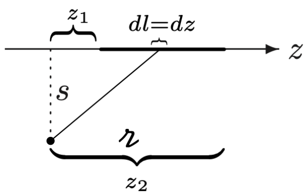
Exercise sheet #10

Problem 1. A solenoid has radius R, current I, and n turns per unit length. Given that the magnetic field is $B = \mu_0 nI$ inside and $B = 0$ outside, find the vector potential $\mathbf{A}$ both inside and outside. Hint: Use the expression for the curl in cylindrical coordinates to find the forms of $\mathbf{A}$ that yield the correct values of $\mathbf{B} = \nabla \times \mathbf{A}$ in the two regions.

Problem 2. Find the magnetic vector potential of a finite segment of straight wire carrying a current I. Calculate the magnetic field generated by that potential. Hint: Put the wire on the z axis, from z_1 to z_2 as shown in the schematic below.



Problem 3. Magnetic scalar potential

(a) Consider an infinite straight wire carrying current I. We know that the magnetic field outside the wire is $\mathbf{B} = (\mu_0 I/2\pi r)\,\hat{\boldsymbol{\theta}}$. There are no currents outside the wire, so $\nabla \times \mathbf{B} = 0$; verify this by explicitly calculating the curl.

(b) Since $\nabla \times \mathbf{B} = 0$, we should be able to write $\mathbf{B}$ as the gradient of a function, $\mathbf{B} = \nabla\psi$. Find ψ, but then explain why the usefulness of ψ as a potential function is limited.

Problem 4. Consider an infinite solenoid with circular cross section. The current is I, and there are n turns per unit length. Show that the magnetic field is zero outside and $B = \mu_0 nI$ (in the longitudinal direction) everywhere inside. Do this in three steps as follows.

(a) Show that the field has only a longitudinal component. Hint: Consider the contributions to the field from rings that are symmetrically located with respect to a given point.

(b) Use Ampère's law to show that the field has a uniform value outside and a uniform value inside, and that these two values differ by $\mu_0 nI$.

(c) Show that $B \to 0$ as $r \to \infty$. There are various ways to do this. One is to obtain an upper bound on the field contribution due to a given ring by unwrapping the ring into a straight wire segment, and then finding the field due to this straight segment.

Problem 5. A long circular cylinder of radius R carries a magnetization $\mathbf{M} = ks^2\hat{\phi}$, where k is a constant, s is the distance from the axis, and $\hat{\phi}$ is the usual azimuthal unit vector. Find the magnetic field due to $\mathbf{M}$, for points inside and outside the cylinder.