Final Exam

31.01.2019

The time available for the exam is 3h. No calculators, books or slides are allowed, only one doublesided A4 handwritten paper with notes.
Write the correct answer (A,B,C,D, or E) of each multiple-choice question in the following table. Write the final answers to the open questions in the provided space of each problem page.
Each question gives 1 point, as well as each step of the two open questions (total 30 points).

NAME S. OLution

N°SCIPER.........................

TABLE OF ANSWERS							
1	A	7	C	13	E	19	C
2	C	8	A	14	B	20	C
3	D	9	A	15	C	21	A
4	D	10	A/D	16	D	22	C
5	B	11	C	17	A	23	A
6	A	12	D	18	A	24	B

$$\text{grade} = 1 + \frac{p \cdot 5}{29}$$

$\langle 4.4 \rangle$

PHYSICAL CONSTANTS

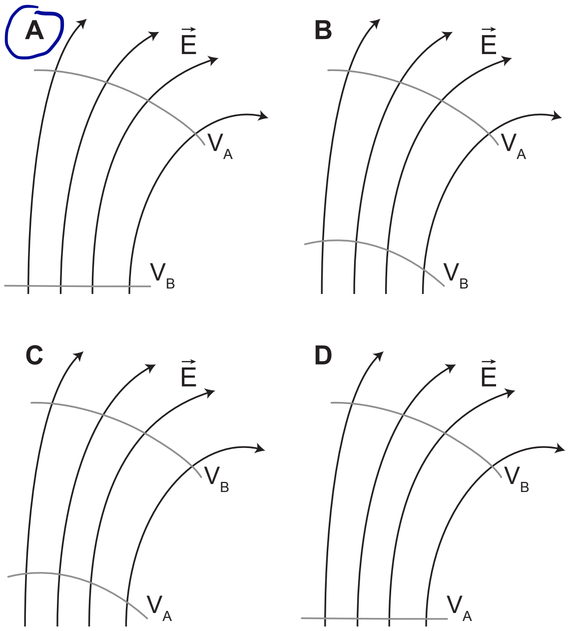
$$q_e = 1.6 \times 10^{-19}\ C$$
$$m_e = 9.1 \times 10^{-31}\ kg$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$
$$k = (4\pi\epsilon_0)^{-1} = 8.9 \times 10^{9}\ Nm^2/C^2$$
$$\mu_0 = 4\pi \times 10^{-7}\ Vs/(Am)$$
$$h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s$$
$$c = 3 \times 10^{8}\ m/s$$

TRIGONOMETRY

$$\cos 30^\circ = \sin 60^\circ = \sqrt{3}/2$$
$$\cos 45^\circ = \sin 45^\circ = \sqrt{2}/2$$
$$\cos 60^\circ = \sin 30^\circ = 1/2$$

Question 1 76 %

In the figure below electric field lines are shown. Which is the correct representation of the isopotential lines given that $V_B > V_A$?



Question 2 92 %

Two positive charges $+q$ and one positive charge $+2q$ are fixed on the corners of an equilateral triangle of side a, as shown in the figure below. Evaluate the magnitude of the Coulomb force on a negative charge $-q$ placed in the middle of one leg of the triangle as shown in the figure.

A. $\frac{q^2}{2\pi\epsilon_0 a^2}$

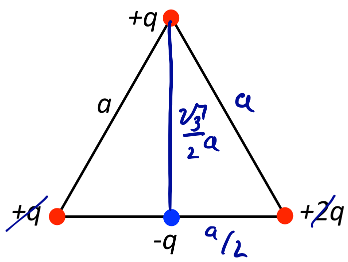
B. $\frac{q^2\sqrt{37}}{3\pi\epsilon_0 a^2}$

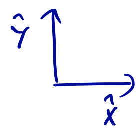
C. $\frac{q^2\sqrt{10}}{3\pi\epsilon_0 a^2}$

D. $\frac{q^2\sqrt{3}}{2\pi\epsilon_0 a^2}$

$$F = \frac{-q^2}{4\pi\varepsilon_0 \left(\frac{a}{2}\right)^2}\hat{x} - \frac{q^2}{4\pi\varepsilon_0 \left(\frac{\sqrt{3}}{2}a\right)^2}\hat{y} = \frac{-q^2\hat{x}}{\pi\varepsilon_0 a^2} - \frac{q^2\hat{y}}{3\pi\varepsilon_0 a^2}$$

$$= \frac{-2q^2}{\pi\varepsilon_0 a^2}\left(\sqrt{1+\left(\tfrac{1}{3}\right)^2}\right) = \frac{-\sqrt{10}\,q^2}{3\pi\varepsilon_0 a^2}$$





Question 3 36%

A charge $+Q$ and a charge $-Q$ are uniformly distributed along the two halves of a circular wire with radius R. What is the modulus of the electric field in the centre of the circle.

