Exercise Sheet 3

Discussion 25.09.2024

Exercise 1 - Airplane take-off

a) Estimate what speed an Airbus 380 needs to take off if the wings are designed such that the air velocity above the wing is 20% larger as below the wing. The area of each wing is $A = 425\ \mathrm{m}^2$ and the maximum take off weight $m = 560$ tons. Take $\rho = 1.3\ \mathrm{kg/m}^3$ for the air density.

b) Do you think the real take-off speed is lower or higher, and why?

Discussion 1 - Streamlines

Sketch possible streamlines around and in a house when there is a slight wind that causes the door to slam when the windows at opposite sides are open. How do you interpret the streamline density and its spatial variations?

Exercise 2 - Streamlines in 2D

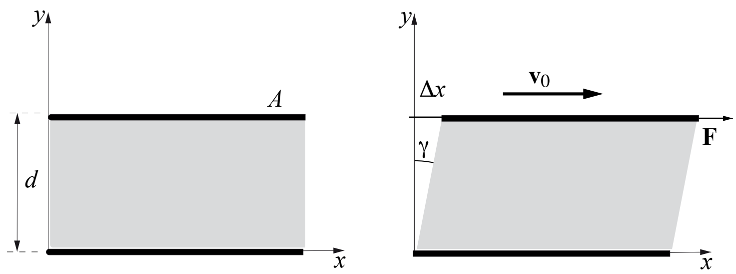
Determine the analytical expression for the streamlines and the acceleration $\vec{a}(x, y)$ for a stationary bidimensional flow described by the velocity field :

$$\vec{v} = (\frac{v_0}{l})(x\vec{e}_x - y\vec{e}_y) \ .$$

Make a sketch of the streamlines and the lines of constant acceleration.

Exercise 3 - Viscosity, shear stress, and shear strain

Consider a fluid with viscosity η between two plates (distance between the plates d, in the y direction). A force F is applied to the upper plate, which moves at constant velocity v_0 in the x direction. We observe a constant velocity gradient in the fluid between the bottom stationary plate and the moving plate at the top.

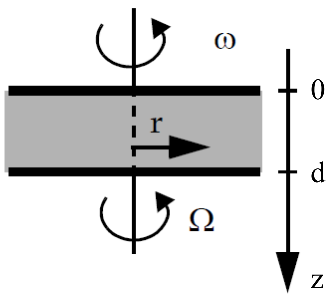


a) Show that the shear stress S_{yx} is proportional to the gradient of the velocity in the y direction.

b) The shear strain is given by $e_{yx} = \dfrac{\Delta x}{h}$. Rewrite the expression for S_{yx} in terms of $\dot{e}$, i.e. the shear rate.

Exercise 4 - Viscous drag

Two identical disks of radius R are able to rotate without friction around their axis: see figure. They are separated by a small distance d and the fluid between the disks has a viscosity η.
The top disk rotates at constant angular velocity ω and the bottom one is initially at rest.



Determine the temporal dependency $\Omega(t)$ describing the rotation of the second disk if we assume that the shear rate $\frac{\partial v}{\partial z}$ depends only on r (distance from the axis) and possibly on t.

Hints:
- the non-slip boundary condition is satisfied: the velocity of the fluid in contact with each disk is equal to that of the disk;
- the moment of inertia of each disk with respect to its axis is I.
- note that here Ω is not the vorticity.