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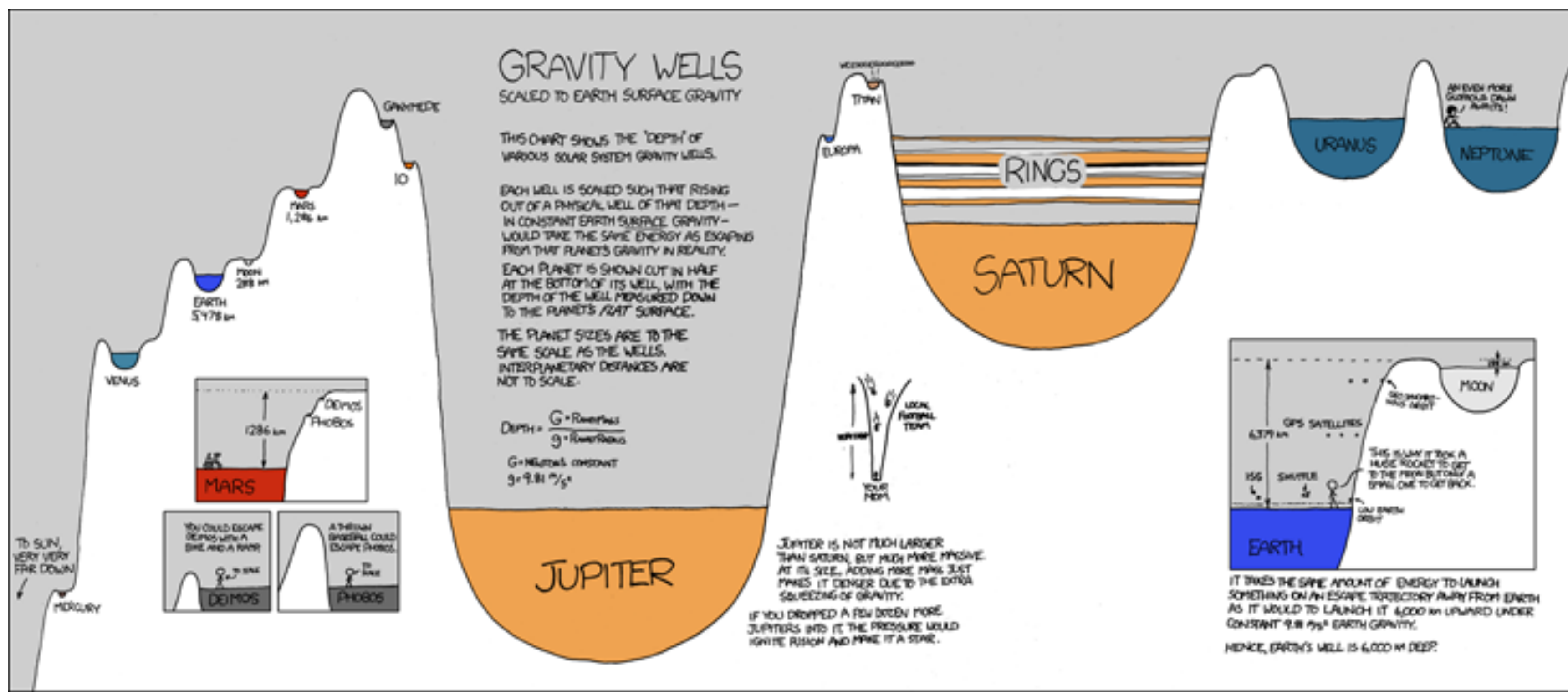
General Physics: Mechanics

PHYS-101(en)

Lecture 9a:
Potential energy, energy conservation



https://xkcd.com/681

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November 11th, 2024

Today's agenda (Serway 7-8, MIT 13-14)

1. A few words on Mock exam 1

2. Power

3. Energy and work

   - Potential energy

   - Conservation of mechanical energy

Mock exam takeaways

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- A total of 182 mock exams were graded
- Average score of graded exams: 10.6/15

Mock exam takeaways

- Common pitfall 1: Recognizing that displacement is a vector.
  - Important to distinguish between vectors and scalars.
  - Read carefully what you are being asked!

$x = 5$

$y = 10$

$\Rightarrow \cancel{\vec{r} = x + y = 15m}$

$\vec{r} = x\hat{x} + y\hat{y} = 5\hat{x} + 10\hat{y}$ ✓

Mock exam takeaways

- Common pitfall 2: Not describing your answer with enough level of detail to show that you understand what you are doing.
  - Physics is **not** about just showing some formulas.
  - You will miss possible partial credit.
  - As an example of what you are expected to do, check the solutions on the Moodle.

Mock exam takeaways

- Common pitfall 3: Not checking whether your answer makes sense.
  - We saw the following answer for problem 2b which ***cannot*** be correct:

$$a = \frac{g(M_P - \mu_k M_T)}{M_P - M_T}$$

  - An important outcome of this course should be that you develop tools to test whether what you think is right is ***actually*** right.

