm^{-1} What is the amplitude of the oscillating magnetic field?

i) $2.7 \times 10^{-7}T$ ii) $1.6 \times 10^{-7}T$ iii) $1.6 \times 10^{-8}T$ iv) $2.7 \times 10^{-8}T$

Problem 3. Which of the following is not correct about the magnetic field lines?

i) The magnetic field lines of a magnet form continuous closed loops
ii) The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.
iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B.
iv) The magnetic field lines may intersect to each other in certain conditions.

Problem 4. Two short bar magnets of magnetic moments m each are arranged at the opposite corners of a square of side d such that their centers coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnetic induction at any of the other corners of the square is:

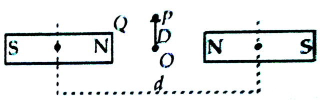
i) $\frac{\mu_0}{4\pi}\frac{m}{d^3}$ ii) $\frac{\mu_0}{4\pi}\frac{2m}{d^3}$ iii) $\frac{\mu_0}{4\pi}\frac{m}{2d^3}$ iv) $\frac{\mu_0}{4\pi}\frac{m^3}{2d^3}$

Problem 5. If two charged particles traverse identical helical paths in a opposite sense in a uniform magnetic field $\vec{B} = B_0\widehat{K}$, what must be true about them?

i) They have equal z - components of momenta.
ii) They necessarily represent a particle-antiparticle pair.
iii) They must have equal charges.
iv) Their charge to mass ratio satisfies:

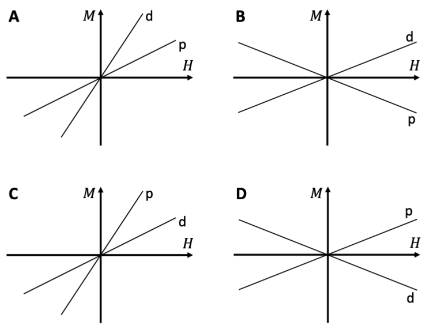
$$\left(\frac{q_1}{m_1}\right) + \left(\frac{q_2}{m_2}\right) = 0.$$

Problem 6. Two identical bar magnets are fixed with their centres at a distance d apart. A stationary charge Q is placed at P in between the gap of the two magnets at distance D from the centre O as shown in the figure. The force on the charge Q is:



i) zero ii) directed along OP iii) directed along PO
iv) directed perpendicular to the plane of paper

Problem 7. Consider a paramagnetic (p) material and a diamagnetic (d) material. Which of the following plots describes the correct trend of magnetization $\vec{M}$ versus applied $\vec{H}$ field for the two materials?
The correct answer is D



Problem 8. Consider the function $\psi = A(Bx + Ct)^2 + D\sin(Ex + Ft)$. For which of the following conditions is v a solution of the wave equation?

i) $C = F$ and $A = D$ ii) $\frac{C}{B} = \frac{F}{E}$ iii) $B = E$ and $A = D$ iv) $\frac{C}{B} = \frac{E}{F}$

Problem 9. Compute the intensity of the standing electromagnetic wave given by

$$E_y(x,t) = 2E_0 \cos kx \cos \omega t, \quad B_z(x,t) = 2B_0 \sin kx \sin \omega t$$

Solution. The Poynting vector for the standing wave is:

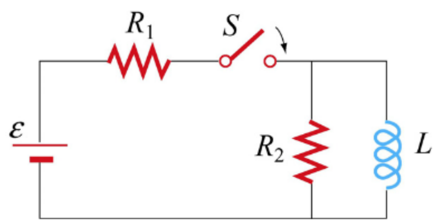
$$\begin{aligned}
\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} &= \frac{1}{\mu_0}\left(2E_0 \cos kx \cos \omega t \hat{\mathbf{j}}\right) \times \left(2B_0 \sin kx \sin \omega t \hat{\mathbf{k}}\right) \\
&= \frac{4E_0 B_0}{\mu_0}(\sin kx \cos kx \sin \omega t \cos \omega t)\hat{\mathbf{i}} \\
&= \frac{E_0 B_0}{\mu_0}(\sin 2kx \sin 2\omega t)\hat{\mathbf{i}}
\end{aligned}$$

The time average of S is

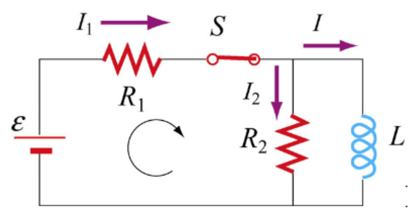
$$\langle S \rangle = \frac{E_0 B_0}{\mu_0} \sin 2kx \langle \sin 2\omega t \rangle = 0$$

The result is to be expected since the standing wave does not propagate. Alternatively, we may say that the energy carried by the two waves traveling in the opposite directions to form the standing wave exactly cancel each other, with no net energy transfer. □

Problem 10. In the circuit shown below, suppose the circuit is initially open. At time $t = 0$ it is thrown closed. What is the current in the inductor at a later time t ?



Solution: Let the currents through R_1, R_2 and L be I_1, I_2 and I, respectively, as shown here:



From Kirchhoff's junction rule, we have $I_1 = I_2 + I$. Similarly, applying Kirchhoff's loop rule to the left loop yields

$$\varepsilon - (I + I_2) R_1 - I_2 R_2 = 0$$

Similarly, for the outer loop, the modified Kirchhoff's loop rule gives

$$\varepsilon - (I + I_2) R_1 = L\frac{dI}{dt}$$

The two equations can be combined to yield

$$I_2 R_2 = L\frac{dI}{dt} \quad \Rightarrow \quad I_2 = \frac{L}{R_2}\frac{dI}{dt}$$

Substituting into the first equation the expression obtained above for I_2, we have

$$\varepsilon - \left(I + \frac{L}{R_2}\frac{dI}{dt}\right) R_1 - L\frac{dI}{dt} = \varepsilon - IR_1 - \left(\frac{R_1 + R_2}{R_2}\right) L\frac{dI}{dt} = 0$$

Dividing the equation by $(R_1 + R_2)/R_2$ leads to

$$\varepsilon' - IR' - L\frac{dI}{dt} = 0$$

where

$$R' = \frac{R_1 R_2}{R_1 + R_2}, \quad \varepsilon' = \frac{R_2 \varepsilon}{R_1 + R_2}$$

The differential equation can be solved and the solution is given by

$$I(t) = \frac{\varepsilon'}{R'}\left(1 - e^{-R't/L}\right)$$

Since

$$\frac{\varepsilon'}{R'} = \frac{\varepsilon R_2/(R_1 + R_2)}{R_1 R_2/(R_1 + R_2)} = \frac{\varepsilon}{R_1}$$

the current through the inductor may be rewritten as

$$I(t) = \frac{\varepsilon}{R_1}\left(1 - e^{-R't/L}\right) = \frac{\varepsilon}{R_1}\left(1 - e^{-t/\tau}\right)$$

where $\tau = L/R'$ is the time constant. $\square$