Exercise sheet #1

Problem 1. Starting from a point A, a person bicycles 10 miles due east to point B, stops for lunch, sells his bicycle, and then walks 10 miles due north to point C. Another person starts from B, walks 4 miles due north and 12 miles due east and then, feeling tired, and having brought along a surplus of travellers' checks, buys a car and drives 6 miles due north and 2 miles due west, ending at point G in the pouring rain. What is the magnitude of the displacement that each person undergoes?

Problem 2. A pinball moving in a plane with velocity $\mathbf{s}$ bounces, in a purely elastic impact, from a baffle whose endpoints are $\mathbf{p}$ and $\mathbf{q}$. What is the velocity vector after the bounce?

Problem 3. Prove that $[\mathbf{A} \times (\mathbf{B} \times \mathbf{C})] + [\mathbf{B} \times (\mathbf{C} \times \mathbf{A})] + [\mathbf{C} \times (\mathbf{A} \times \mathbf{B})] = 0$. Under what conditions does $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$?

Problem 4. Prove that the two-dimensional rotation matrix preserves dot products. (That is, show that $\bar{A}_y \bar{B}_y + \bar{A}_z \bar{B}_z = A_y B_y + A_z B_z$). What constraints must the elements (R_{ij}) of the three-dimensional rotation matrix satisfy, in order to preserve the length of $\mathbf{A}$ (for all vectors $\mathbf{A}$) ?

$$\text{Rotation matrix applied to A:} \quad \mathbf{RA} = \begin{pmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

Problem 5. (a) If $\mathbf{A}$ and $\mathbf{B}$ are two vector functions, what does the expression $(\mathbf{A} \cdot \nabla)\mathbf{B}$ mean? That is, what are its x,y and z components, in terms of the Cartesian components of $\mathbf{A}$,$\mathbf{B}$ and ∇?
(b)Compute $(\hat{\mathbf{r}} \cdot \nabla)\hat{\mathbf{r}}$ for $\mathbf{r} = \frac{\mathbf{r}}{r} = \frac{x\hat{\mathbf{x}}+y\hat{\mathbf{y}}+z\hat{\mathbf{z}}}{\sqrt{x^2+y^2+z^2}}$
(c)Evaluate $(\mathbf{v_a} \cdot \nabla)\mathbf{v_b}$ for the functions $\mathbf{v}_a = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} - 2xz\hat{\mathbf{z}}$ and $\mathbf{v}_b = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}$

Problem 6. Prove that the divergence of a curl is always zero.