Exercise sheet #13

Problem 1. Verify that, for $\omega = kc$,

$$E(x,t) = E_0 \cos(kx - \omega t)$$
$$B(x,t) = B_0 \cos(kx - \omega t)$$

satisfy the one-dimensional wave equation:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left\{\begin{array}{c} E(x,t) \\ B(x,t) \end{array}\right\} = 0$$

Problem 2. If the electric field in free space is $\mathbf{E} = E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin[(2\pi/\lambda)(z + ct)]$ with $E_0 = 20$ volts /m, then the magnetic field, not including any static magnetic field, must be what?

Problem 3. Write out formulas for **E** and **B** that specify a plane electromagnetic sinusoidal wave with the following characteristics. The wave is traveling in the direction $-\hat{\mathbf{x}}$; its frequency is 100 megahertz (MHz), or 10^8 cycles per second; the electric field is perpendicular to the $\hat{\mathbf{z}}$ direction.

Problem 4. Consider the two oppositely traveling electric-field waves,

$$\mathbf{E}_1 = \hat{\mathbf{x}}E_0\cos(kz - \omega t) \quad \text{and} \quad \mathbf{E}_2 = \hat{\mathbf{x}}E_0\cos(kz + \omega t).$$

The sum of these two waves is the standing wave, $2\hat{\mathbf{x}}E_0 \cos kz \cos \omega t$.

(a) Find the magnetic field associated with this standing electric wave by finding the B fields associated with each of the above traveling **E** fields, and then adding them.

(b) Find the magnetic field by instead using Maxwell's equations to find the **B** field associated with the standing electric wave, $2\hat{\mathbf{x}}E_0 \cos kz \cos \omega t$.

Problem 5. A wave propagating through a string in the z directions with the vertical $\mathbf{f}_v(z,t) = A\cos(kz - \omega t + \delta_v)\hat{\mathbf{x}}$ and horizontal components $\mathbf{f}_h(z,t) = A\cos(kz - \omega t + \delta_h)\hat{\mathbf{y}}$ is circularly polarized if the two components are of equal amplitude but out of phase by 90 degrees.

(a) Show that at a fixed point z, the string moves in a circle about the z axis. Does it go clockwise or counterclockwise, as you look down the axis toward the origin? How would you construct a wave circling the other way? (In optics, the clockwise case is called right circular polarization, and the counterclockwise, left circular polarization. Hint: set $\delta_v = 0, \delta_h = 90°$ in the equations for $\mathbf{f}_v(z,t)$ and $\mathbf{f}_h(z,t)$.

(b) Sketch the string at time $t = 0$

(c) How would you shake the string in order to produce a circularly polarized wave?