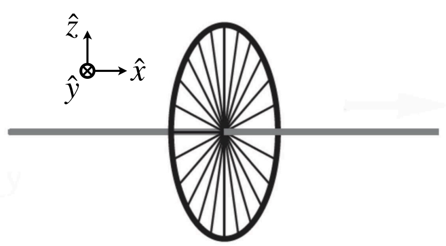
Problem Set 13

Harmonic motion and gyroscopes

PHYS-101(en)

1. Gyroscope

A gyroscope consists of a thin metal hoop of radius R and of mass M. Inside the metal hoop there are massless spokes, which radiate from the hoop's center. These connect to a massless bar of total length L, which is centered such that it extends a distance $\ell = L/2$ on each side of the hoop. The hoop and spokes are free to rotate about the bar. A demonstrator holds each end of the bar in a horizontal position, while the hoop is rotating at an angular velocity $\omega_h \hat{x}$. Find the magnitude and direction of the additional force exerted by each of the demonstrator's hands in order to cause the following motion. Note that you can neglect gravity in this problem as the demonstrator is preventing the gyroscope from falling and you are only concerned with finding the *additional* force (i.e. in addition to that needed to keep the bar at rest).



1. The bar is accelerated with a constant acceleration $a\hat{x}$ in the direction along the length of the bar.
2. The bar is rotated at a constant angular velocity $\omega_b \hat{z}$ in a horizontal plane around its center.

2. Simple pendulum

A simple pendulum consists of a massless string of length L hanging vertically and a point mass of mass m. The bottom of the string is attached to the mass, while the top position is fixed (but the string is free to pivot about this point). Suppose the string is initially pivoted by a small angle ϕ_0 from the vertical position and then released from rest.

1. Using the small angle approximation ($\sin \phi \approx \phi$) and either energy conservation or Newton's second law, show that the angle that the point mass makes with the vertical axis ϕ satisfies the differential equation of a simple harmonic oscillator.
2. What is the frequency and angular frequency of the oscillation?
3. How long will the pendulum take to return to its initial position, i.e. what is the period of its oscillation?
4. What are the angular and translational speeds of the point mass at the bottom of its swing?
5. Is the pendulum's angular speed the same as its angular frequency? Why or why not?
6. Does the period of the pendulum depend on m?