A. $\frac{Q}{4\pi\epsilon_0 R}$

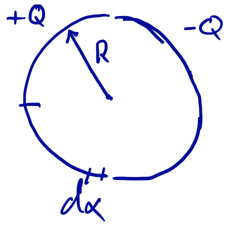
B. $\frac{Q}{4\pi\epsilon_0 R^2}$

C. $\frac{Q}{\pi^2\epsilon_0 R}$

D. $\frac{Q}{\pi^2\epsilon_0 R^2}$

E. 0

Superposition (2 times)



$\lambda = \frac{Q}{\pi R}$

$dq = \lambda R \, d\alpha = \frac{Q}{\pi} d\alpha$

$dE = \frac{Q}{4\pi^2 \varepsilon_0 R^2} \cos\alpha \, d\alpha$

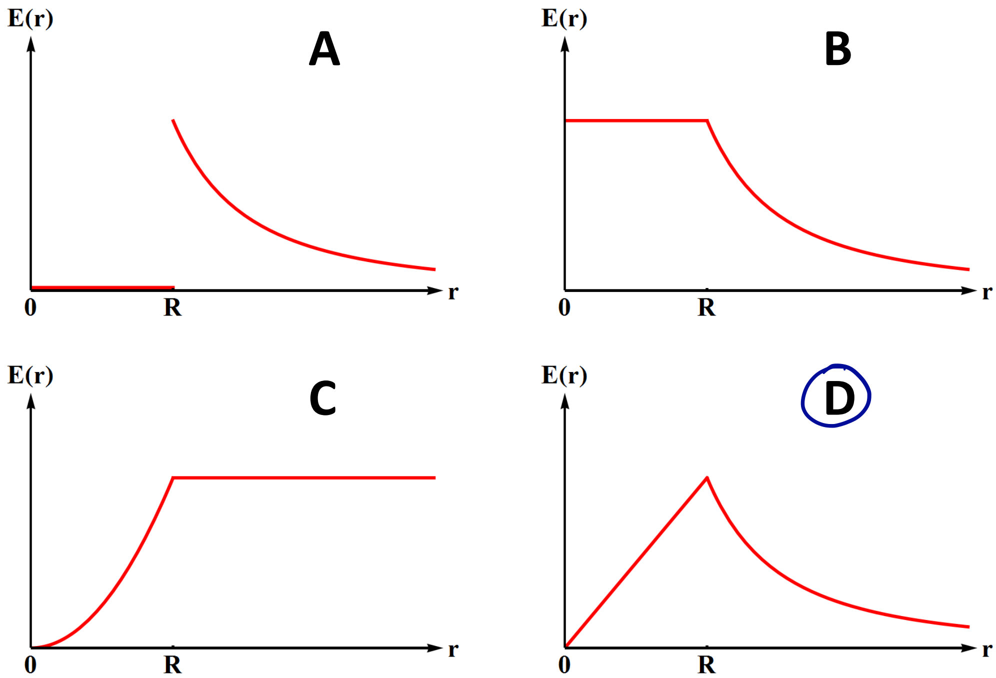
$E_1 = \int_0^{\pi/2} \frac{Q}{4\pi^2\varepsilon_0 R^2} \cos\alpha \, d\alpha = \frac{Q}{4\pi^2\varepsilon_0 R^2}$

[Exercise 3 sheet 3]

$E_{tot} = 2 \cdot 2 \cdot E_1 = \frac{Q}{\pi^2 \varepsilon_0 R^2}$

Question 4 79%

Consider an infinitely long cylinder of radius R with uniform volume charge distribution. Choose the correct plot of the electric field as function of distance from the cylinder axis.



Question 5 81 %

Four positive charges $+q$ are placed at the corners of a square with side d. Evaluate the total work done for moving two of the charges to a point halfway along the side, as shown in the figure below.

