Final Exam

30.01.2018

The time available for the exam is 3h. No calculators, books or slides are allowed, only one A4 handwritten paper with notes.
Write the correct answer (A,B,C,D or E) of each multiple-choice question in the following table. Write the final answers to the open questions in the provided space of each problem page.
Each question gives 1 point, as well as each step of the two open questions (total 30 points).

NAME..

N°SCIPER.........................

TABLE OF ANSWERS							
1		7		13		19	
2		8		14		20	
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4		10		16		22	
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PHYSICAL CONSTANTS

$$q_e = 1.6 \times 10^{-19}\ C$$

$$m_e = 9.1 \times 10^{-31}\ kg$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$

$$k = (4\pi\epsilon_0)^{-1} = 8.9 \times 10^{9}\ Nm^2/C^2$$

$$\mu_0 = 4\pi \times 10^{-7}\ Vs/(Am)$$

$$h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s$$

$$c = 3 \times 10^{8}\ m/s$$

TRIGONOMETRY

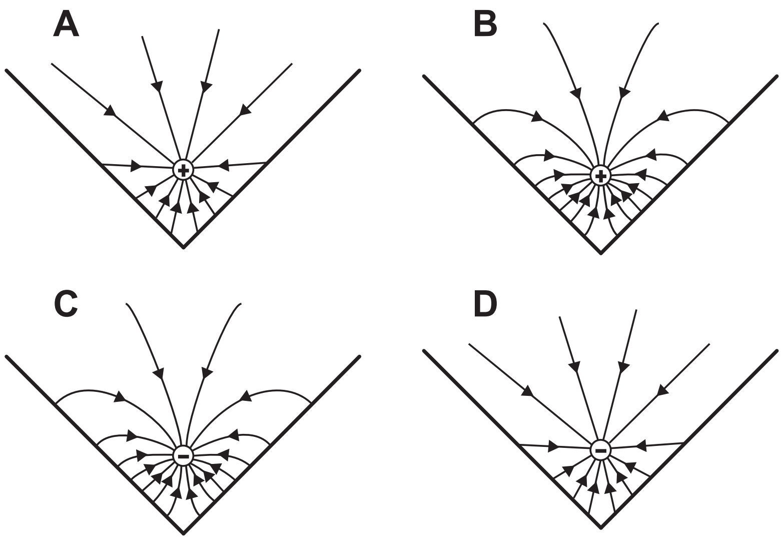
$$\cos 30^\circ = \sin 60^\circ = \sqrt{3}/2$$

$$\cos 45^\circ = \sin 45^\circ = \sqrt{2}/2$$

$$\cos 60^\circ = \sin 30^\circ = 1/2$$

Question 1

A charge is placed next to a grounded conductive plane shaped at an angle of 90°. Which of the following is the correct representation of the $\vec{E}$ field lines?



Question 2

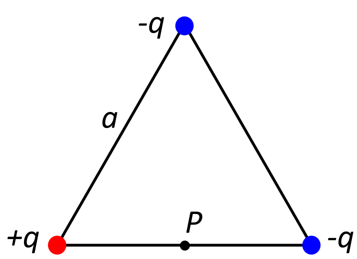
A positive charge $+q$ and two negative charges $-q$ are fixed on the corners of an equilateral triangle of side a, as shown in the figure below. Evaluate the magnitude of the electric field in the point P shown in the figure.

A. $\frac{q}{3\pi\epsilon_0 a^2}$

B. $\frac{q\sqrt{37}}{3\pi\epsilon_0 a^2}$

C. $\frac{3\sqrt{13}}{\pi\epsilon_0 a^2}$

D. $\frac{q\sqrt{3}}{\pi\epsilon_0 a^2}$



Question 3

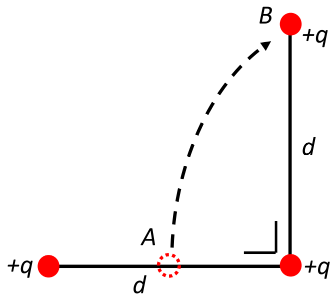
Two positive charges $+q$ are fixed at distance d. Evaluate the work done for moving a third charge $+q$ from point A halfway between the two charges to point B at $90°$ at distance d from one of the charges, as shown in the figure below.

