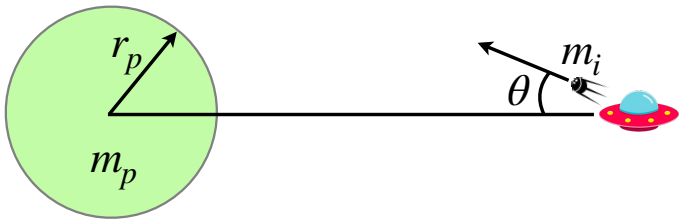
# Problem Set 12

Angular momentum

PHYS-101(en)

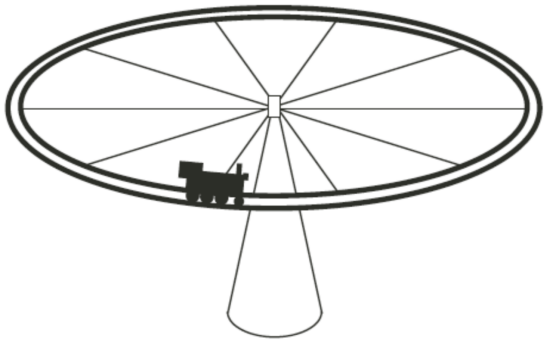
## 1. Planetary survey

A spaceship is sent to investigate a planet of mass $m_p$ and radius $r_p$. The ship launches an instrument with mass $m_i$ when it is a distance $5r_p$ from the center of the planet. The instrument has an initial speed $v_0$ (with respect to the planet), while it is initially traveling at an angle $\theta$ with respect to a radial line between the center of the planet and the launch position of the instrument. For what angle $\theta$ will the instrument just graze the surface of the planet?




## 2. Toy locomotive

A toy locomotive of mass $m_L$ runs on a uniform horizontal circular track of radius $R_T$ and total mass $m_T$. The track forms the rim of an otherwise massless wheel, which is free to rotate without friction about a vertical axis. The locomotive starts from rest and accelerates without slipping to a final speed of $v$ *relative to the track*. What is the locomotive's final speed $v_f$ *relative to the ground*?

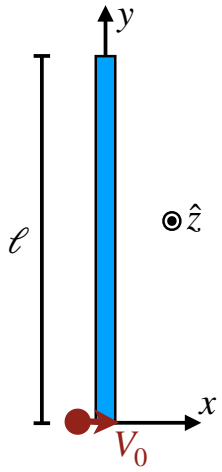


### 3. Particle-rod collision revisited

We will revisit problem 2 of problem set 6, and you are encouraged to use those results. Note that we use $\ell$ instead of $L$ to distinguish it from the magnitude of the angular momentum.

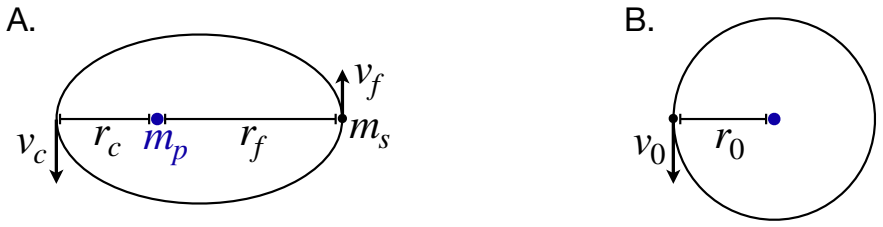
A slender uniform rod of length $\ell$ and mass $M$ rests along the $y$-axis on a frictionless, *horizontal* table. A particle of equal mass $M$ is moving along the $x$-axis at a speed $V_0$. At $t = 0$, the particle strikes the end of the rod and sticks to it. Note that gravity would be acting in the $\hat{z}$ direction, but does not need to be considered.

1. Calculate the angular velocity $\vec{\omega}$ of the rod-particle system about its *center of mass* after the collision. Express your answer in terms of $V_0$, $\ell$, $\hat{x}$, $\hat{y}$, and $\hat{z}$.
2. Using this result, calculate the position of the particle (stuck at one end of the rod) $\vec{r}_p(t)$ for $t \geq 0$. Express your answer in terms of $\vec{R}_{CM}(t)$, $\ell$, $\omega = |\vec{\omega}|$, $t$, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



**4. Elliptic Orbit**

A satellite of mass $m_s$ is in an elliptical orbit around a planet of mass $m_p$, which is located at one focus of the ellipse. It is not burning any fuel nor expelling any propellant. The satellite has a speed $v_f$ at the distance $r_f$ when it is furthest from the planet.




1. What is the speed $v_c$ of the satellite when it is at its closest distance to the planet? What is this distance $r_c$? Express your answer in terms of $m_s$, $m_p$, $G$, $v_f$, and $r_f$ as needed.
2. If the satellite were in a circular orbit of radius $r_0 = r_c$, would its speed $v_0$ be greater than, equal to, or less than the speed $v_c$ in the original elliptic orbit?