A. $(4+\sqrt{2})\frac{q^2}{4\pi\epsilon_0 d}$

B. $(1+\overset{4}{\sqrt{5}}-\frac{1}{\sqrt{2}})\frac{q^2}{4\pi\epsilon_0 d}$

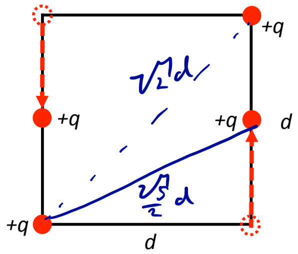
C. $(1-\frac{1}{\sqrt{2}})\frac{q^2}{4\pi\epsilon_0 d}$

D. $\frac{q^2}{\pi\epsilon_0 d}$

$$U_1 = \frac{4q^2}{4\pi\mathcal{E}_0 d} + \frac{2q^2}{4\pi\mathcal{E}_0 \sqrt{2}d}$$

$$U_2 = \frac{2q^2}{4\pi\mathcal{E}_0 \frac{\sqrt{5}}{2}d} + \frac{2q^2}{4\pi\mathcal{E}_0 d/2} + \frac{q^2}{4\pi\mathcal{E}_0 \sqrt{2}d} + \frac{q^2}{4\pi\mathcal{E}_0 d}$$

$$\left(\frac{4}{\sqrt{5}} + 4 + \frac{1}{\sqrt{2}} + 1\right) - \left(4 + \frac{2}{\sqrt{2}}\right) = 1 + \frac{4}{\sqrt{5}} - \frac{1}{\sqrt{2}}$$



Question 6 64 %

A sphere with radius R has a volume charge distribution given by $\rho(r) = \rho_0/r$. What is the value of the electrostatic potential at a distance $2R$ from the centre of the sphere? $(V(\infty) = 0)$

A. $\frac{\rho_0 R}{4\epsilon_0}$

B. $\frac{\rho_0}{4\epsilon_0 R}$

C. $\frac{\rho_0 R}{4\pi\epsilon_0}$

D. $\frac{\rho_0}{4\pi\epsilon_0 R}$

$$Q = \int_0^R \frac{\rho_0}{r} 4\pi r^2 dr = \int_0^R 4\pi\rho_0 r dr$$

$$Q = 2\pi\rho_0 R^2$$

$$V = \frac{Q}{4\pi\mathcal{E}_0 2R} = \frac{2\pi\rho_0 R^2}{8\pi\mathcal{E}_0 R} = \frac{\rho_0 R}{4\mathcal{E}_0}$$

Question 7 92%

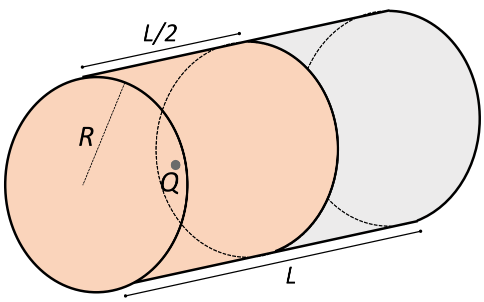
A positive charge Q is placed in the centre of a cylinder with radius R and length L at a distance $L/4$ from the left face, as illustrated in the figure. The flux of the electric field through the cylinder is Φ_0. What is the flux through the cylinder if its length is reduced to $L/2$ while keeping the left face fixed, as illustrated in the figure.

A. $\frac{\Phi_0}{\epsilon_0}$

B. $\frac{\Phi_0\sqrt{2}R}{\epsilon_0 L}$

C. $\frac{Q}{\epsilon_0}$

D. $\frac{Q\sqrt{2}R}{\epsilon_0 L}$



Question 8 70%

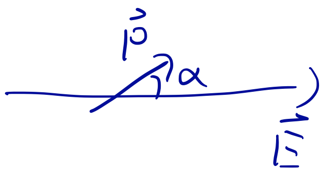
Two charges $+q$ and $-q$ are rigidly connected at very small distance d in a region of space where there is an homogeneous electric field $\vec{E}$. The angle between the field lines and the line connecting the charges is α. Evaluate the potential energy U of the system of charges, using the convention for zero potential energy from the lecture.

A. $-qdE\cos\alpha$

B. $-qdE\sin\alpha$

C. $-q^2dE\cos\alpha$

D. $-q^2dE\sin\alpha$



$\vec{p} = q\vec{d}$

$U = -\vec{p}\cdot\vec{E}$

$U = -qdE\cos\alpha$

Question 9 81%

Oil has a dielectric strength, or breakdown potential gradient, of 12×10^6 V/m. A device is operating at 30 kV while submerged in oil. What is the smallest external curvature radius you should use for the device to avoid a discharge?

A. 2.5 mm

B. 4 mm

C. 5 mm

D. 8 mm

$$r = \frac{V}{E_{max}} = \frac{30 \times 10^3}{12 \times 10^6} = \frac{1}{4} \times 10^{-2} = 2.5 \text{ mm}$$

Question 10 70%

A cylindrical capacitor with length L and radii R_1 and R_2 as illustrated in the figure is charged to a potential V_0. Afterwards the capacitor is isolated from the power source and a dielectric (ϵ_r) is completely inserted in the capacitor. What is the change in the electric energy stored in the system?

