XI. Application du solide indéformable

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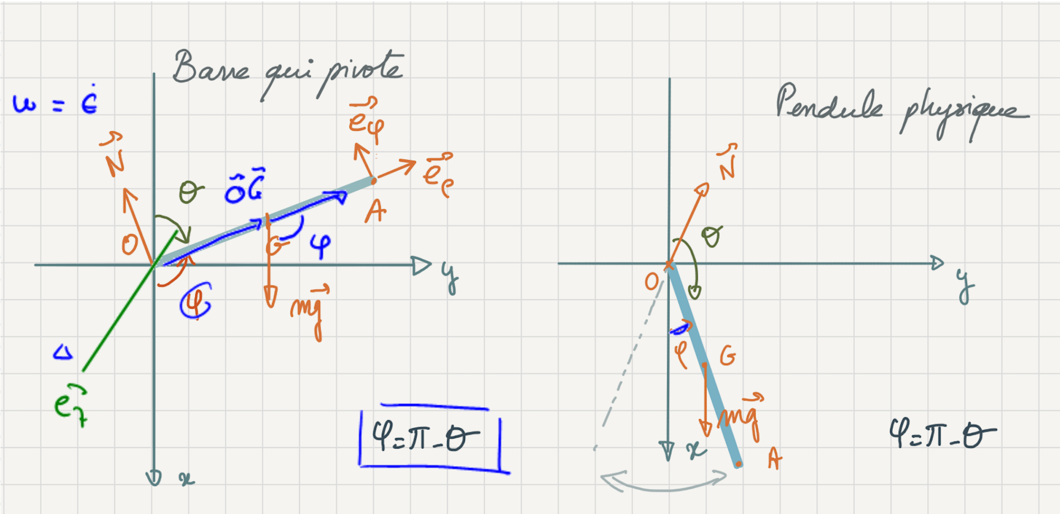
EPFL

Plan du cours

Table des matières

Barre homogène de masse *m* et de longueur *l* pivotant autour de 0, point fixe.



$$I_\Delta = \left(\frac{l}{2}\right)^2 \cdot m + I_G$$

Forces : $\vec{N} + m\vec{g} = M\vec{a}_G$ $\qquad \vec{M}_O = \frac{d\vec{L}_O}{dt} \qquad \vec{L}_O = I_a\,\vec{\omega}$

Moments de forces : $\vec{M}_O = \vec{OG} \times m\vec{g} = \frac{\ell}{2}\cdot mg \cdot \sin\varphi\,(-\vec{e}_z)$

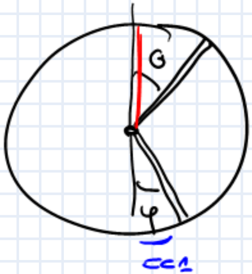
θ	φ
	$\varphi = \pi - \theta$
$\vec{M}_O = -\frac{\ell m g}{2}\sin\theta\,\vec{e}_z$	$\vec{M}_O = -\frac{\ell}{2}\,mg\,\sin(\varphi)\,\vec{e}_z \qquad \dot{\varphi} = -\dot{\theta}$
$\vec{\omega} = -\dot{\theta}\,\vec{e}_z$	$\vec{\omega} = \dot{\varphi}\,\vec{e}_z$

$$I_a = \left(\frac{\ell}{2}\right)^2 \cdot m + I_G = \frac{1}{2}m\ell^2 + \frac{\ell^2}{4}m = \frac{1}{3}m\ell^2$$