A. $\frac{3q^2}{8\pi\epsilon_0 d}$

B. $\frac{q^2}{4\pi\epsilon_0 d}$

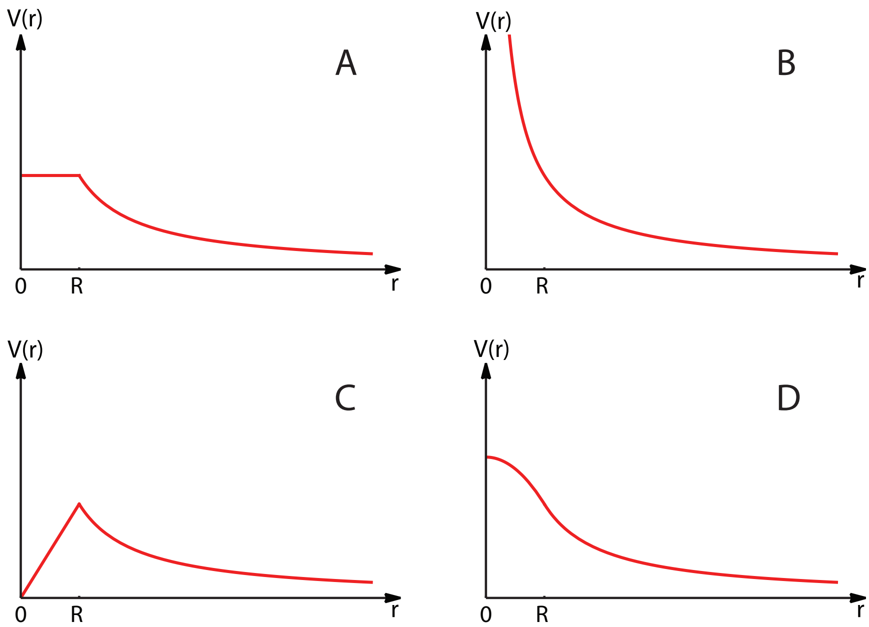
C. $\frac{(6-\sqrt{2})q^2}{8\pi\epsilon_0 d}$

D. $\frac{(3-\sqrt{2})q^2}{2\pi\epsilon_0 d}$



Question 4

A metallic sphere of radius R is charged with total charge Q. Which of the following is the correct plot of the electric potential as a function of distance from the center $V(r)$?



Question 5

A mixed beam of electrons e^- and positrons e^+ is sent with velocity $\vec{v} = v_0\hat{x}$ in a device of length ℓ with homogeneous electric field $\vec{E}$ along $\hat{y}$. Evaluate the magnitude of $\vec{E}$ that is required in order to have a separation d in the y direction at the end of the device between the position of the e^- and the position of the e^+. Neglect all the interactions between the particles (a e^+ has same mass but opposite charge of an e^-).

A. $\frac{dmv_0^2}{e\ell^2}$

B. $\frac{2dmv_0^2}{e\ell^2}$

C. $\frac{d^2mv_0^2}{e\ell}$

D. $\frac{2d^2mv_0^2}{e\ell}$

Question 6

Two charges $+q$ and $-q$ are rigidly connected at very small distance d in a region of space where there is an homogeneous electric field $\vec{E}$. The angle between the field lines and the line connecting the charges is α. Evaluate the torque acting on the system of charges.

A. $qdE\sin\alpha$

B. $2qdE\sin\alpha$

C. $qdE\cos\alpha$

D. $2qdE\cos\alpha$

E. 0

Question 7

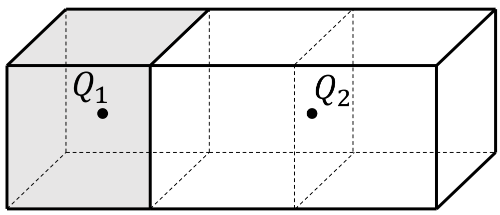
A charge Q_1 is placed in the center of a cube, as shown in the figure below. The flux of electric field through the surface of the cube is Φ_1. A rectangular cuboid made of two cubes identical to the previous one is placed next to the cube, as shown in the figure below. A second charge $Q_2 = -2Q_1$ is placed in the center of the cuboid. How large is the flux of electric field through the surface of the big rectangular cuboid made of three cubes?

A. $-\frac{7}{3}\Phi_1$

B. $-\frac{5}{3}\Phi_1$

C. $-\Phi_1$

D. $-2\Phi_1$



Question 8

The conductor shown in the figure below is at potential V. At what point just outside the object is the electric field the highest?