A. $(1 - \frac{1}{\epsilon_r})\frac{\pi\epsilon_0 L V_0^2}{\ln\left(\frac{R_2}{R_1}\right)}$

B. $(\epsilon_r - 1)\frac{\pi\epsilon_0 L V_0^2}{\ln\left(\frac{R_2}{R_1}\right)}$

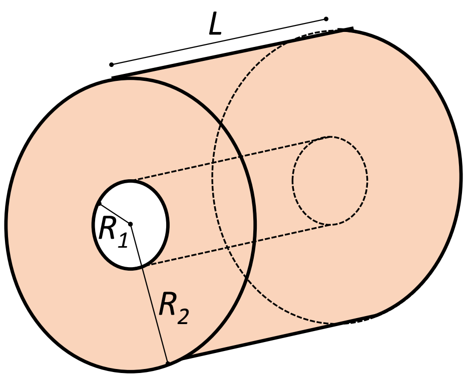
C. $(1 - \frac{1}{\epsilon_r})\frac{\pi\epsilon_0\epsilon_r L V_0^2}{\ln\left(\frac{R_2}{R_1}\right)}$

D. $(\epsilon_r - 1)\frac{\pi\epsilon_0 L V_0^2}{\epsilon_r \ln\left(\frac{R_2}{R_1}\right)}$

$$U_E^0 = \frac{1}{2} C_0 V_0^2 = \frac{\pi \varepsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)} V_0^2$$

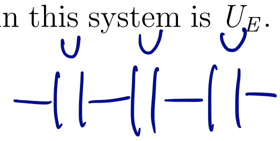
$$U_E = \frac{1}{\varepsilon_r} U_E^0$$

$$\rightarrow \Delta U_E = \left(1 - \frac{1}{\varepsilon_r}\right) \frac{\pi \varepsilon_0 L V_0^2}{\ln\left(\frac{R_2}{R_1}\right)}$$



Question 11 65%

Three identical and equally changed capacitors with capacitance C are placed in series. The total energy stored in this system is U_E. What is the magnitude of the charge on the middle capacitor?



A. $\sqrt{\frac{4}{3} U_E C}$

B. $\sqrt{6 U_E C}$

C. $\sqrt{\frac{2}{3} U_E C}$

D. $\sqrt{\frac{2}{27} U_E C}$

$V' = 3V$ $\quad \frac{1}{C'} = \frac{3}{C} \rightarrow C' = \frac{C}{3}$

$U_E = \frac{1}{2} C' V'^2 = \frac{1}{2} \frac{C}{3} 9 V^2 = \frac{3}{2} C V^2$

$\rightarrow V = \sqrt{\frac{2}{3} \frac{U_E}{C}}$, $\quad Q = CV = \sqrt{\frac{2}{3} U_E C}$

Question 12 52%

The same current I flows in the opposite direction through two infinite parallel wires placed at distance $2d$ as shown in the figure below (not to scale). Evaluate the magnitude of the force per unit of length acting on a wire placed in P with current $\frac{I}{2}\hat{z}$.

A. $\frac{\mu_0 I^3}{4\pi d}$

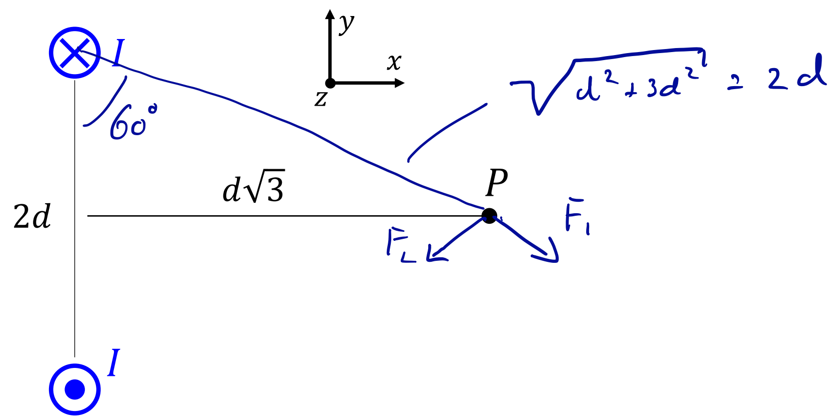
B. $\frac{\mu_0 I^3}{8\pi d}$

C. $\frac{\mu_0 I^2}{4\pi d}$

D. $\frac{\mu_0 I^2}{8\pi d}$

$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{\mu_0 I^2}{2 \cdot 2 \cdot 2\pi d} = \frac{\mu_0 I^2}{8\pi d}$

$F_{tot} = F_1 \cos 60° + F_2 \cos 60° = 2 \cdot \frac{1}{2} \cdot \frac{\mu_0 I^2}{8\pi d}$



Question 13 59%

A current of I passes through a circular wire with radius R placed in the xy-plane. A cube with side L, centred along the axis of the wire loop, is placed at a distance $2R$ along the z-axis. What is the total flux of the magnetic field through the surface of the cube.