$\vec{L}_O = -\frac{1}{3}m\ell^2\dot{\theta}\,\vec{e}_z$	$\vec{L}_O = \frac{1}{3}m\ell^2\dot{\varphi}\,\vec{e}_z$
$\frac{d\vec{L}_O}{dt} = -\frac{1}{3}m\ell^2\ddot{\theta}\,\vec{e}_z$	$\frac{d\vec{L}_O}{dt} = \frac{1}{3}m\ell^2\ddot{\varphi}\,\vec{e}_z$
$-\frac{\cancel{\ell}\cancel{m}g}{2}\sin\theta\,\cancel{\vec{e}_z} = -\frac{1}{3}\cancel{m}\ell^{\cancel{2}}\ddot{\theta}\,\cancel{\vec{e}_z}$	$-\frac{\cancel{\ell}}{2}\cancel{m}g\sin\varphi\,\cancel{\vec{e}_z} = \frac{1}{2}\cancel{m}\ell^{\cancel{2}}\ddot{\varphi}\,\cancel{\vec{e}_z}$
$\ddot{\theta} - \frac{3}{2}\frac{g}{\ell}\sin\theta = 0$	$\ddot{\varphi} + \frac{3}{2}\frac{g}{\ell}\sin\varphi = 0$

$$\boxed{\ddot{\theta} - \frac{3}{2}\frac{g}{\ell}\sin\theta = 0}$$

$$\boxed{\ddot{\varphi} + \frac{3}{2}\frac{g}{\ell}\sin\varphi = 0}$$



$$\ddot{\theta} - \frac{3}{2}\frac{g}{\ell}\theta = 0$$

$$\ddot{\theta} = \underbrace{\frac{3}{2}\frac{g}{\ell}}_{\Omega}\theta$$

$$\theta(t) = e^{\Omega t}$$

$$\ddot{\theta} = \Omega^2 e^{\Omega t} = \Omega^2\theta$$

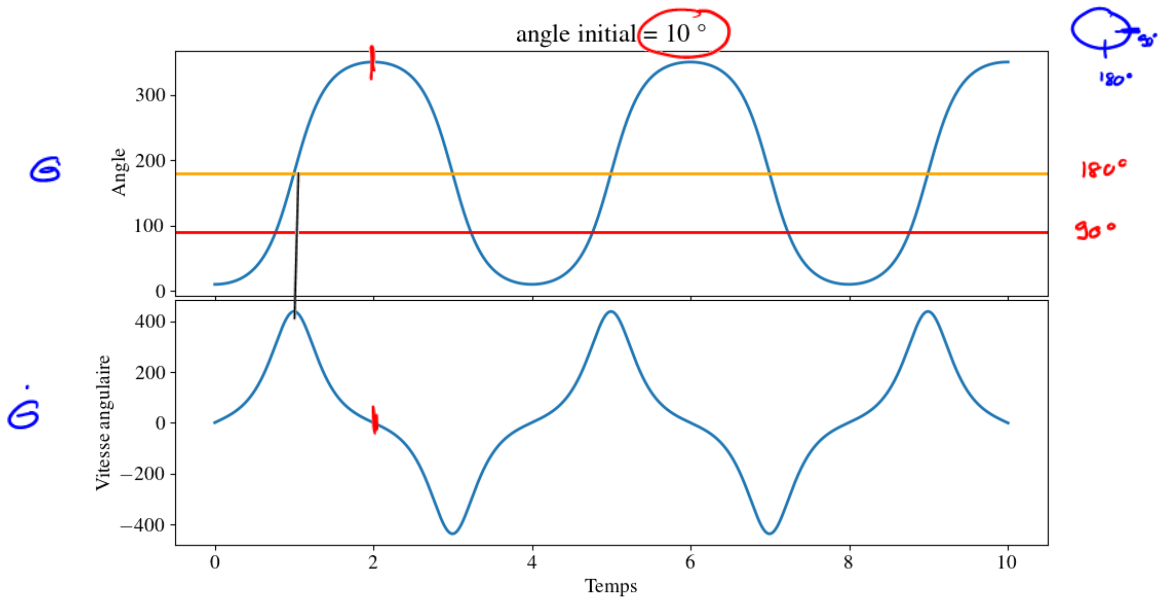
$$\varphi \ll 1$$

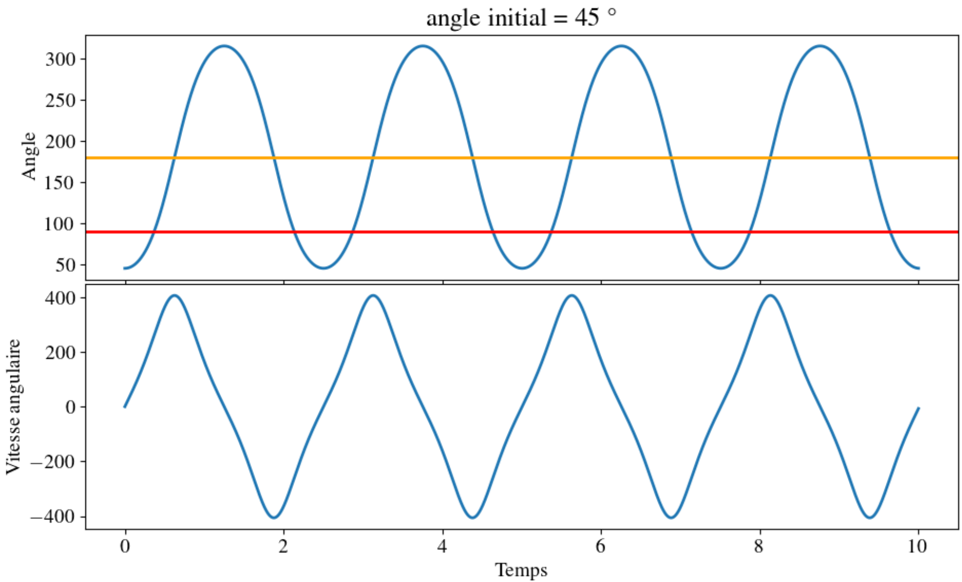
$$\sin\varphi \cong \varphi$$

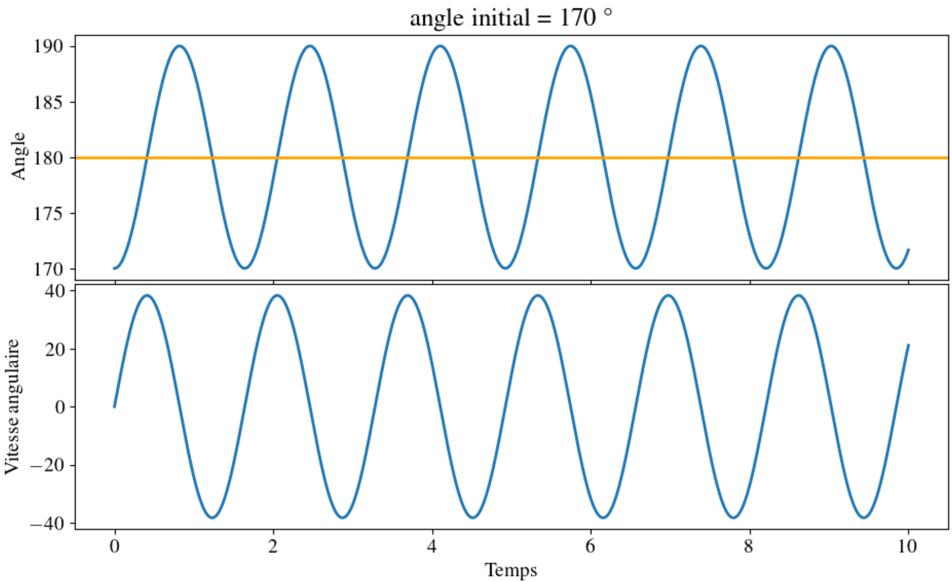
$$\ddot{\varphi} + \underbrace{\frac{3}{2}\frac{g}{\ell}}_{\Omega^2}\varphi = 0$$

