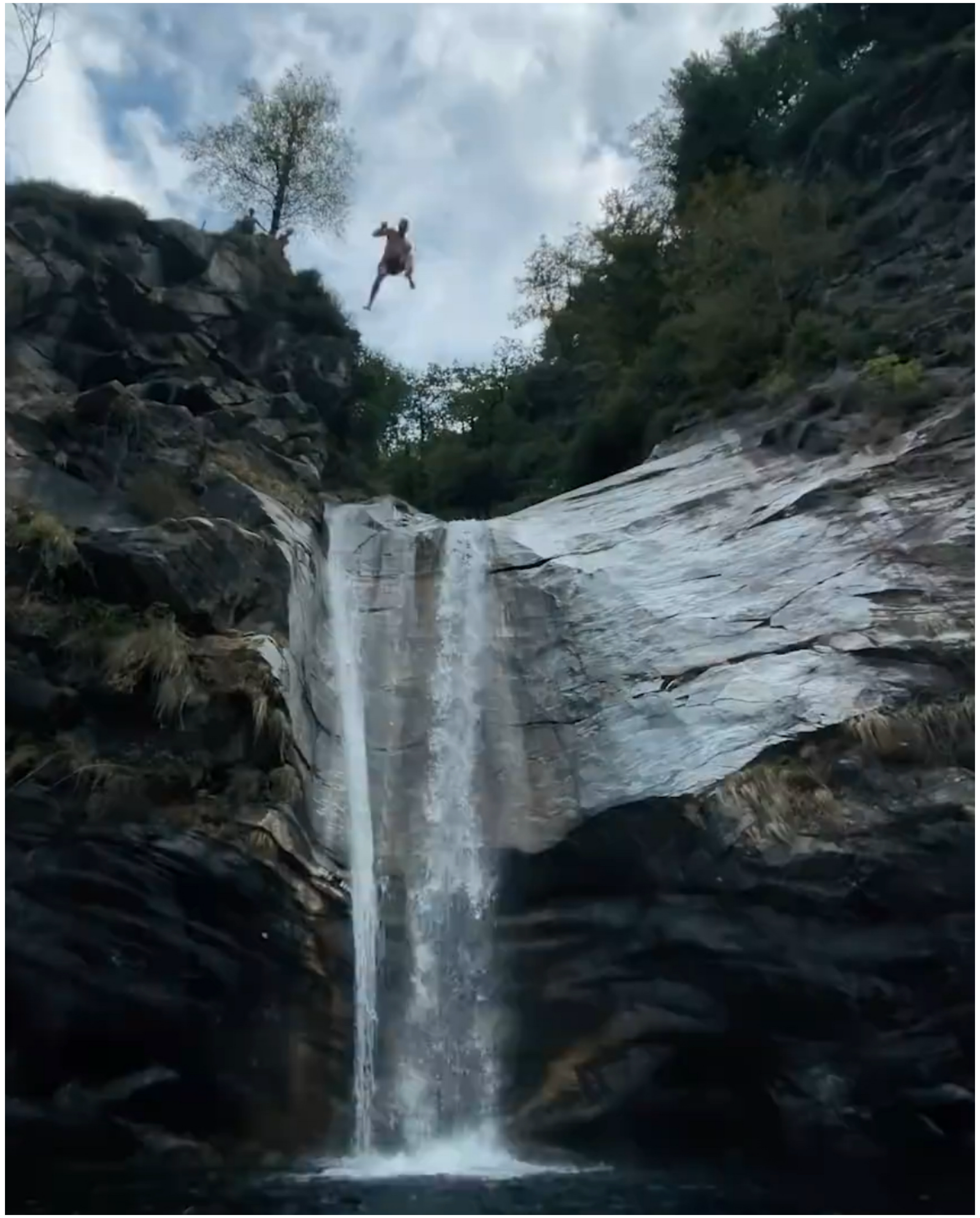
General Physics: Mechanics



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PHYS-101(en)