A. $\frac{\mu_0 I R^2 L^2}{(L^2+R^2)^{\frac{3}{2}}}$

B. $\frac{\mu_0 I R^2 L^2}{4\pi(L^2+R^2)^{\frac{3}{2}}}$

C. $\frac{3\mu_0 I R^2 L^2}{2(L^2+R^2)^{\frac{3}{2}}}$

D. $\frac{\sqrt{3}\mu_0 I R^2 L^2}{2(L^2+R^2)^{\frac{3}{2}}}$

E. 0

$$\oiint \vec{B}\cdot d\vec{S} = 0$$

Question 14 79%

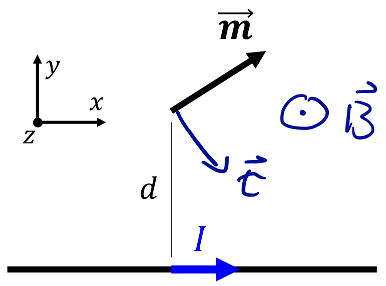
A current I passes through an infinite straight wire. A magnetic dipole $\vec{m}$ is placed at distance d from the wire with a 45° angle with respect to the wire itself, as shown in the figure below. Evaluate the torque acting on the dipole.

A. $\frac{\mu_0 I m}{2\pi d}\frac{(\hat{z}-\hat{y})}{\sqrt{2}}$

B. $\frac{\mu_0 I m}{2\pi d}\frac{(\hat{x}-\hat{y})}{\sqrt{2}}$

C. $\frac{\mu_0 I m}{2\pi d}\frac{(\hat{y}-\hat{x})}{\sqrt{2}}$

D. $\frac{\mu_0 I m}{2\pi d}\frac{(-\hat{x}-\hat{y})}{\sqrt{2}}$



Right hand Rule

Question 15 72%

A beam of electrons with various velocities $v_0 \pm \Delta v$ enters a device with constant electric field E and magnetic field B applied as shown in the figure below. Evaluate the velocity of the electrons at point P, given that $v_0 = 20$ km/s,$\Delta v = 7$ km/s,$E = 2500$ N/C,$B = 0.1$ T.

A. 13 km/s

B. 20 km/s

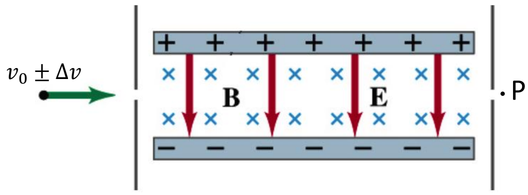
C. 25 km/s

D. 27 km/s

E. No electrons make it through

$$V = \frac{E}{B} = \frac{2500}{0.1} = 25 \times 10^3$$

within range of $v_0 \pm \Delta v$



Question 16 58%

A mixed beam of α,β and γ particles enters a bubble chamber with constant magnetic field pointing out of the page as shown in the figure below. A photo of the chamber is taken, and the trajectory are shown. Label the three particles (in nuclear physics, the α particle is the Helium nucleus, the β particle is the electron and the γ particle is the X-ray photon).

A. 1α , 2β , 3γ

B. 1α , 2γ , 3β

C. 1β , 2α , 3γ

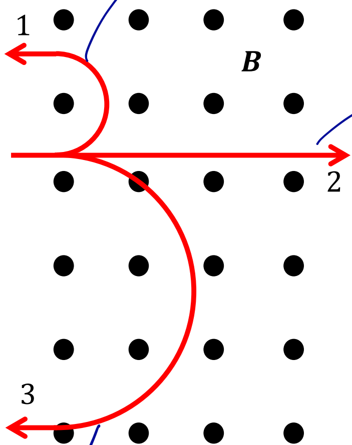
D. 1β , 2γ , 3α

E. 1γ , 2α , 3β

$r = \frac{mv}{qB}$ and direction

α is + β is -

$q = -$



γ

$q = +$

Question 17 65%

Two infinite parallel wires are placed at distance d. The current through the wires flows in the same direction, but is different in magnitude: $I_1 \neq I_2$. At what distance x from the wire number 1 between the wires is the magnetic field equal to zero.

