1	**C**	7	**C**	13	**B**	19	**E**
2	**B**	8	**D**	14	**B**	20	**D**
3	**C**	9	**D**	15	**C**	21	**B**
4	**A**	10	**B**	16	**D**	22	**B**
5	**A**	11	**D**	17	**D**	23	**D**
6	**A**	12	**C**	18	**C**	24	**C**

1 point per correct answer
No punishment for wrong answer/guessing
1 point per correct part of open problem (also 1/2 points)

Total 30 points

Grade=1+(5*points/30)

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Problem 1

Consider a very long cylinder of radius a made by a material of relative permittivity ϵ_r and relative permeability μ_r. A beam of electrons (charge $-|q_e|$, mass m_e) travels in the cylinder (without being scattered by the material) with velocity v. The electrons are homogeneously distributed in the cylinder with volume density n_e.

a) Determine the electric field $\vec{E}$ in cylindrical coordinates inside the material.

$$\oiint \vec{D}\cdot d\vec{S} = \sum Q = -n_e q_e \pi r^2 L$$

$$2\pi r L D = -n_e q_e \pi r^2 L \longrightarrow \vec{D} = -\frac{n_e q_e r}{2}\hat{r}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$$

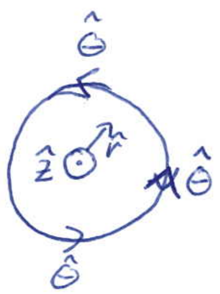
$$\boxed{\vec{E} = -\frac{n_e q_e r}{2\varepsilon_0 \varepsilon_r}\hat{r}}$$

b) Determine the magnetic field $\vec{B}$ in cylindrical coordinates inside the material.

$$\oint \vec{H}\cdot d\vec{L} = -n_e q_e v \pi r^2 \quad (= I)$$

$$2\pi r H = -n_e q_e v \pi r^2 \longrightarrow \hat{H} = -\frac{1}{2} n_e q_e v r \hat{\theta}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$



$$\boxed{\vec{B} = -\frac{1}{2}\mu_0 \mu_r n_e q_e v r \hat{\theta}}$$

(don't judge sign)

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Because of Coulomb and Lorentz forces, each electron will interact with the other electrons, so that the shape of the beam can change.

c) For what velocity v will the radius a of the electron beam not change?

$$F_c = qE = \frac{n_e q_e^2 r}{2 \varepsilon_0 \varepsilon_r}$$

$$F_L = q v \times B = \frac{n q^2 v^2 r \mu_0 \mu_r}{2}$$

$$|F_c| = |F_L|$$

$$\frac{\cancel{n} q^{\cancel{2}} \cancel{r}}{\cancel{2} \varepsilon_0 \varepsilon_r} = \frac{\cancel{n} q^{\cancel{2}} v^2 \cancel{r} \mu_0 \mu_r}{\cancel{2}}$$

$$v^2 = \frac{1}{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}$$

$$v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$\boxed{v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} \left(= \frac{c}{\sqrt{\varepsilon_r \mu_r}} \right)}$$

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Problem 2

A current I is passed through a square wire loop of side d as shown in the figure (a) below.

a) Determine the magnetic dipole moment associated to the loop (specify the direction).

$$\vec{m} = AI\,\hat{z} = d^2 I\,\hat{z}$$

$\boxed{\vec{m} = d^2 I\,\hat{z}}$

The loop is placed in a region with homogeneous magnetic field $\vec{B}$ tilted at an angle $\alpha = 30°$ as shown in the figure (b) below.

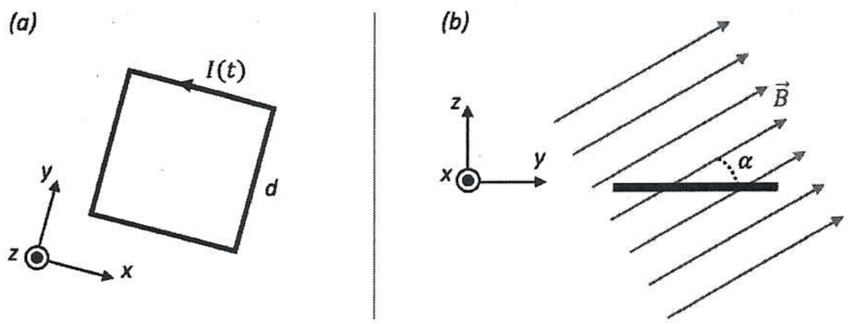
b) Evaluate the torque exerted on the loop (specify the direction).

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$$\vec{\tau} = -mB\cos\alpha\,\hat{x}$$

$$\vec{\tau} = \frac{-\sqrt{3}}{2} d^2 I B\hat{x}$$

$\boxed{\vec{\tau} = \frac{-\sqrt{3}}{2} d^2 I B\hat{x}}$



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The current I in the loop is switched off, the loop is rotated until $\alpha = 90°$, and the magnitude of the magnetic field starts to vary in time *continuously.* Let $d = \sqrt{2}\ m$. By measuring the voltage induced in the loop as a function of time, we obtain the plot shown in the figure below.

c) Make a sketch of the magnitude of the magnetic field as a function of time.

$$\Delta V = -\frac{d\Phi}{dt} = -\frac{d(Bd^2)}{dt} = -2\frac{dB}{dt}$$

$$-2B = \int_0^4 V\,dt = -4 \quad \rightarrow B(t=4) = 2T$$