Lecture 9b: Potential energy, energy conservation

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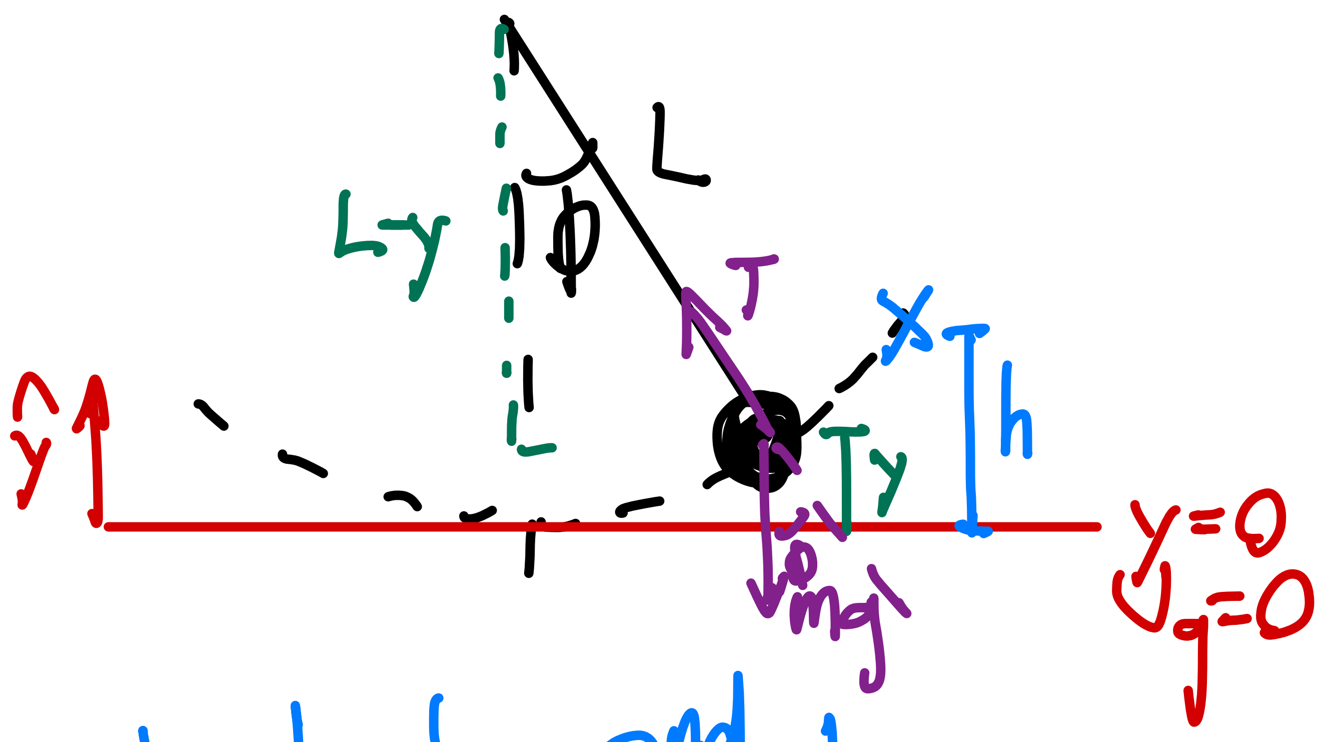
Tension in a pendulum

Calculate the tension in a massless string of a pendulum.



$$\Delta E_m = \cancelto{0}{W_{non}}$$

$$\Rightarrow K(y) + U_g(y) = \cancelto{0}{K(h)} + \underbrace{U(h)}_{=mgh}$$

$$\Rightarrow \frac{1}{2}\cancel{m} v_\phi^2 + \cancel{m} g y = \cancel{m} g h \Rightarrow v_\phi^2 = 2g(h-y)$$

Newton's 2nd law: $T - mg\cos(\phi) = ma_c \Rightarrow T = mg\cos(\phi) + ma_c$

$$a_c = \frac{v_\phi^2}{L} = \frac{1}{L}2g(h-y) \Rightarrow T = mg\underbrace{\cos(\phi)}_{=\frac{L-y}{L}} + m\frac{1}{L}2g(h-y)$$