A. $\frac{I_1 d}{I_1+I_2}$

B. $\frac{I_1 d}{I_1-I_2}$

C. $\frac{I_2 d}{I_1+I_2}$

D. $\frac{I_1+I_2 d}{I_1}$

E. Nowhere between the wires

$B = \frac{\mu_0 I}{2\pi r}$

$I_1 \xleftrightarrow{x} \quad \cdot \; I_2$, $\xleftrightarrow{d}$

$B_1 = \frac{\mu_0 I_1}{2\pi x}$

$B_2 = \frac{\mu_0 I_2}{2\pi (d-x)}$

$B_{tot} = 0 = \frac{I_1}{x} - \frac{I_2}{d-x} \rightarrow \frac{I_1}{x} = \frac{I_2}{d-x}$ Solve for x ...

Question 18 46%

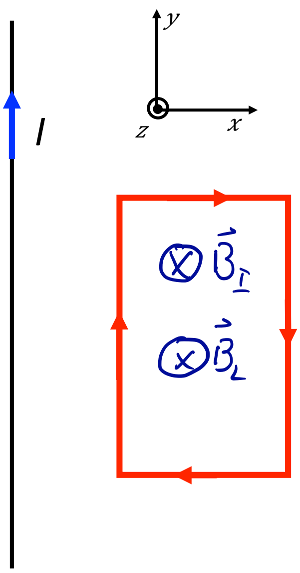
A square loop is placed next to a wire carrying a current along the y-direction. Along which direction should the loop be moved to induce a clockwise current in it as illustrated in the figure?

A. $+x$

B. $-x$

C. $+y$

D. $-y$



Response to $\vec{B}_I$ decreasing

$\rightarrow$ increase distance

$\rightarrow +x$

Question 19 52%

The so-called "μ-metal" is a material with very high relative magnetic permeability μ_r. Consider a magnetic field B in vacuum oriented with an angle α with respect to the axis shown in the figure below. Evaluate the new angle α' inside the μ-metal.

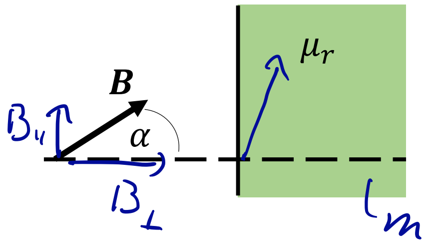
A. $\alpha\mu_r$

B. α/μ_r

C. $\arctan(\mu_r \tan\alpha)$

D. $\arctan(\frac{1}{\mu_r}\tan\alpha)$

$$\tan\alpha' = \frac{\mu_r B_\parallel}{B_\perp} = \mu_r \tan\alpha$$



$B_{\parallel m} = \mu_r B_\parallel$

$B_{\perp m} = B_\perp$

Question 20 40%

A ferromagnetic material is inserted in a solenoid where an alternate current at 50 Hz is passed. In the figure below the hysteresis cycle of the material is shown. Estimate the energy dissipated in the ferromagnet in one minute per unit volume.

A. 3 MJ

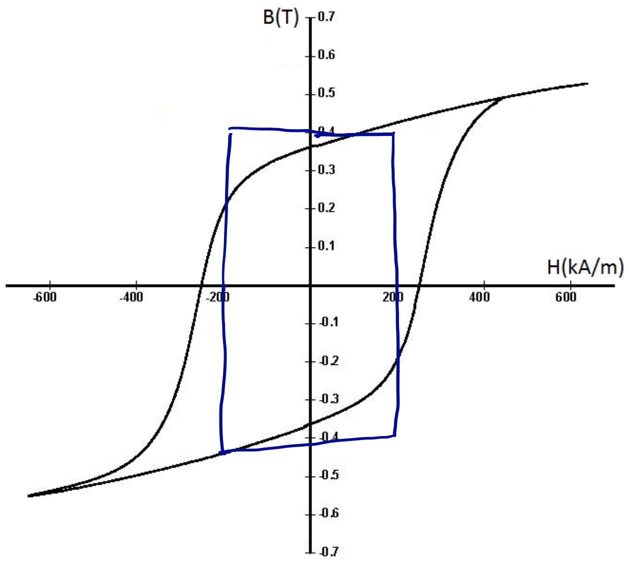
B. 20 MJ

C. 1 GJ

D. 50 GJ

$W \simeq 4 \cdot 0.4 \cdot 2\times10^5 = 3.2\times10^5$

$\text{Total} \simeq 3.2\times10^5 \cdot 50 \cdot 60 = 3.2\times10^5 \cdot 3\times10^3 \simeq 1\times10^9$



Question 21 91%

An infinite cylinder of a paramagnetic material with relative magnetic permeability μ_r has a uniform magnetisation $\vec{M}$. What is the magnitude of the H-field inside the cylinder.