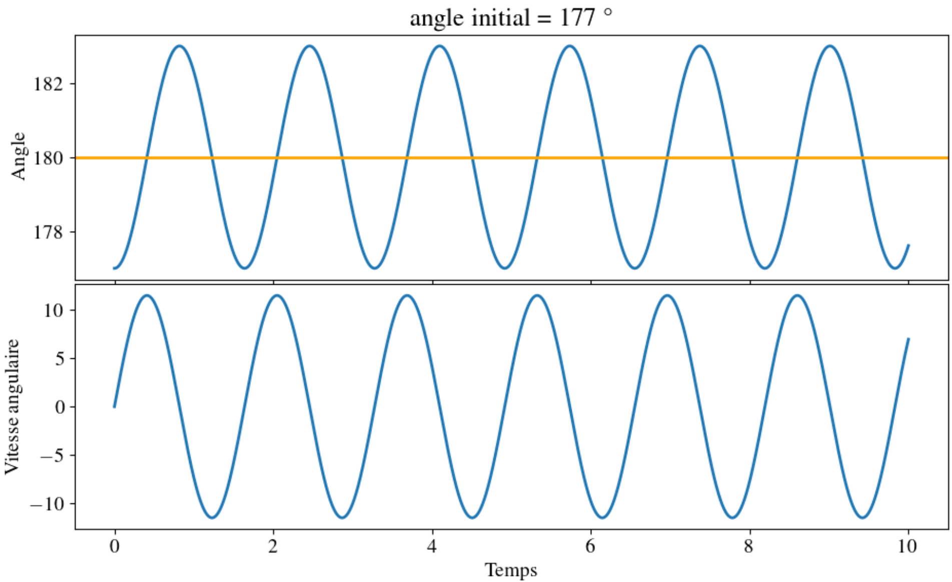
$$\ddot{\varphi} + \Omega^2\varphi = 0$$

$$\sim \sin(\Omega t + \varphi_0)$$

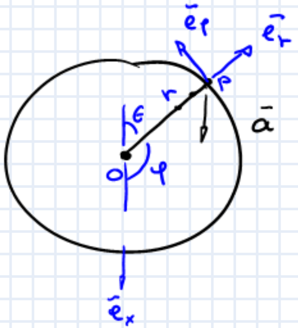

angle initial = 10 °
Angle
Vitesse angulaire
Temps
300
200
100
0
400
200
0
−200
−400
0
2
4
6
8
10
180°
90°
180°
90°


angle initial = 45 °
Angle
300
250
200
150
100
50
Vitesse angulaire
400
200
0
−200
−400
0
2
4
6
8
10
Temps


angle initial = 170 °
Angle
190
185
180
175
170
Vitesse angulaire
40
20
0
−20
−40
0
2
4
6
8
10
Temps


angle initial = 177 °
Angle
182
180
178
Vitesse angulaire
10
5
0
−5
−10
0
2
4
6
8
10
Temps

Analyse du vecteur accélération



$$\ddot{\theta} - \frac{3}{2}\frac{g}{\ell}\sin\theta = 0 \qquad \varphi = \pi - \theta \qquad \dot{\varphi} = -\dot{\theta}$$

$$\ddot{\varphi} = -\ddot{\theta}$$

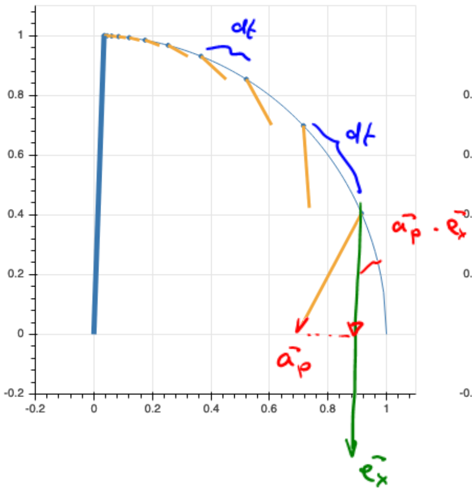
$$\vec{a}_P = -r\dot{\varphi}^2\vec{e}_r + r\ddot{\varphi}\vec{e}_\varphi \qquad r = d = \text{cte}$$

