

In cylindrical coordinates $(\hat{r}, \hat{\varphi}, \hat{z})$ for a scalar field B and a vector field $\vec{A} = (A_r, A_\varphi, A_z)$:

$$\nabla B(r, \varphi, z) = \begin{bmatrix} \frac{\partial B}{\partial r} \\ \frac{1}{r} \frac{\partial B}{\partial \varphi} \\ \frac{\partial B}{\partial z} \end{bmatrix}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial(rA_r)}{\partial r} + \frac{1}{r} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla^2 B = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial B}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 B}{\partial \varphi^2} + \frac{\partial^2 B}{\partial z^2}$$

$$\nabla^2 \vec{A} = \begin{bmatrix} \nabla^2 A_r - \frac{A_r}{r^2} - \frac{2}{r^2} \frac{\partial A_\varphi}{\partial \varphi} \\ \nabla^2 A_\varphi - \frac{A_\varphi}{r^2} + \frac{2}{r^2} \frac{\partial A_r}{\partial \varphi} \\ \nabla^2 A_z \end{bmatrix}$$

$$\nabla \times \vec{A} = \begin{bmatrix} \frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \\ \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \\ \frac{1}{r} \frac{\partial(rA_\varphi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \varphi} \end{bmatrix}$$

$$(\vec{A} \cdot \nabla) \vec{A} = \begin{bmatrix} A_r \frac{\partial A_r}{\partial r} + \frac{A_\varphi}{r} \frac{\partial A_r}{\partial \varphi} - \frac{A_\varphi^2}{r} + A_z \frac{\partial A_r}{\partial z} \\ A_r \frac{\partial A_\varphi}{\partial r} + \frac{A_\varphi}{r} \frac{\partial A_\varphi}{\partial \varphi} + \frac{A_\varphi A_r}{r} + A_z \frac{\partial A_\varphi}{\partial z} \\ A_r \frac{\partial A_z}{\partial r} + \frac{A_\varphi}{r} \frac{\partial A_z}{\partial \varphi} + A_z \frac{\partial A_z}{\partial z} \end{bmatrix}$$