Name (Last, First):

Instructions

1. Write clearly and in ink (pen), and indicate clearly your result (with your reasoning explained).
2. Only responses written on the A4 sheets of the exam (front and back) will be taken into account.
3. Justify your calculations and responses with diagrams and by describing the laws of physics you have used.
4. This exam should be taken in 3 hours.

Allowed:

- one page A4 (front and back) *handwritten by you*
- language dictionary

Not allowed:

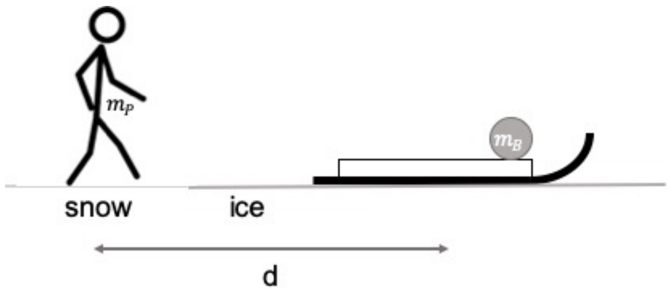
- Any form of electronic device including a calculator
- Leaving the room without authorization

(Do not mark on this page below this line)

NO CONTENT ON THIS PAGE

1. Snowball throw

A person of mass m_P loads their sled with a snowball, of mass m_B. The person then pushes the sled onto a frictionless, horizontal icy lake, leaving it at rest a distance d away, as shown in the diagram. The mass of the person plus the empty sled together is M_S. Ignore air resistance for this exercise. **<u>For (a) and (b), draw the relevant diagram(s).</u>**



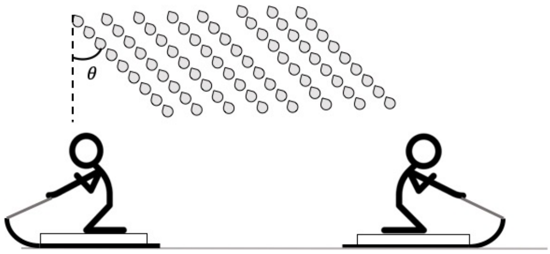
a) The person jumps with initial velocity $\vec{v}_P$. What are the conditions on the vector components of $v_{P,x}$ and $v_{P,y}$ so that the person lands in the sled, then the person+sled slides with speed v_0? Write your answers in terms of g (the gravitational acceleration at the earth's surface), d, m_P, m_B, M_S and v_0. (Hint: Do not use the range equation)

1. Snowball throw

b) The person and sled are sliding with speed v_0. The person throws the snowball horizontally off the back of the sled. The snowball has an initial horizontal velocity v_b with respect to the ice, in the same direction as the motion of the sled. If instead the person throws the snowball horizontally off the front of the sled, in the direction of its motion, what would be the speed v_f of the snowball with respect to the ice? Assume the person throws with the same strength, and thus gives the snowball the same speed relative themselves in each case. Write your answer in terms of only v_0 and v_b.

c) After throwing the snowball, the sled+person has a velocity u_f. The sled runs off the lake onto the snow, where the coefficient of kinetic friction is μ_k. What is the distance L the sled will slide before coming to a stop, in terms of in terms of g, μ_k, and u_f?

2. Sleeting sledding



An icy rain begins to fall. An observer in the ground frame measures that it is falling with a speed v_R at an angle θ with respect to the vertical. **<u>For (a) and (c), draw the relevant diagram(s).</u>**

a) To the person in the sled described in Exercise 1 sliding with speed u_f, the rain appears to fall vertically (Case 1). If the person turns the sled to slide with the same speed but in the opposite direction (Case 2), the rain appears to fall at an angle φ. What is θ in terms of φ?

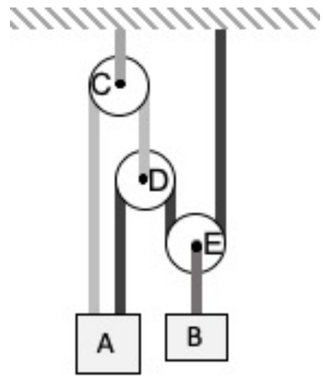
For (b) and (c), suppose the rain falls at a constant rate σ/τ (kg /m^2/s), and accumulates in the sled whose area is A.

b) Before the rain begins, the mass of the person+sled system is M_S. What is the mass of the system as a function of time, $M(t)$?

