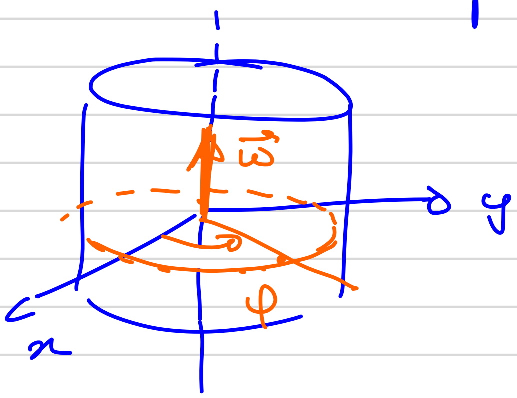
Coordonnés cylindriques : utilisation du vecteur rotation pour retrouver $\dot{\vec{e}}_\rho$ et $\dot{\vec{e}}_\varphi$



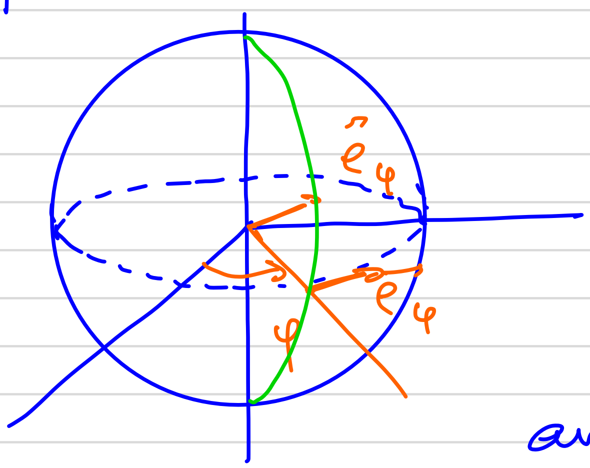
$$\vec{\omega} = \dot{\varphi}\,\vec{e}_3$$

$$\dot{\vec{e}}_\rho = \vec{\omega} \wedge \vec{e}_\rho = \dot{\varphi}\,\vec{e}_3 \wedge \vec{e}_\rho = \dot{\varphi}\,\vec{e}_\varphi$$

$$\dot{\vec{e}}_\varphi = \vec{\omega} \wedge \vec{e}_\varphi = \dot{\varphi}\,\vec{e}_3 \wedge \vec{e}_\varphi = -\dot{\varphi}\,\vec{e}_\rho$$

Coordonnés sphériques

Ici le vecteur rotation est un peu plus difficile à "trouver". On a 2 rotations



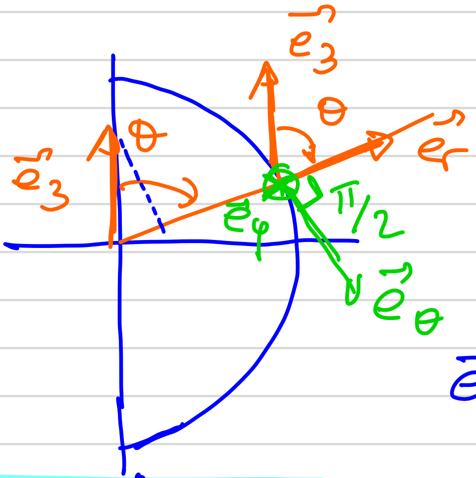
- 1 rotation autour de $\vec{e}_3$ avec $\omega = \dot{\varphi}$ $\Rightarrow \vec{\omega}_z = \dot{\varphi}\,\vec{e}_3$
- 1 rotation autour de $\vec{e}_\varphi$ avec $\omega = \dot{\theta} \Rightarrow \vec{\omega}_\varphi = \dot{\theta}\,\vec{e}_\varphi$

$$\vec{\omega} = \dot{\varphi}\,\vec{e}_3 + \dot{\theta}\,\vec{e}_\varphi$$

$$\dot{\vec{e}}_r = \vec{\omega} \wedge \vec{e}_r = \left(\dot{\varphi}\,\vec{e}_3 + \dot{\theta}\,\vec{e}_\varphi\right) \wedge \vec{e}_r$$

$$= \dot{\varphi}\,\vec{e}_3 \wedge \vec{e}_r + \dot{\theta}\,\underbrace{\vec{e}_\varphi \wedge \vec{e}_r}_{\vec{e}_\theta}$$

"plan de la porte":



$$\vec{e}_3 \wedge \vec{e}_r = \sin\theta\,\vec{e}_\varphi$$

$$\vec{e}_3 \wedge \vec{e}_\theta = \underbrace{\sin\left(\theta + \frac{\pi}{2}\right)}_{\cos\theta}\vec{e}_\varphi$$

$$\vec{e}_3 = \cos\theta\,\vec{e}_r - \sin\theta\,\vec{e}_\theta$$

$$\dot{\vec{e}}_r = \dot{\varphi}\sin\theta\,\vec{e}_\varphi + \dot{\theta}\,\vec{e}_\theta$$

$$\dot{\vec{e}}_\theta = \vec{\omega} \wedge \vec{e}_\theta = \left(\dot{\varphi}\,\vec{e}_3 + \dot{\theta}\,\vec{e}_\varphi\right) \wedge \vec{e}_\theta$$

$$= \dot{\varphi}\,\underbrace{\vec{e}_3 \wedge \vec{e}_\theta}_{\cos\theta\,\vec{e}_\varphi} + \dot{\theta}\,\underbrace{\vec{e}_\varphi \wedge \vec{e}_\theta}_{-\vec{e}_r}$$

$$\dot{\vec{e}}_\theta = -\dot{\theta}\,\vec{e}_r + \dot{\varphi}\cos\theta\,\vec{e}_\varphi$$

$$\dot{\vec{e}}_\varphi = \vec{\omega} \wedge \vec{e}_\varphi = \left(\dot{\varphi}\,\vec{e}_3 + \dot{\theta}\,\vec{e}_\varphi\right) \wedge \vec{e}_\varphi = \dot{\varphi}\,\vec{e}_3 \wedge \vec{e}_\varphi + \underbrace{\cancel{\dot{\theta}\,\vec{e}_\varphi \wedge \vec{e}_\varphi}}_{\vec{0}}$$

$$= \dot{\varphi}\left(\cos\theta\,\vec{e}_r - \sin\theta\,\vec{e}_\theta\right) \wedge \vec{e}_\varphi$$

$$\dot{\vec{e}}_\varphi = -\dot{\varphi}\cos\theta\,\vec{e}_\theta - \dot{\varphi}\sin\theta\,\vec{e}_r$$