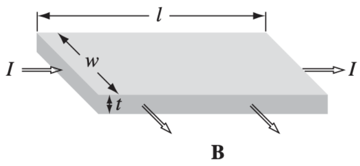
Exercise sheet #9

Problem 1. Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particles. What is their mass ratio?

Problem 2. A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field $\mathbf{B}$ pointing out of the page (See fig. below).



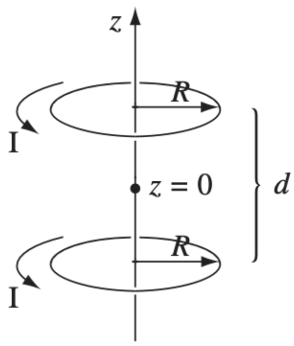
(a) If the moving charges are positive, in which direction are they deflected by the magnetic field? This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electric force to counteract the magnetic one. Equilibrium occurs when the two exactly cancel. (This phenomenon is known as the Hall effect.)

(b) Find the resulting potential difference (the Hall voltage) between the top and bottom of the bar, in terms of B, v (the speed of the charges), and the relevant dimensions of the bar.

(c) How would your analysis change if the moving charges were negative? [The Hall effect is the classic way of determining the sign of the mobile charge carriers in a material.]

Problem 3. Use the Biot-Savart law to find the field inside and outside an infinitely long solenoid of radius R, with n turns per unit length, carrying a steady current I

Problem 4. The magnetic field on the axis of a circular current loop is far from uniform (it falls off sharply with increasing z). You can produce a more nearly uniform field by using two such loops a distance d apart (Fig. below).



(a) Find the field (B) as a function of z, and show that $\partial B/\partial z$ is zero at the point midway between them ($z = 0$).

(b) If you pick d just right, the second derivative of B will also vanish at the midpoint. This arrangement is known as a Helmholtz coil; it's a convenient way of producing relatively uniform fields in the laboratory. Determine d such that $\partial^2 B/\partial z^2 = 0$ at the midpoint, and find the resulting magnetic field at the center. [Answer: $8\mu_0 I/5\sqrt{5}R$]

Problem 5. A volume current density $\mathbf{J} = J\hat{\mathbf{z}}$ exists in a slab between the infinite planes at $x = -b$ and $x = b$. (So the current is coming out of the page, see figure below) Additionally, a surface current density $\mathcal{J} = 2bJ$ points in the $-\hat{\mathbf{z}}$ direction on the plane at $x = b$. (a) Find the magnetic field as a function of x, both inside and outside the slab. (b) Verify that $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ inside the slab. (Don't worry about the boundaries.)