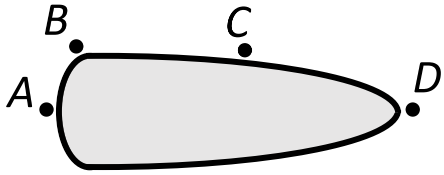
A. Point A

B. Point B

C. Point C

D. Point D

E. The electric field is the same at all points



Question 9

A charge $-q$ is placed at distance d from a large grounded conductive plane. What is the magnitude of the force experienced by the charge?

A. $\frac{q^2}{2\pi\epsilon_0 d^2}$

B. $\frac{q^2}{4\pi\epsilon_0 d^2}$

C. $\frac{q^2}{8\pi\epsilon_0 d^2}$

D. $\frac{q^2}{16\pi\epsilon_0 d^2}$

E. 0

Question 10

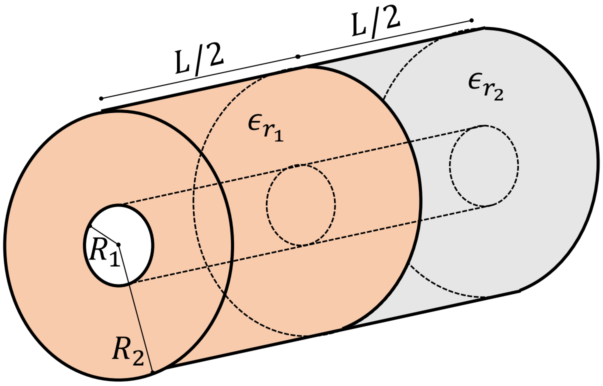
Consider a coaxial cable of length L and radii $R_1 < R_2$, where the two cylinders are homogeneously charged with linear charge density $+\lambda$ and $-\lambda$. The two halves of the cable are filled with dielectric materials with relative permittivity ϵ_{r_1} and ϵ_{r_2}, as shown in the figure below. Evaluate the capacitance of the cable.

A. $\frac{2\pi\epsilon_0 L\left(\epsilon_{r_1}+\epsilon_{r_2}\right)}{\ln\left(\frac{R_2}{R_1}\right)}$

B. $\frac{\pi\epsilon_0 L\left(\epsilon_{r_1}+\epsilon_{r_2}\right)}{\ln\left(\frac{R_2}{R_1}\right)}$

C. $\frac{2\pi\epsilon_0\epsilon_{r_1}\epsilon_{r_2}L}{\epsilon_{r_1}\ln\left(\frac{2R_2}{R_1+R_2}\right)+\epsilon_{r_2}\ln\left(\frac{R_1+R_2}{2R_1}\right)}$

D. $\frac{\pi\epsilon_0\epsilon_{r_1}\epsilon_{r_2}L}{\epsilon_{r_1}\ln\left(\frac{2R_2}{R_1+R_2}\right)+\epsilon_{r_2}\ln\left(\frac{R_1+R_2}{2R_1}\right)}$



Question 11

Consider 8 identical spherical metallic droplets of radius R that are made to coalesce into a single spherical droplet. Only three of the droplets were charged, with a potential of 1 V, 3 V and 4 V with respect to ground respectively. Evaluate the electric potential of the big droplet after the merging.

A. 1 V

B. 3 V

C. $\frac{8}{3}$ V

D. 4 V

Question 12

Consider an isolated parallel plate capacitor. The two plates have area A and are charged with surface charge density $+\sigma$ and $-\sigma$. Evaluate the magnitude of the force acting on the two plates.

A. $\frac{\sigma A}{2\epsilon_0}$

B. $\frac{2\epsilon_0}{\sigma A}$

C. $\frac{\sigma^2 A}{2\epsilon_0}$

D. $\frac{2\epsilon_0}{\sigma^2 A}$

Question 13

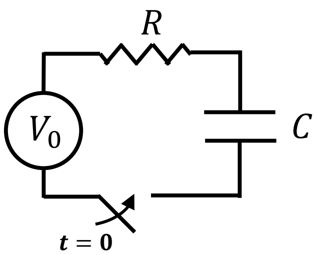
The RC circuit shown in the figure below is closed at the time $t = 0$ s. The values of the resistance, capacitance and voltage source are $R = 3$ kΩ, $C = 5$ μF and $V_0 = 2$ V respectively. What is the charge accumulated on the plates of the capacitor after a time $t = 15$ ms? (*Hint*: $e \approx 2.7$, $\frac{1}{e} \approx 0.37$)