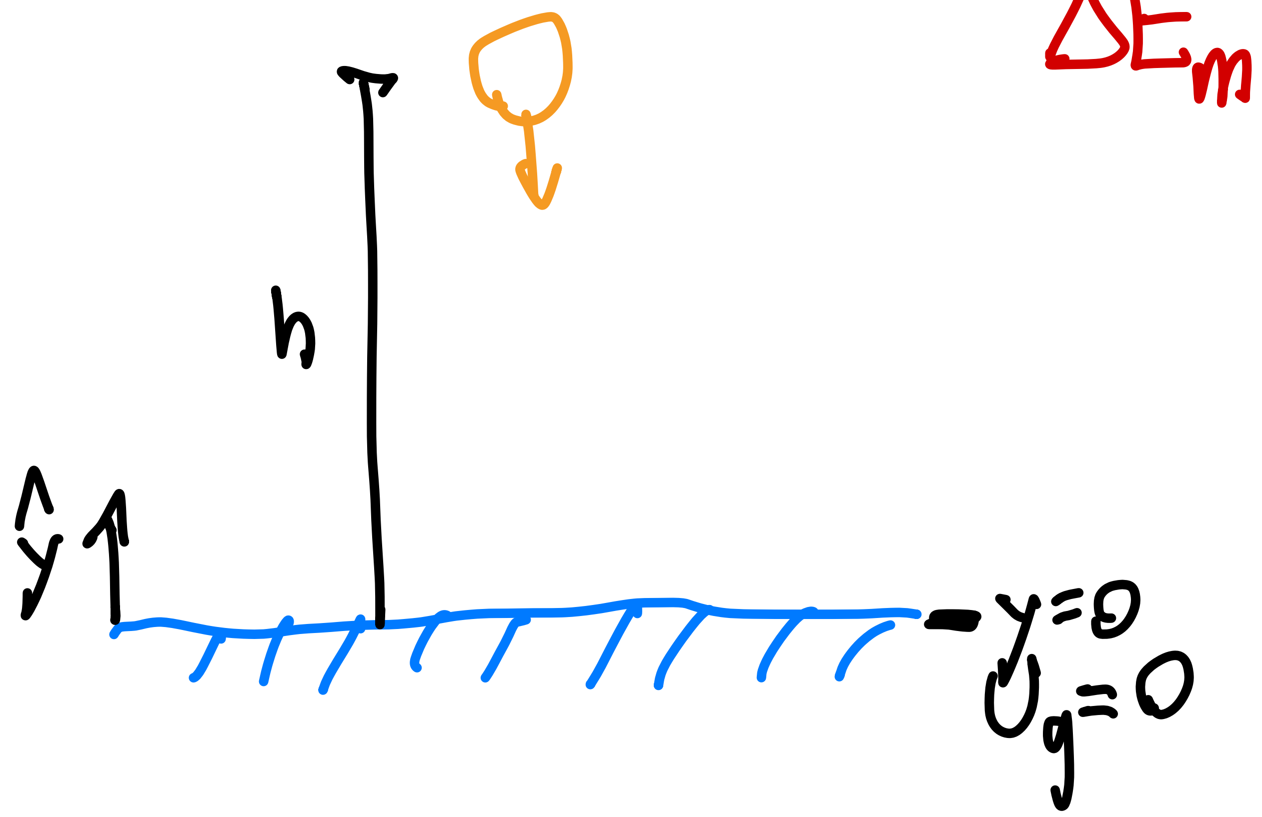
$$T = mg\frac{1}{L}(L-y) + mg\frac{2}{L}(h-y) = \frac{mg}{L}(L-y) + \frac{mg}{L}(2h-2y) = \boxed{\frac{mg}{L}(L+2h-3y)}$$

Example: Cliff jumping

You're on vacation and want to jump from a cliff into water. If humans can temporarily withstand about 10 g's of force without much discomfort, how high can you jump from? How deep must the water be? Neglect gravity when you're in the water and ignore *air* resistance, but assume your drag coefficient in water is $\beta \approx 15$ kg/m.

Cliff jumping safety

1. Be a good swimmer
2. Never go alone
3. Make sure the water is deep enough
4. Prepare for intense water pressure
5. Wear shoes
6. Don't jump headfirst
7. Keep your body streamlined
8. Blow out through your nose
9. Be of sound mind
10. Confidence is key



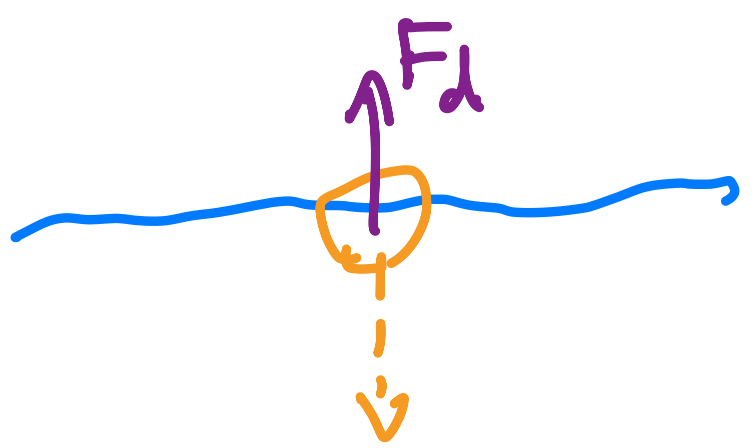
$$\Delta E_m = 0 \Rightarrow \cancel{K_i} + U_{gi} = K_f + \cancel{U_{gf}}$$

$$\Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v_w^2$$

$$\Rightarrow v_w^2 = 2gh$$

Example: Cliff jumping

If humans can temporarily withstand about 10 g's of force without much discomfort, how high can you jump from?



$$\vec{F}_d = \beta v^2 \hat{y} \Rightarrow |\vec{F}_d|^{max} = \beta v_w^2 = \beta(2gh) = 2\beta g h$$

$$|\vec{F}_d|^{max} \leq 10 mg$$

$$\Rightarrow \cancel{2}\beta g h \leq \overset{5}{\cancel{10}} m \cancel{g} \Rightarrow \boxed{h \leq 5\frac{m}{\beta}}$$

