Exercise Sheet 1

Discussion 11.09.2024

Exercise 1 - Vector calculus with ∇

With the vector field $\vec{A} = (A_x, A_y, A_z)$ and scalar field B write out the expression for

a) the divergence $(\nabla \cdot \vec{A})$;

b) the curl $(\nabla \times \vec{A})$

c) $(\vec{A} \cdot \nabla)\vec{A}$

d) $\nabla \times (\nabla B)$

Exercise 2 - Divergence and curl of fields

Consider the vector fields $\vec{A}$ drawn in the figure on the next page and described by the expressions below. Indicate whether the divergence $(\nabla \cdot \vec{A})$ and curl $(\nabla \times \vec{A})$ are zero or not. Assume that $\vec{A}$ is 2D, or shows no dependence on x_3.
As indicated in the lecture you can use the following expressions to help visualise the divergence and curl: $\nabla \cdot \vec{A} = \oiint \vec{A} \cdot d\vec{S}$ and $\nabla \times \vec{A} = \oint \vec{A} \cdot d\vec{l}$

e) $\vec{A} = (x_2 - x_1, -x_2)$

f) $\vec{A} = (x_2, x_1)$

Exercise 3 - Pressure in hydrostatics

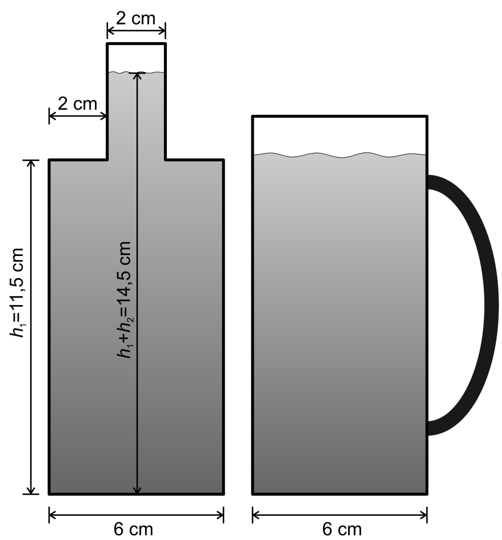
Use the absence of shear stress (cisaillement) in a static fluid to show that the pressure is isotropic even if it is not homogeneous (Pascal law). Suggestion : consider the equilibrium of a corner of fluid with an infinitesimal size.

Exercise 4 - Hydrostatic pressure in a bottle

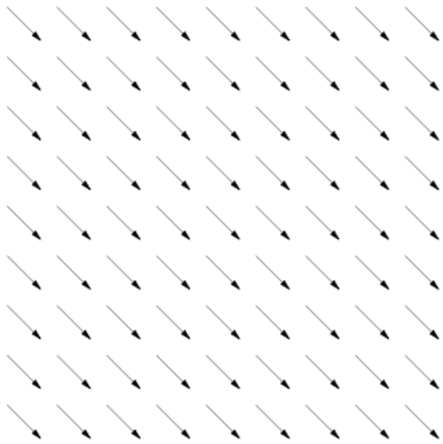
To celebrate the first physics lecture of the year, you decide to reward yourself with a drink. While pouring the beer, or water, from the bottle you think about the hydrostatic pressure. You realise that the hydrostatic pressure exerts a larger force on the bottom of the bottle as on the bottom of the glass.

a) Considering the drawing, calculate this difference in force.

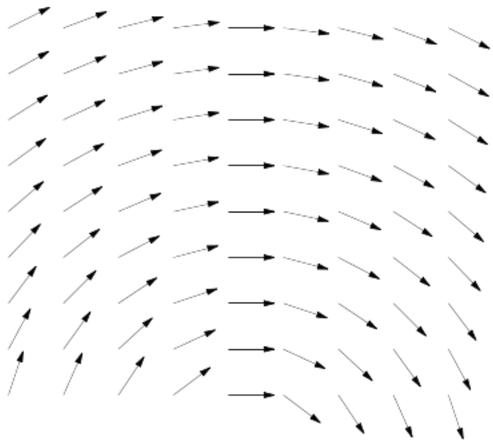
b) Your intuition (rightfully) tells you that in both cases it should show the same weight on a scale, if the weight of the empty glass/bottle is accounted for. Explain this apparent paradox, and also prove it mathematically.



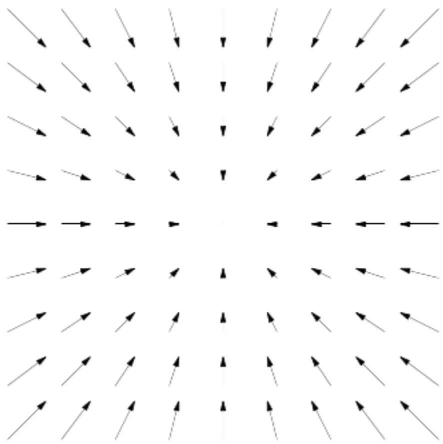
a) $\vec{A} = \overrightarrow{\text{constante}}$



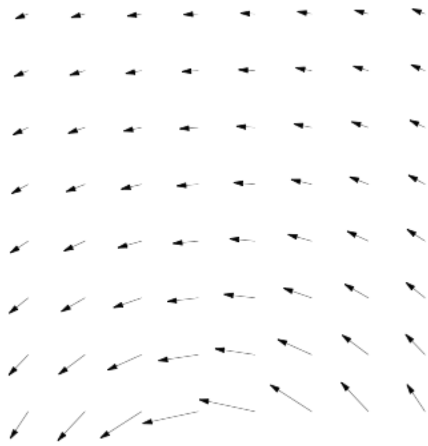
b) $\|\vec{A}\| = \text{constante}$



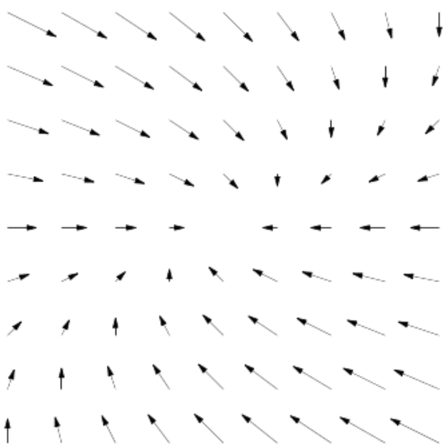
c) $\|\vec{A}\| \propto \sqrt{x_1^2 + x_2^2}$



d) $\|\vec{A}\| \propto 1/\sqrt{x_1^2 + x_2^2}$



e)



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