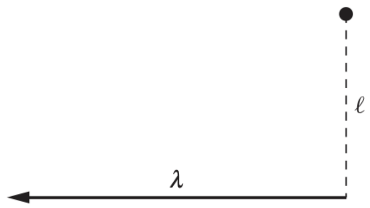
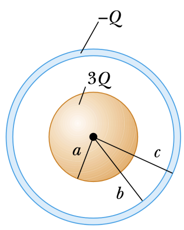
Quiz #1

Problem 1. A half-infinite line has linear charge density λ. Find the electric field at a point that is "even" with the end, a distance ℓ from it, as shown in the figure below. You should find that the field always points up at a 45° angle, independent of ℓ.



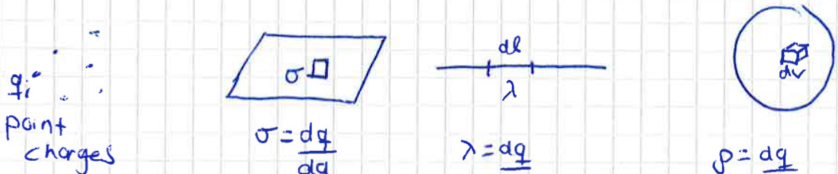
Problem 2. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in the figure below. (i) Construct a spherical gaussian surface of radius $r > c$ and find the net charge enclosed by this surface. (ii) What is the direction of the electric field at $r > c$? (iii) Find the electric field at $r \geq c$. (iv) Find the electric field in the region with radius r where $b < r < c$ (v) Construct a spherical gaussian surface of radius r, where $b < r < c$, and find the net charge enclosed by this surface. (vi) Construct a spherical gaussian surface of radius r, where $a < r < b$, and find the net charge enclosed by this surface. (vii) Find the electric field in the region $a < r < b$. (viii) Construct a spherical gaussian surface of radius $r < a$, and find an expression for the net charge enclosed by this surface, as a function of r. Note that the charge inside this surface is less than $3Q$. (ix) Find the electric field in the region $r < a$. (x) Determine the charge on the inner surface of the conducting shell. (xi) Determine the charge on the outer surface of the conducting shell. (xii) Make a plot of the magnitude of the electric field versus r.



Problem 3. Imagine the xy plane, the xz plane, and the yz plane all made of metal and soldered together at the intersections. A single point charge Q is located a distance d from each of the planes. Sketch the configuration of image charges you need to satisfy the boundary conditions. What is the direction and magnitude of the force that acts on the charge Q?

Problem 4. If a point charge is located outside a hollow conducting shell, there is an electric field outside, but no electric field inside. On the other hand, if a point charge is located inside a hollow conducting shell, there is an electric field both inside and outside (although the external field would be zero in the special case where the shell happened to have charge exactly equal and opposite to the point charge). The situation is therefore not symmetric with respect to inside and outside. Explain why this is the case, by considering where electric field lines can begin and end.

Electrostatics $\longrightarrow$ I want to know the electric field generated by a distribution of charges



$\sigma = \frac{dq}{da}$ $\qquad$ $\lambda = \frac{dq}{d\ell}$ $\qquad$ $\rho = \frac{dq}{dV}$

What we have done in the past few weeks is develop tools towards doing this.

What are a few tricks that we've learned to calculate $\underline{E}$?

① Use coulomb's law + the principle of superposition (Lecture 2)

Force on a charge q: $\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i Q}{r_i^2}\hat{r}_i \;\Big|\; \frac{Q}{4\pi\varepsilon_0}\int_A \frac{\sigma\, da\,\hat{r}}{r^2} \;\Big|\; \frac{Q}{4\pi\varepsilon_0}\int_C \frac{\lambda\, d\ell\,\hat{r}}{r^2} \;\Big|\; \frac{Q}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV\,\hat{r}}{r^2}$

② Or equivalently using the definition of $\underline{E} = \frac{F}{Q}$ (Lecture 3)

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i\hat{r}_i}{r_i^2} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_A \frac{\sigma\, da\,\hat{r}}{r^2} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_C \frac{\lambda\, d\ell\,\hat{r}}{r^2} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV\,\hat{r}}{r^2}$$

* In lecture 3 we also discussed Gauss's law which gives us a tool to calculate $\underline{E}$ when the problem has symmetries (i.e when we can find a surface that encloses our source charge where $\underline{E}$ is constant over the surface and $\underline{E} \parallel d\underline{A}$)

③ $$\oint \underline{E}\cdot d\underline{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

④ Equivalently in lecture 4 we defined the electric potential $\underline{E} = -\nabla\varphi$ or $\varphi(\underline{r}) = -\int_{\infty}^{P} \underline{E}\cdot d\underline{s}$

electric potential: $\varphi(\underline{r})$ $\quad \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_A \frac{\sigma\, dA}{r} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_C \frac{\lambda\, d\ell}{r} \;\Big|\; \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{r}$

⑤ We recasted the potential problem (i.e finding an integral) into a differential problem, where we found two differential equations satisfied by the potential: (Lecture 5)

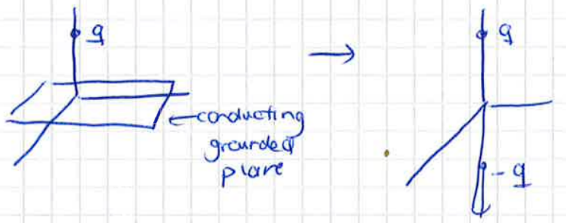
$\nabla^2\varphi = -\frac{1}{\varepsilon_0}\rho$ $\qquad$ Poisson's eq. (Regions w/charge)

$\nabla^2\varphi = 0$ $\qquad$ Laplace's eq. (Regions w/no charge)

\+ we proved the uniqueness theorem that states that if we find φ that satisfies the BC of a problem $\Rightarrow$ we found a solution.

⑥ We used the uniqueness theorem to come up w/ yet another tool to calculate $\underline{E}$, in the specific case of conductors where knowing the charge distribution is <u>not</u> trivial. (lecture 6)

Method of Images



\+ done.

⑦ Today we come up w/ yet another tool to approximate the potential of a charge distribution at "far away" points. (Lecture 8. TODAY!)

The multipole expansion

$$\varphi(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}Q + \frac{\underline{p}\cdot\hat{r}}{r^2} + \frac{Q_{ij}\hat{r}_i\hat{r}_j}{r^3} + \dots\right]$$

So now you have a lot of tools at your disposal to solve any electrostatic problem that comes your way.