A. $\frac{M}{\mu_r - 1}$

B. $\frac{M}{\mu_r}$

C. $(\mu_r - 1)M$

D. $\mu_r M$

$$M = \chi_m H = (\mu_r - 1) H$$

$$H = \frac{M}{\mu_r - 1}$$

Question 22 54%

The capacitor shown in the figure below is made of two parallel circular plates of radius R at distance d in vacuum and the potential between the plates is ramped up as $V(t) = V_0 t$. Evaluate the magnitude of the magnetic field at the point P at distance a from the capacitor axis.

A. $\frac{\mu_0 \epsilon_0 a V_0 R}{d}$

B. $\frac{2\mu_0 \epsilon_0 a V_0}{Rd}$

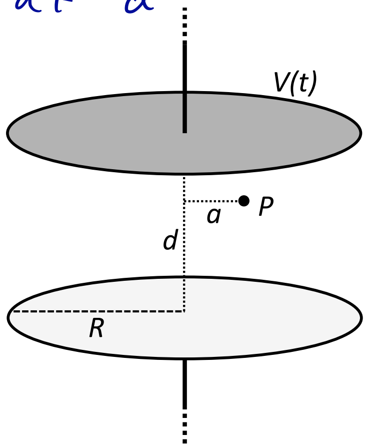
C. $\frac{\mu_0 \epsilon_0 a V_0}{2d}$

D. $\frac{2\mu_0 \epsilon_0 a V_0}{d}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{s}$$

$$2\pi a B = \mu_0 \varepsilon_0 \pi a^2 \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{dV}{dt} \cdot \frac{1}{d} = \frac{V_0}{d} \quad \rightarrow B = \frac{\mu_0 \varepsilon_0 a V_0}{2d}$$



Question 23 52%

The magnitude of the electric field of a monochromatic plane wave polarized along $\hat{x}$ is $E_0 \cos(y + ct)$. In which direction is the B field pointing at $t = 0$?

A. $\hat{z}$

B. $-\hat{z}$

C. $\hat{x}$

D. $\hat{y}$

E. $-\hat{y}$

$\vec{E} \times \vec{B} = -\hat{y}$ ← from $(y + ct)$

$\vec{E} \to \hat{x}$

Question 24 71%

In a non-conductive medium so-called Lecher lines can be used to determine the wavelength of radio waves. For a given frequency ν a wavelength of λ_0 is measured in air, whereas in another medium λ_r is measured for the same source of radio waves. What is is the relative permittivity ϵ_r of this medium.

A. $\frac{\lambda_0}{\lambda_r}$

B. $\left(\frac{\lambda_0}{\lambda_r}\right)^2$

C. $\sqrt{\frac{\lambda_0}{\lambda_r}}$

D. $\frac{\lambda_0 - \lambda_r}{\lambda_0 + \lambda_r}$

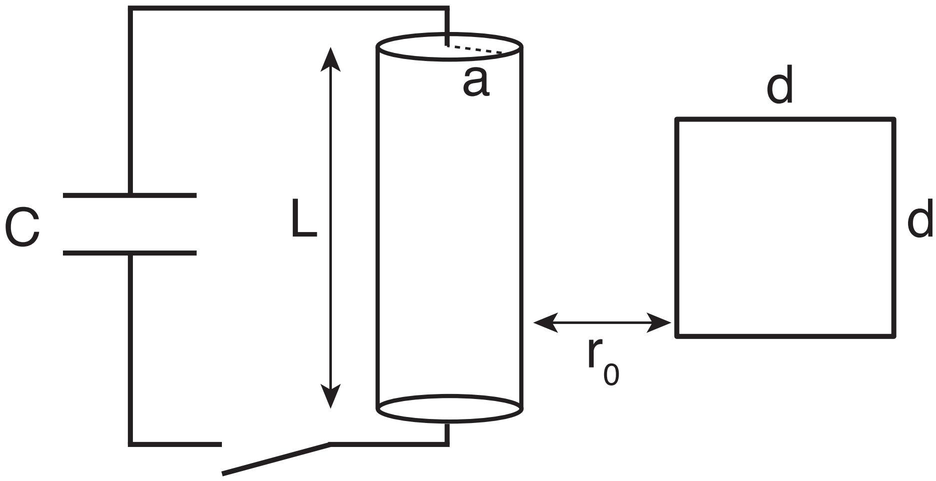
$\lambda = \frac{c}{\nu}$

$\varepsilon_r = \left(\frac{c}{c_m}\right)^2 = \left(\frac{\lambda_0}{\lambda_m}\right)^2$

NAME... N°SCIPIER.........................