A. 3.7 μC

B. 6.3 μC

C. 3.7 mC

D. 6.3 mC



Question 14

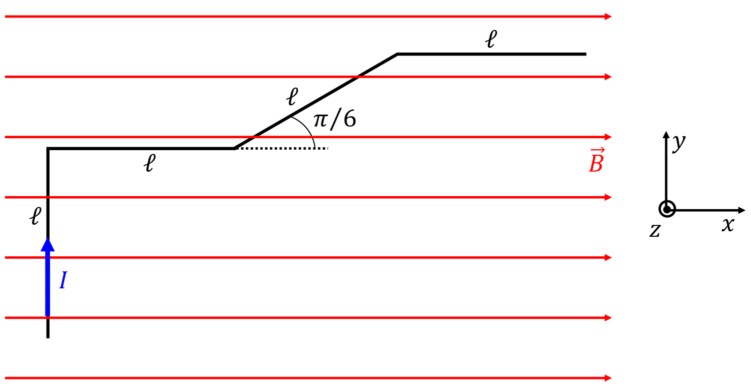
A current I is passed through a wire made of four segments of length ℓ shaped as shown in the figure below. Evaluate the force acting on the wire placed in a homogeneous magnetic field $\vec{B} = B_0\hat{x}$.

A. $\frac{3I\ell B_0}{2}\,(+\hat{z})$

B. $\frac{3I\ell B_0}{2}\,(-\hat{z})$

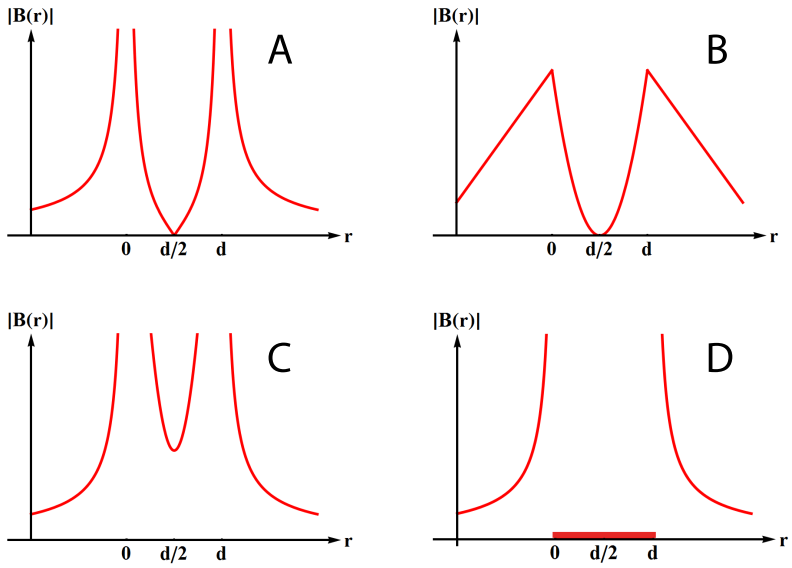
C. $\frac{(2+\sqrt{3})I\ell B_0}{2}\,(+\hat{z})$

D. $\frac{(2+\sqrt{3})I\ell B_0}{2}\,(-\hat{z})$



Question 15

The same current I flows in opposite directions through two infinite parallel wires placed at distance d. Choose the correct plot of the module of the magnetic field in the plane of the two wires placed at 0 and d.



Question 16

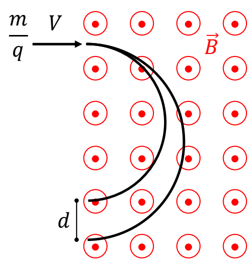
A beam of particles of charge q and mass m are accelerated by a voltage V and sent into a region with $\vec{B}$ field perpendicular to their trajectory, as shown in the figure below. A separation in space d is measured at the end of the magnetic field region when two different fields are used, $\vec{B}_1$ and $\vec{B}_2 = \vec{B}_1/2$. Determine the acceleration voltage V as a function of B_1, d, m, q.

