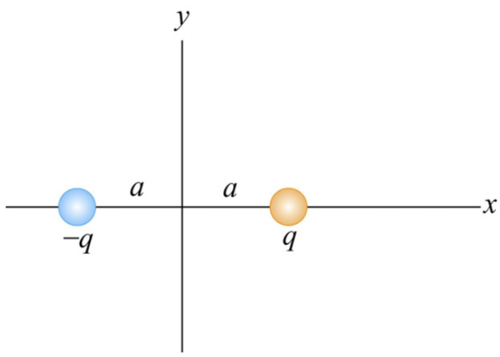
Exercise sheet #4

Problem 1. Suppose the electric field in some region is found to be $E = kr^3\hat{r}$ (where k is a constant), in spherical coordinates.

(a) Find the charge density ρ

(b) Find the total charge contained in a sphere of radius R, centered at the origin.

Problem 2. Consider a system of two charges shown in the figure below. Find the electric potential at an arbitrary point on the x axis and make a plot of the electric potential as a function of x/a.

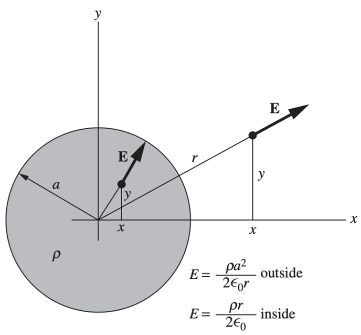


Problem 3. We have two metal spheres, of radii R_1 and R_2, quite far apart from one another compared with these radii. Given a total amount of charge Q which we have to divide between the spheres, how should it be divided so as to make the potential energy of the resulting charge distribution as small as possible? When you have found the optimum division of the charge, show that with that division the potential difference between the two spheres is zero.

Problem 4. For the cylinder of uniform charge density in the figure below:

(a) Show that the expression there given for the field inside the cylinder follows from Gauss's law;

(b) Find the potential ϕ as a function of r, both inside and outside the cylinder, taking $\phi = 0$ at $r = 0$.



Problem 5. A square sheet has uniform surface charge density σ. Letting the electric potential ϕ be zero at infinite distance from the square, denote by ϕ_0 the potential at the center of the square and by ϕ_1 the potential at a corner. Determine the ratio ϕ_0/ϕ_1. The answer can be found with very little calculation by combining a dimensional argument with superposition. (Think about the potential at the center of a square with the same charge density and with twice the edge length.)