2. Sleeting sledding

c) Use the concept of momentum transfer to find the differential equation for the derivative of the velocity of the sled as a function of time in Case 1 and in Case 2, dV_{S1}/dt and dV_{S2}/dt. **Do not** solve the differential equations by integrating. Write your answers in terms of some or all of v_R, θ, V_{S1}, V_{S2}, σ, τ, M_S and A. (Hint: Terms that are a product of two differential elements are small, and can be set equal to zero.)

3. Pulling pulleys



Two blocks, A and B, with masses m_A and m_B can move along the vertical axis, connected by a system of ropes and pulleys (C, D, E). Assume the pulleys are massless and frictionless, and the ropes are massless and inextensible. Rope 1 connects block A to pulley D (light grey), and rope 2 connects block A to the ceiling (black).

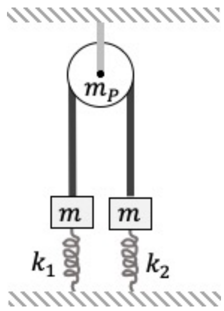
a) Derive the constraint equation that relates the vertical component of the acceleration of block A, a_A, to that of block B, a_B.

b) At a time t^*, the velocity of block B is v_B^*, downward. What are the velocities of block A and pulley D, v_A^* and v_D^*?

3. Pulling pulleys

c) Find the tension in rope 1 and rope 2, T_1 and T_2, as a function of m_A, m_B, and g, the gravitational acceleration at the earth's surface. Under what condition will the system be in static equilibrium? Check that your answer makes sense when compared with part (a).

4. Spring fling



Two blocks of equal mass m are attached by a massless, inextensible string. The string wraps over a massless pulley, which hangs from the ceiling. Each mass is attached to a spring anchored to the floor. The spring constants are k_1 and k_2 as shown in the diagram.

a) Initially, the system is at rest, and both springs are at their equilibrium lengths. The block on the left is pulled down by a distance y_0 and released from rest. Assume $k_1 < k_2$. What is the acceleration of each block, $a_{1,i}$ (left block), and $a_{2,i}$ (right block) immediately after the release? Specify both the magnitude and direction.

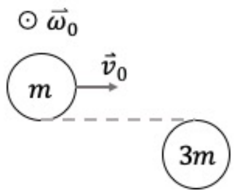
4. Spring fling

b) What is the tension in the string, T? Under what constraint on the spring constants would the accelerations of the two blocks be independent of each other? In this case, what values would $a_{1,i}$ and $a_{2,i}$ take on?

c) What natural oscillatory frequency, ω_0, does the system take on in case (b)? Graph the position of both blocks with the axes provided below, be sure to clearly distinguish the motion of each block, and to note key features or values particular to the motion.

5. Sticky disks

A uniform disk (disk 1) of mass m and radius R slides on a frictionless table with center of mass velocity $\vec{v}_0$, while spinning with an angular velocity $\vec{\omega}_0$ about its center of mass. It contacts a second disk (disk 2) of mass $3m$ and radius R which is at rest, and instantly sticks, so that the two move as a single, rigid object. Note: the moment of inertia of disk 1 about an axis through its center of mass (out of the page as drawn) is $\frac{1}{2}mR^2$. Write your answers in terms of the given variables m, R, $\vec{v}_0$, and $\vec{\omega}_0$.



a) What is the vertical position of the center of mass of the two-disk system, y_{cm}? What are the translational and rotational velocities about the system's center of mass after the collision, $\vec{v}_f$ and $\vec{\omega}_f$? Indicate both magnitudes and directions.

5. Sticky disks

b) For what value of $\vec{\omega}_0$ would the system after the collision not rotate?

c) How much mechanical energy, ΔE, is lost in the collision, under the condition that the system does not rotate after the collision? Where did this mechanical energy likely go?