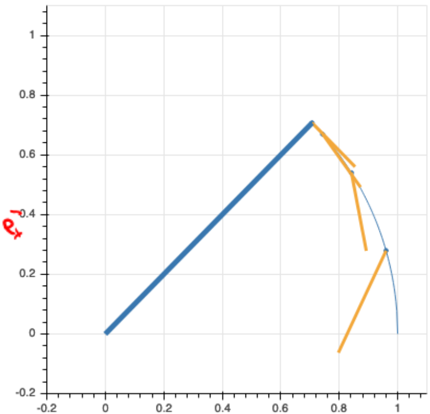
$$= -d\dot{\varphi}^2\vec{e}_r + d\ddot{\varphi}\vec{e}_\varphi$$

$$= -d\dot{\theta}^2\vec{e}_r - d\ddot{\theta}\vec{e}_\varphi$$

$$\boxed{\vec{a}_P = -d\dot{\theta}^2\vec{e}_r - d\left[\frac{3}{2}\frac{g}{\ell}\sin\theta\right]\vec{e}_\varphi}$$

$$\vec{a}_P\left(\theta(t), \dot{\theta}(t)\right)$$

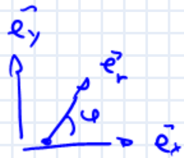

dt
dt
$\vec{a}_p \cdot \vec{e}_x$
$\vec{a}_p$
$\vec{e}_x$



$$\vec{a}_P \cdot \vec{e}_x = \left(\vec{a}_P = -d\dot{\theta}^2 \vec{e}_r - d\left[\frac{3}{2}\frac{g}{\ell}\sin\theta\right] \vec{e}_\varphi \right) \cdot \vec{e}_x$$

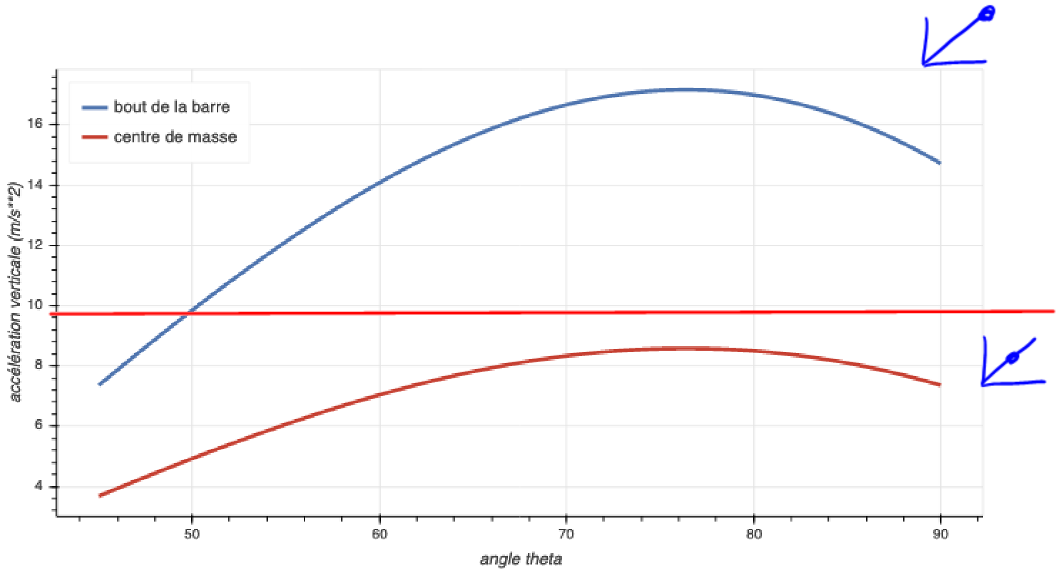
$$\vec{e}_r = \cos\varphi\, \vec{e}_x + \sin\varphi\, \vec{e}_y$$

