# Problem Set 14

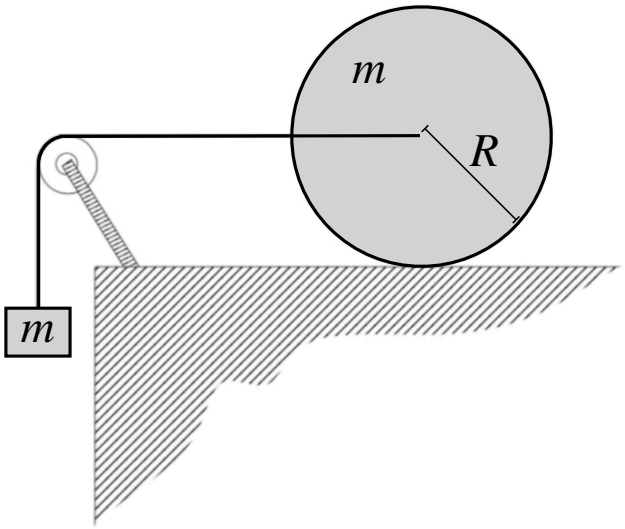
Rotation, translation, and rolling

PHYS-101(en)

### 1. Wheel pulled by a block

A uniform solid wheel, of mass $m$ and radius $R$, rolls without sliding on a horizontal table. It is pulled at its axis by an inextensible wire, which passes around a pulley and connects to a suspended block (see figure) with the same mass $m$. The wire and pulley have negligible masses and the wire always remains tight. The system is initially at rest.

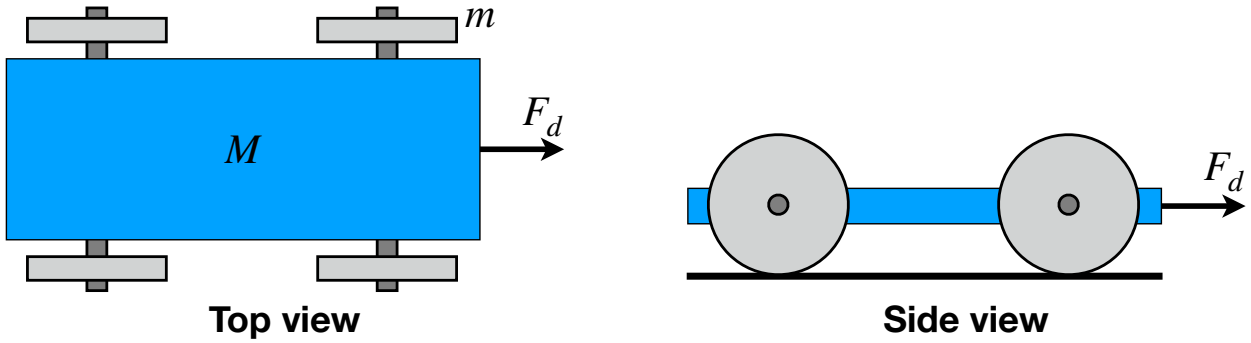
1. What is the speed of the center of mass of the wheel after it has moved forwards by a distance $d$?
2. Since the wheel does not slip, what can we say about the value of the static friction coefficient $\mu_s$ between the wheel and the table?



## 2. Donkey cart

A donkey starts pulling the cart shown below with a constant horizontal force of $F_d = 180$ N. The cart has four solid wheels, each of which have a diameter of $d = 70$ cm, a mass of $m = 12$ kg, and a uniform mass density. The mass of the cart without the wheels is $M = 120$ kg. The cart starts from rest, the axles are very thin, and the wheels roll without slipping.

1. Draw a free body diagram for the wheels.
2. Find the acceleration and speed of the cart after it has been pulled for 3 seconds.
3. Find the minimum value of the coefficient of static friction in order for the wheels to roll without slipping.




## 3. The hanging spider

A spider of mass $m$ is hanging from the ceiling by its thread, which has spring constant $k$ and equilibrium length $L$. It oscillates vertically and is subject to gravity.

1. Draw a free body diagram for the spider. In your chosen coordinate system, what is the equilibrium position of the spider?
2. Write the equation of motion for the spider.
3. Find the conditions for which the general solution to the equation of motion is given by

$$x(t) = A\cos(\omega_0 t + \varphi) + \overline{x}.$$

   In other words, determine the values of $\omega_0$ and $\overline{x}$.
4. Find the particular solution given the initial conditions that $v(t_0) = 0$ and that the force exerted by the spring at $t = t_0$ is zero. In other words, determine the values of the amplitude $A$ and phase $\varphi$.
5. What is the maximum speed of the spider?
6. For the general case (i.e. unspecified initial conditions), calculate the value of

$$E = \frac{k}{2}\left(x(t) - \overline{x}\right)^2 + \frac{m}{2}v(t)^2.$$

   What does this quantity represent?