Today's agenda (Serway 7-8, MIT 13-14)

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1. A few words on Mock exam 1
2. **Power**
3. Energy and work
   - Potential energy
   - Conservation of mechanical energy

From last week — kinetic energy and work

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- Kinetic energy is $K = mv^2/2$
- Work done by a <u>constant</u> force is $W = \vec{F} \cdot \vec{\ell} = F\ell\cos\theta$
- Work done by a <u>variable</u> force $\vec{F}$ along a trajectory C is

$$W = \int_C \vec{F} \cdot d\vec{\ell}$$

- **Work-kinetic energy theorem**: The total (net) work done on a system equals the change in its kinetic energy

$$W_{net} = \Delta K$$

- Units of work and energy are Joules (J)

Power from work

- **Power** is defined as the rate at which work is done

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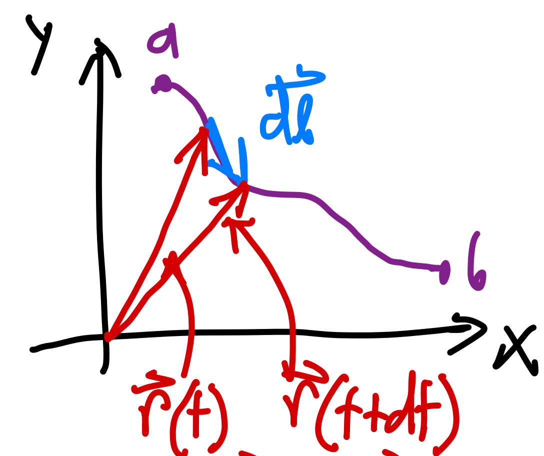
- Has units of Watts (W) = Joules per second = [kg·m^2/s^3]
- Using the work-kinetic energy theorem, power is also the amount of energy transferred or converted per unit time
- Humans operate at roughly 100 W

Power from force

- For a constant force, power can be written as

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F} \cdot \vec{\ell}\right) = \vec{F} \cdot \frac{d\vec{\ell}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \boxed{\vec{F} \cdot \vec{v}}$$



$$d\vec{\ell} = d\vec{r}$$

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\Rightarrow \quad d\vec{r} = \vec{v}\,dt$$

Power from force

- For a constant force, power can be written as

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F}\cdot\vec{\ell}\right) = \vec{F}\cdot\frac{d\vec{\ell}}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

- What about a variable force?

$$P = \frac{d}{dt}W = \frac{d}{dt}\left[\int_C \vec{F}\cdot \underline{d\vec{l}}\right] \qquad = \vec{v}\,dt$$

$$= \frac{d}{dt}\left[\int_{t_0}^{t}\vec{F}\cdot\vec{v}\,dt\right]$$

$$= \boxed{\vec{F}\cdot\vec{v}}$$

Conceptual question

A particle starts from rest at $x = 0$ and moves to $x = L$ while experiencing the variable force $F(x)$ shown in the figure.
What is the particle's kinetic energy at $x = L/2$ and at $x = L$?