A. $\frac{qdB_1}{8m}$

B. $\frac{3qdB_1}{4m}$

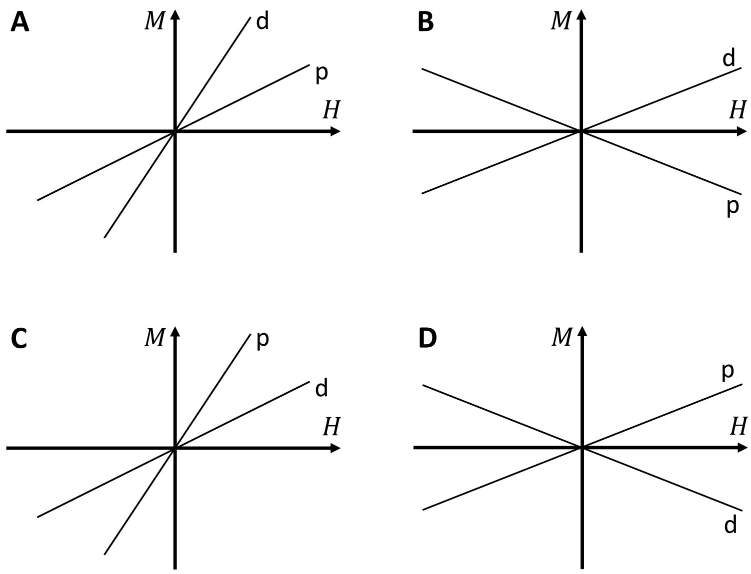
C. $\frac{3qd^2B_1^2}{4m}$

D. $\frac{qd^2B_1^2}{8m}$



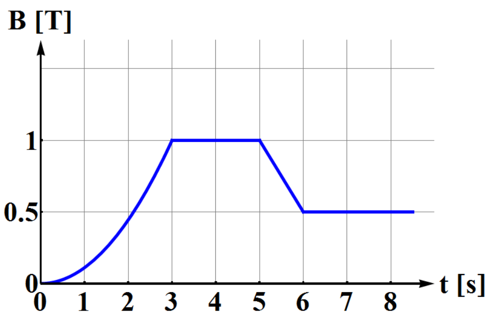
Question 17

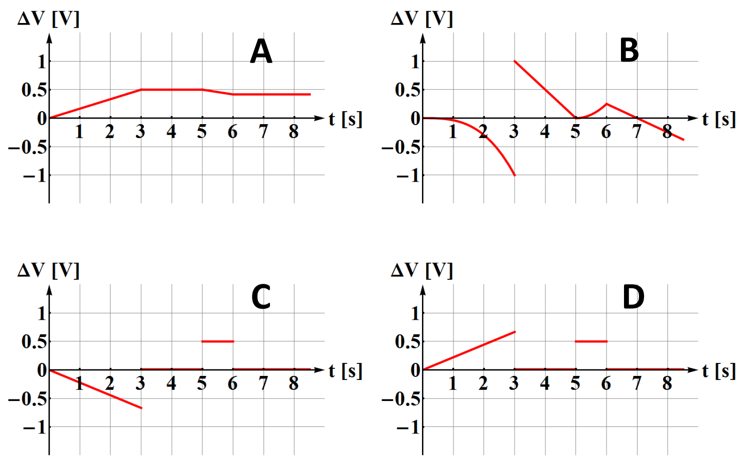
Consider a paramagnetic (p) material and a diamagnetic (d) material. Which of the following plots describes the correct trend of magnetization $\vec{M}$ versus applied $\vec{H}$ field for the two materials?



Question 18

In the figure below the magnitude of the magnetic field perpendicular to a wire loop of unitary area as a function of time is shown. Which is the correct plot of the potential difference induced in the wire?





Question 19

A wire loop of resistance R shaped as an equilateral triangle of side d is placed in the same plane as a straight long wire without touching it, as shown in the figure below. Evaluate the current that flows in the loop when a time-varying current $I(t)$ flows through the long wire.

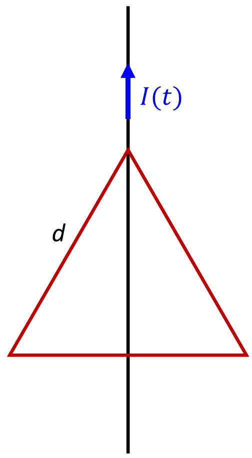
A. $-\frac{\mu_0 d\sqrt{3}}{2R}\frac{d}{dt}I(t)$

B. $-\frac{\mu_0 \pi d^2\sqrt{3}}{2R}\frac{d}{dt}I(t)$

C. $-\frac{\mu_0 d\sqrt{3}}{2R}\int_0^t I(t)\,dt$

D. $-\frac{\mu_0 \pi d^2\sqrt{3}}{2R}\int_0^t I(t)\,dt$

E. 0



Question 20

A straight solenoid of length L is made of N windings. A magnetic material is inserted in the solenoid. When a current I is passed in the solenoid, a Hall sensor measures a magnetic field B_m in the material. Evaluate the relative permeability μ_r of the material.

