Exercise Sheet 2

Discussion 18.09.2024

Exercise 1 - Jet d'eau

The water jet of Geneva is 140 m high.

a) Calculate the water velocity at bottom of the jet. We will suppose that the nozzle has an adapted profile, such that the velocity vector is vertical and constant on the whole jet section.

b) What is the water pressure at the bottom of the jet?

Discussion 1 - Steady flow and acceleration

Explain how for a steady flow the acceleration, as described in the lecture, can be non-zero. What would be a flow with zero acceleration ? Can you imagine a flow that is not steady but has zero acceleration?

Exercise 2 - Tank with a hole

The bottom of a cylindrical tank of radius R is perforated with a circular hole of radius r. The tank is filled up to a certain height h with an ideal and incompressible fluid of density ρ.

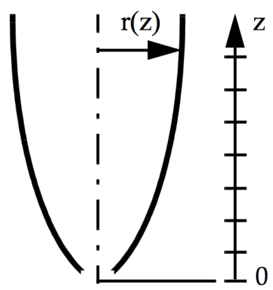
a) Derive the time T_0 necessary to empty the tank for $r \ll R$

b) Derive T_0 for $0 < r < R$

Hypothesis: the flow is quasi-steady.

Exercise 3 - Egyptian water clock

In ancient Egypt, time was measured with a water clock made from a vessel filled with an ideal and incompressible fluid - and pierced with a hole on the bottom. The level of the fluid decreases at equal time intervals with respect to the equidistant graduations marked on the vertical axis. Derive the geometry $r(z)$ of the vessel - knowing that it has a vertical revolution axis.



Exercise 4 - Vorticity vector

Consider the following velocity profiles:

a) $\vec{v} = (v_x(z),0,0)$, with $v_x(z) = Cz$, in cartesian coordinates $(x,\ y,\ z)$

b) $\vec{v} = (0,v_\phi(r),0)$, with $v_\phi(r) = \omega r$, in cylindrical coordinates $(\hat{e}_r,\ \hat{e}_\phi,\ \hat{e}_z)$

c) $\vec{v} = (0,v_\phi(r),0)$, with $v_\phi(r) = C/r$, in cylindrical coordinates $(\hat{e}_r,\ \hat{e}_\phi,\hat{e}_z)$

For each case, draw the velocity field and derive the vorticity $\vec{\Omega}$. Is the flow rotational or irrotational ? Is there a rotation of the fluid on a local and/or global scale ?