Problem 1 < ? = 2 points

A cylindrical wire of a material with with conductivity σ, length L, and radius a is connected via a switch to a capacitor with capacitance C, as illustrated in the figure below.



a) Determine the resistance R of the wire.

$$R = \frac{\rho L}{A} = \frac{L}{\sigma \pi a^2}$$

$$R = \boxed{\frac{L}{\sigma \pi a^2}}$$

NAME... N°SCIPIER.........................

In the following the dimensions of the wire can be neglected and it can be treated as infinitely long with resistance R.

b) The capacitor is initially charged to a potential V_0 and at time $t = 0$ the switch is closed. Determine the magnitude of the magnetic field B as a function of time at a distance $r \gg a$ from the wire.

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{\tau}} \quad , \quad \tau = RC$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 V_0 e^{-\frac{t}{\tau}}}{R\, 2\pi r}$$

$$\boxed{B = \frac{\mu_0 V_0 e^{-\frac{t}{RC}}}{R\, 2\pi r}}$$

At a distance $r_0 \gg a$ from the wire a square loop with side length d is placed as shown in the figure.

c) What is the voltage $V(t)$ induced in the loop after the switch is closed at $t = 0$? (Hint: ignore relativistic effects and self-induction)

$$\Phi = d \int_{r_0}^{r_0+d} B\, dr = \frac{\mu_0 V_0 e^{-\frac{t}{\tau}} d}{R\, 2\pi} \int_{r_0}^{r_0+d} \frac{dr}{r} = \frac{\mu_0 V_0 e^{-\frac{t}{\tau}} d}{R\, 2\pi} \ln\left(\frac{r_0+d}{r_0}\right)$$

$$V = \frac{d\Phi}{dt} = \frac{1}{RC} \frac{\mu_0 V_0 e^{-\frac{t}{\tau}} d}{R\, 2\pi} \ln\left(\frac{r_0+d}{r_0}\right)$$

$$\boxed{V = \frac{\mu_0 V_0 d\, e^{-\frac{t}{RC}} \ln\left(\frac{r_0+d}{r_0}\right)}{R^2 C\, 2\pi}}$$

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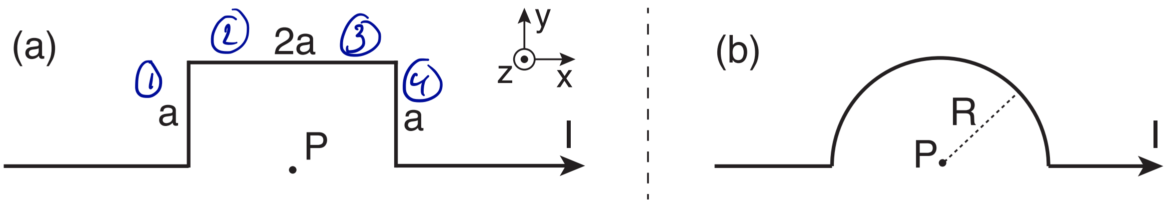
Problem 2 $< 7 = 1.5$ points

A current I is passed through a wire shaped as half a square of side $2a$ placed in the xy-plane as shown in the figure (a) below.



a) Determine the direction of the B-field at point P at the centre of the square.

$\hat{B} = -\hat{z}$

b) Evaluate the magnitude of the B-field at point P. (Hint: $\int \frac{dy}{(y^2+a^2)^{\frac{3}{2}}} = \frac{y}{a^2\sqrt{y^2+a^2}}$)

$$d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2} \qquad r = \sqrt{y^2+a^2}$$

$$B_1 = \frac{\mu_0 I a}{4\pi} \int_0^a \frac{dy}{(y^2+a^2)^{3/2}} = \frac{\mu_0 I}{4\pi \sqrt{2}\, a}$$

$$B = 4 \cdot B_1$$

$B = \dfrac{\mu_0 I}{\sqrt{2}\, \pi a}$

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c) We now replace the square wire with a semi circle as shown in figure (b) on the previous page. What should be the radius R of this semi circle in order to have the same magnitude of the B-field as for the square wire? (Use B_{sq} for the field of the square wire in case you did not manage to answer problem (b).)

$$B_e = \frac{\mu_0 I}{4R} \quad \left(\frac{1}{2} \text{ of single loop}\right)$$

$$\frac{\mu_0 I}{4R} = \frac{\mu_0 I}{\sqrt{2}\pi a} \longrightarrow R = \frac{\sqrt{2}}{4}\pi a$$

$$\frac{\mu_0 I}{4R} = B_{sq}$$

$$R = \frac{\mu_0 I}{4 B_{sq}}$$

$R = \dfrac{\sqrt{2}}{4}\pi a$

SCRAP PAPER. It will **not** be considered for grading.

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