$$\vec{e}_\varphi = -\sin\varphi\, \vec{e}_x + \cos\varphi\, \vec{e}_x$$

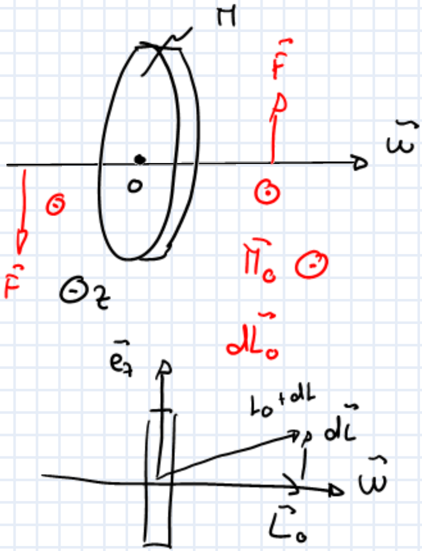


$$\varphi \sim \theta$$

$$\vec{a}_P \cdot \vec{e}_x = d\ddot{\theta}\cos\theta + d\,\frac{3}{2}\frac{g}{\ell}\sin^2\theta$$


bout de la barre
centre de masse
accélération verticale (m/s**2)
16
14
12
10
8
6
4
50
60
70
80
90
angle theta

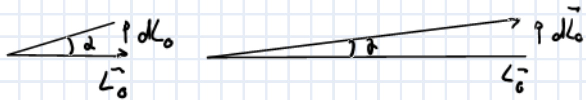
XI-2. Mouvement gyroscopique

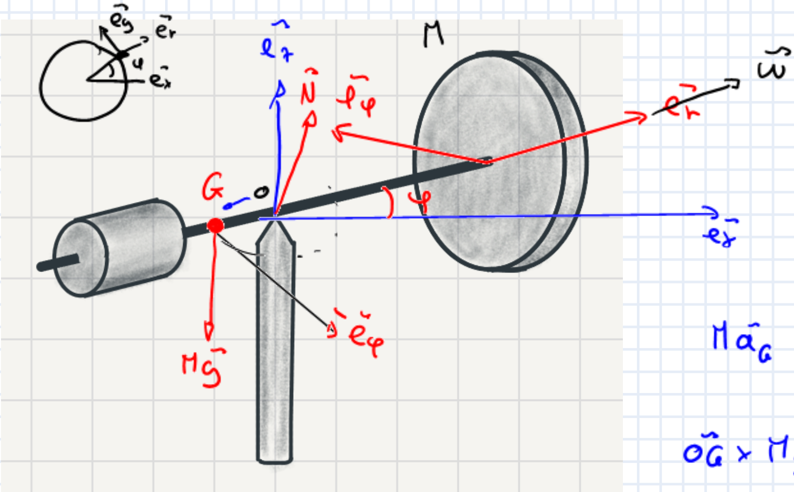


$$\vec{L}_o = I_\Delta \vec{\omega}$$

$$M r^2$$

$$\boxed{\frac{d\vec{L}_o}{dt} = \vec{M}_o}$$





$$\vec L = I_a \vec \omega$$

$$M \vec a_G = M \vec g + \vec N$$

$$\vec{OG} \times M\vec g = \frac{d\vec L_O}{dt}$$

$$M g \, OG(-\vec e_\varphi) = \frac{d}{dt}\left(I_a \omega \vec e_r\right) \qquad \vec L_O = I_a \vec e_r$$

$$= I_a \omega \underbrace{\frac{d}{dt}\vec e_r} = I_a \omega \dot\varphi \vec e_\varphi \qquad \frac{d}{dt}\vec e_r = \dot\varphi \vec e_\varphi$$

$$M_g\, OG\left(-\cancel{\vec{e}_\varphi}\right) = \frac{d}{dt}\left(I_a\, \omega\, \vec{e}_r\right) \qquad |\vec{OG}| = d_G$$

$$= I_a\, \omega\, \frac{d}{dt}\vec{e}_r = I_a\, \omega\, \dot{\varphi}\, \cancel{\vec{e}_\varphi}$$

$$\boxed{\dot{\varphi} = -\frac{M_g\, d_G}{I_a\, \omega}}$$

- précession ($\dot{\varphi}$)
- nutation