A. $\frac{\mu_0 LI}{B_m N}$

B. $\frac{B_m N}{\mu_0 LI}$

C. $\frac{\mu_0 NI}{B_m L}$

D. $\frac{B_m L}{\mu_0 NI}$

Question 21

Consider the function $\psi = A\left(Bx + Ct\right)^2 + D\sin\left(Ex + Ft\right)$. For which of the following conditions is ψ a solution of the wave equation?

A. $C = F$ and $A = D$

B. $\frac{C}{B} = \frac{F}{E}$

C. $B = E$ and $A = D$

D. $\frac{C}{B} = \frac{E}{F}$

Question 22

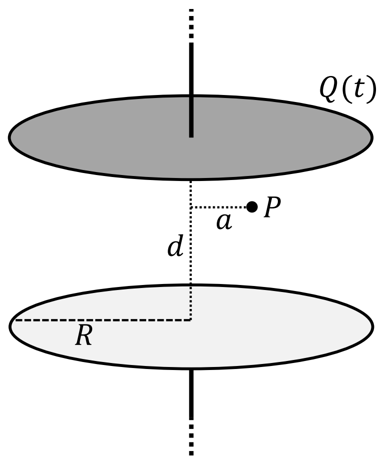
The capacitor shown in the figure below is made of two parallel circular plates of radius R at distance d in vacuum and is connected to a AC source, so that the charge on a plate varies as $Q(t) = Q_0 \sin\left(\omega t\right)$. Evaluate the magnitude of the magnetic field at the point P at distance a from the capacitor axis.

A. $\frac{\mu_0 Q_0 \omega \cos\left(\omega t\right) R}{2\pi a^2}$

B. $\frac{\mu_0 Q_0 \omega \cos\left(\omega t\right) a}{2\pi R^2}$

C. $\frac{\mu_0 Q_0 \omega \cos\left(\omega t\right) R}{2\pi a d}$

D. $\frac{\mu_0 Q_0 \omega \cos\left(\omega t\right) a}{2\pi d R}$



Question 23

A monochromatic plane wave is linearly polarized along the $\hat{x}$ direction, and the magnitude of its electric field is $E = E_0 \cos(z + ct)$. What is the expression of the magnetic field $\vec{B}$ of the wave?

A. $\frac{E_0}{c} \cos(y + ct)\hat{x}$

B. $\frac{E_0}{c} \cos(y + ct)\hat{z}$

C. $\frac{E_0}{c} \cos(z + ct)\hat{x}$

D. $\frac{E_0}{c} \cos(z + ct)\hat{y}$

Question 24

An electric dipole oscillates in time according to $p(t) = 2 \cdot 10^{-20} \sin(2\pi 200\,[MHz]\,t)\,[C \cdot m]$. Evaluate the energy of the emitted electromagnetic wave.

A. $3.31 \cdot 10^{-7}$ eV

B. $4.14 \cdot 10^{-7}$ eV

C. $8.28 \cdot 10^{-7}$ eV

D. $13.2 \cdot 10^{-7}$ eV

NAME... N°SCIPIER.........................

Problem 1

Consider a very long cylinder of radius a made by a material of relative permittivity ϵ_r and relative permeability μ_r. A beam of electrons (charge $-|q_e|$, mass m_e) travels in the cylinder (without being scattered by the material) with velocity v. The electrons are homogeneously distributed in the cylinder with volume density n_e.

a) Determine the electric field $\vec{E}$ in cylindrical coordinates inside the material.

$\vec{E} =$

b) Determine the magnetic field $\vec{B}$ in cylindrical coordinates inside the material.

$\vec{B} =$

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Because of Coulomb and Lorentz forces, each electron will interact with the other electrons, so that the shape of the beam can change.

c) For what velocity v will the radius a of the electron beam not change?

$v =$

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Problem 2

A current I is passed through a square wire loop of side d as shown in the figure (a) below.

