Exercise Sheet 10

Discussion 20.11.2024

Exercise 1 - Semicircle Wire

A closed wire is made up with two semicircles and two straight wires as shown in Fig. 1. The two semicircles have radii $2R$ and R. The straight wires connect the two semicircles and are placed in the diameter's direction. A current I is circulating through the wire. Calculate the magnetic field at the centre of the semicircles.

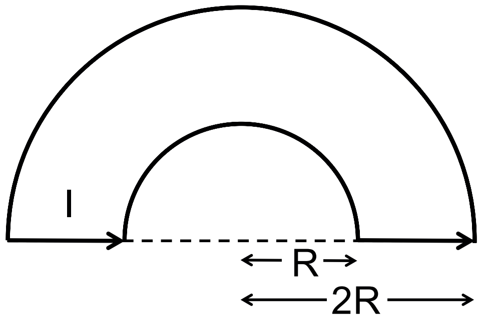


Figure 1: A closed wire made up of two semicircles..

Exercise 2 - Moving Charge

A wire is shaped in a circle of radius a and is charged with charge Q uniformly distributed along the wire.

a) The wire rotates around the axis through the center of the circle with angular velocity ω. Evaluate the $\vec{B}$ field at the center.

A thin disk of radius a is charged with charge Q uniformly distributed on the surface.

b) The disk rotates around the axis through its center with angular velocity ω. Evaluate the $\vec{B}$ field at the center.

Exercise 3 - Helmholtz coils

A Helmholtz coil consists of two rings separated by a distance $2h$ and traversed by the same current I.

a) Determine the B-field on the axis of the ring as a function of I, h and their radius R.

b) For what distance between the rings is the B-field around the centre of the system uniform?

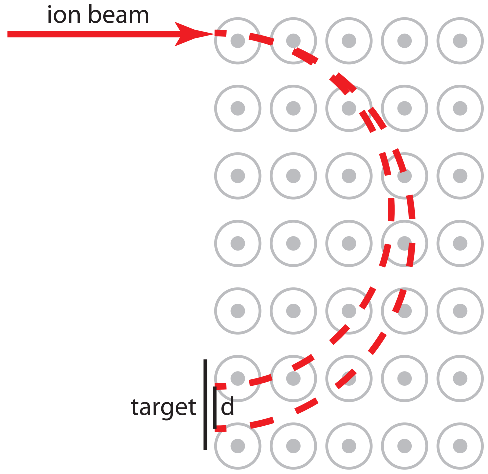
Exercise 4 - Mass Spectrometer

In order to separate the fissile Uranium isotope U_{235} from the heavier most common non-fissile U_{238}, a mixed beam of U_{235} and U_{238} ions is sent into a mass spectrometer. Here the ions are first accelerated and then shot into a homogeneous magnetic field region, with $B = 0.5\ T$ perpendicular to the beam, as shown in the figure below. The charge of both U isotope ions is $q = 3.2 \cdot 10^{-19}\ C$, and the acceleration voltage is $100\ kV$ ($m_{U_{235}} = 3.903 \cdot 10^{-25}\ kg$, $m_{U_{238}} = 3.953 \cdot 10^{-25}\ kg$).

a) Evaluate the different speeds of the two ion species $v_{U_{235}}$ and $v_{U_{238}}$ when they enter the magnetic field region.

b) Evaluate the spatial separation d between the two isotopes when they exit the mass spectrometer after the semicircular path in the magnetic field region.

c) What acceleration voltage would be needed in order to get a separation of $d = 2\ cm$?



Exercise 5 - Electron beam

Recall the exercise 1 of sheet n. 5: a beam of electrons with cylindrical shape, infinitely long and with radius a, with a given electron volume density n_e.

a)] By using Gauss's law (re)determine the $\vec{E}$ field at distance $r < a$.

Now we should take into account that the electrons are moving with uniform velocity v along the length of the cylinder.

b) By using Ampere's law determine the $\vec{B}$ field at distance $r < a$.

b) Evaluate the *total* force acting on each electron in the beam. For what conditions can a beam with constant radius be obtained?