For $\left.\begin{array}{l} \beta = 15\,\frac{kg}{m} \\ m = 60\,kg \end{array}\right\} \Rightarrow h \leq 5\,\frac{60}{15}\,m = 20\,m$

Example: Cliff jumping

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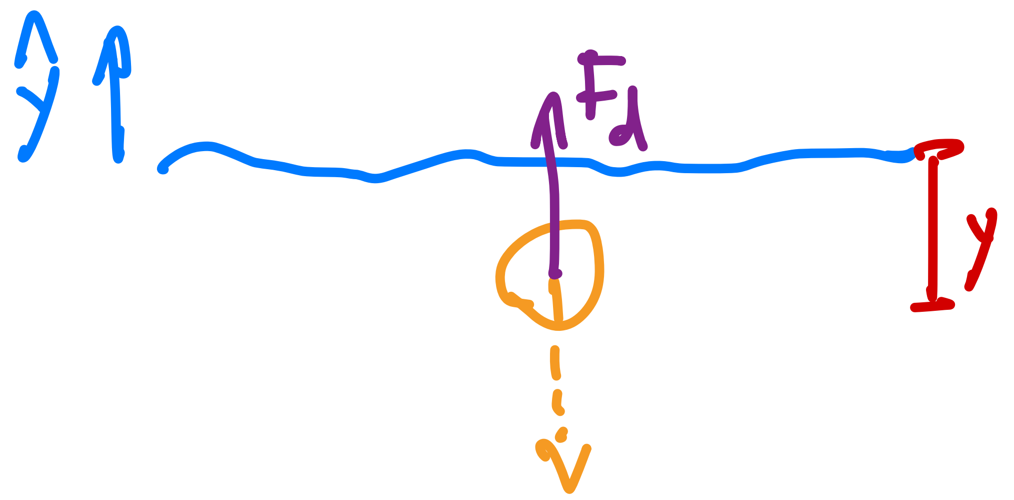
How deep must the water be? Neglect gravity when you're in the water and ignore *air* resistance, but assume your drag coefficient in water is $\beta \approx 15$ kg/m.



$$\Delta K = W_{net} = \cancel{W_g} + \cancel{W_b} + W_d$$

$$\Delta K = K(y) - K(0) = \frac{1}{2} m v_y^2 - \frac{1}{2} m v_w^2$$

$$\vec{F}_d = \beta v_y^2 \hat{y} \qquad d\vec{l} = -\hat{y}\, dy \quad \Rightarrow \quad W_d = \int_0^y \vec{F}_d \cdot d\vec{l} = \int_0^y \beta v_y^2 \hat{y} \cdot (-\hat{y}\, dy)$$

$$= -\beta \int_0^y v_y^2\, dy$$

$$\Rightarrow \frac{1}{2} m v_y^2 - \frac{1}{2} m v_w^2 = -\beta \int_0^y v_y^2\, dy$$

$$\frac{d}{dy}\left[\frac{1}{2} m v_y^2 - \frac{1}{2} m v_w^2\right] = \frac{1}{2} m \frac{d}{dy}(v_y^2)$$

$$=$$

$$\frac{d}{dy}\left[-\beta \int_0^y v_y^2\, dy\right] = -\beta v_y^2$$

$$\frac{d}{dy}\overbrace{(v_y^2)}^{u} = -\frac{2\beta}{m}\overbrace{(v_y^2)}^{u}$$

$$\frac{du}{dy} = -\frac{2\beta}{m} u$$

Example: Cliff jumping

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$\frac{du}{dy} = -\frac{2\beta}{m} u \Rightarrow \int \frac{du}{u} = \int -\frac{2\beta}{m} dy \Rightarrow \ln|u| = -\frac{2\beta}{m} y + \underbrace{C}_{\text{Integration constant}} \Rightarrow \overbrace{u}^{=v^2} = e^{-\frac{2\beta}{m} y} e^{C} = \overbrace{C_1}^{=e^C} e^{-\frac{2\beta}{m} y}$

$v^2 = C_1 e^{-\frac{2\beta}{m} y}$

- To find the value of C_1, we use the initial conditions:

We know that $v^2(0) = v_w^2 \Rightarrow v^2(0) = C_1 e^0 = C_1 = v_w^2 = 2gh \Rightarrow v^2 = 2gh\, e^{-\frac{2\beta}{m} y}$

$$\Rightarrow \boxed{|v| = \sqrt{2gh \cdot e^{-\frac{2\beta}{m} y}} = \sqrt{2gh}\, e^{-\frac{\beta}{m} y}}$$

Typical "decay" distance $\frac{m}{\beta} = \frac{60\,\text{kg}}{15\,\frac{\text{kg}}{\text{m}}} = 4\,\text{m}$

If we know a max. $|v|$ at which the diver can touch the floor at the bottom of the pond, we can find the correspon-ding y.