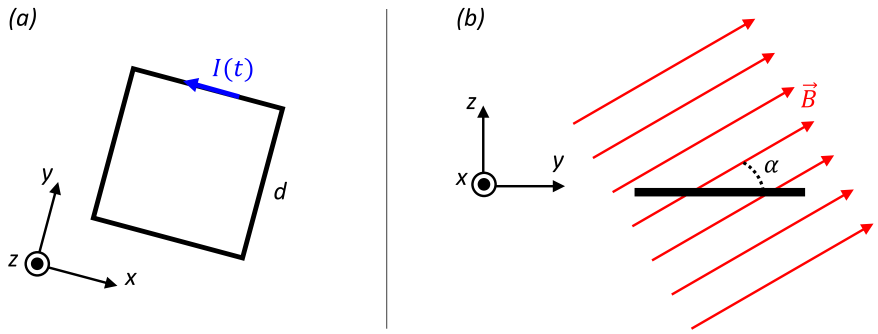
a) Determine the magnetic dipole moment associated to the loop (specify the direction).

$\vec{m} =$

The loop is placed in a region with homogeneous magnetic field $\vec{B}$ tilted at an angle $\alpha = 30°$ as shown in the figure (b) below.

b) Evaluate the torque exerted on the loop (specify the direction).

$\vec{\tau} =$



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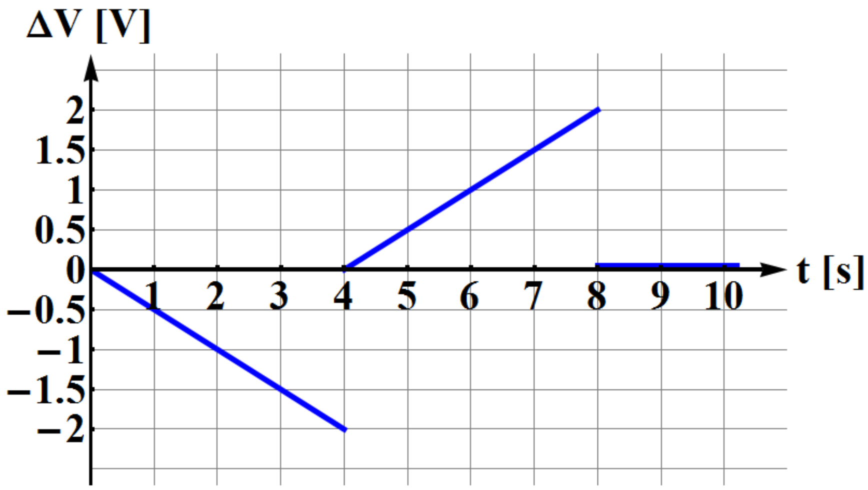
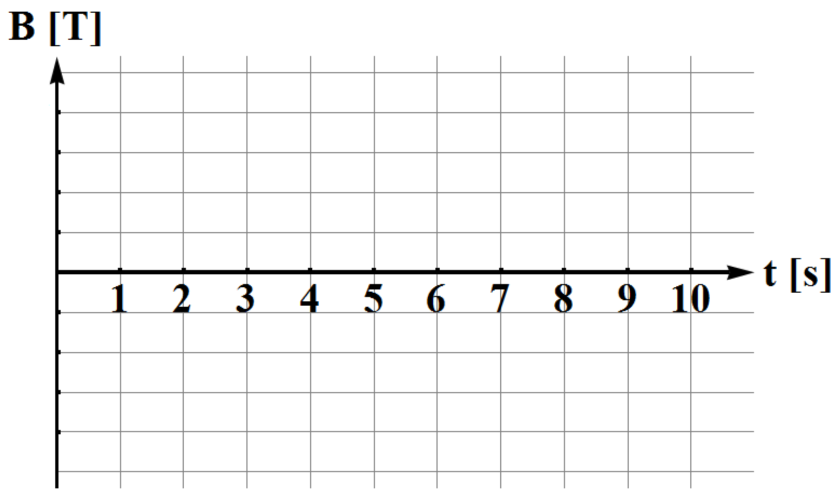
The current I in the loop is switched off, the loop is rotated until $\alpha = 90°$, and the magnitude of the magnetic field starts to vary in time *continuously*. Let $d = \sqrt{2}\ m$. By measuring the voltage induced in the loop as a function of time, we obtain the plot shown in the figure below.

c) Make a sketch of the magnitude of the magnetic field as a function of time.





SCRAP PAPER. It will **not** be considered for grading.

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