A. $\frac{F_{max}L}{2}, F_{max}L$

B. $\frac{F_{max}L}{2}, 0$

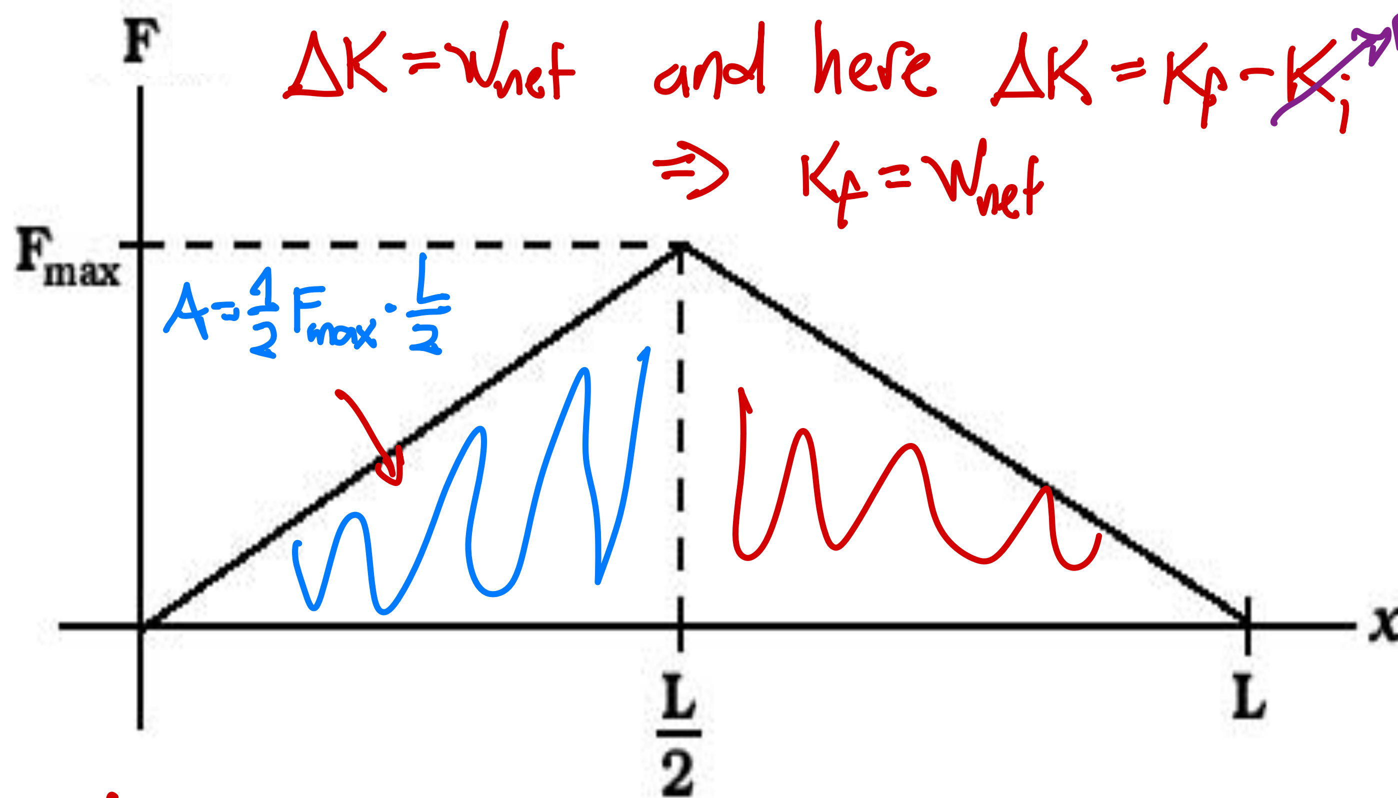
C. $F_{max}L, 0$

D. $\frac{F_{max}L}{4}, \frac{F_{max}L}{2}$

E. $\frac{F_{max}L}{2}, \frac{F_{max}L}{4}$

$\Delta K = W_{net}$ and here $\Delta K = K_f - \cancel{K_i}^{\,0}$

$\Rightarrow K_f = W_{net}$



Between 0 and $\frac{L}{2}$:

$W_{net} = \int_0^{\frac{L}{2}} F(x)\,dx = \frac{1}{4} F_{max} L$

Between 0 and L:

$W_{net} = \int_0^L F\,dx = \int_0^{\frac{L}{2}} F\,dx + \int_{\frac{L}{2}}^{L} F\,dx = \frac{1}{2} F_{max} L$

Conceptual question

A sports car accelerates from zero to 30 km/h in 1.5 s. How long does it take for it to accelerate from zero to 60 km/h, assuming the power of the engine to be independent of velocity and neglecting friction?

A. 2 s
B. 3 s
C. 4.5 s
D. 6 s
E. 9 s

$$W = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}^{\,0} = \frac{1}{2} m v_f^2$$

$$P = \frac{dW}{dt} = \frac{\Delta W}{\Delta t} = \frac{1}{\Delta t} \cdot \frac{1}{2} m v_f^2$$

- For 0 to 30 km/h
- For 0 to 60 km/h

$$P = \frac{1}{\underbrace{\Delta t_1}_{=1.5\,s}}\left(\frac{1}{2} m \underbrace{v_f^2}_{=30\,\frac{km}{h}}\right) = P = \frac{1}{\Delta t_2}\left(\frac{1}{2} m \underbrace{v_{f2}^2}_{v_{f2} = 2 v_f}\right)$$

$$\Rightarrow \frac{1}{\Delta t_1}\left(\cancel{\frac{1}{2} m v_f^2}\right) = \frac{1}{\Delta t_2}\left(\frac{1}{2} m (2 v_f)^2\right) = \frac{1}{\Delta t_2}\left(\frac{1}{2} \cdot 4 m v_f^2\right) = \frac{4}{\Delta t_2}\left(\cancel{\frac{1}{2} m v_f^2}\right)$$

$$\Rightarrow \frac{1}{\Delta t_1} = \frac{4}{\Delta t_2} \Rightarrow \Delta t_2 = 4 \Delta t_1 = 6\,s$$

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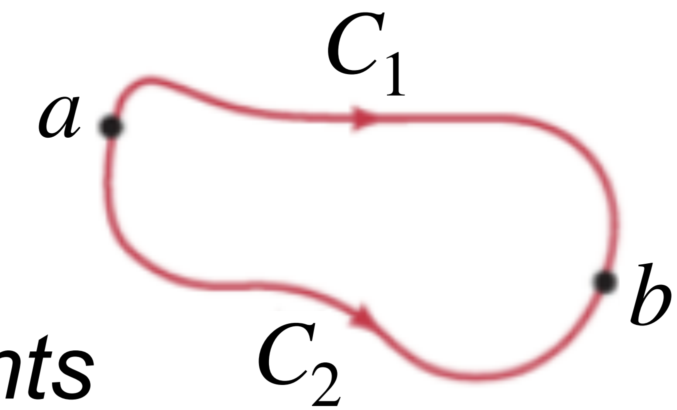
3. **Energy and work**
   - **Potential energy**
   - Conservation of mechanical energy

From last week — conservative forces

- A force is called *conservative* if

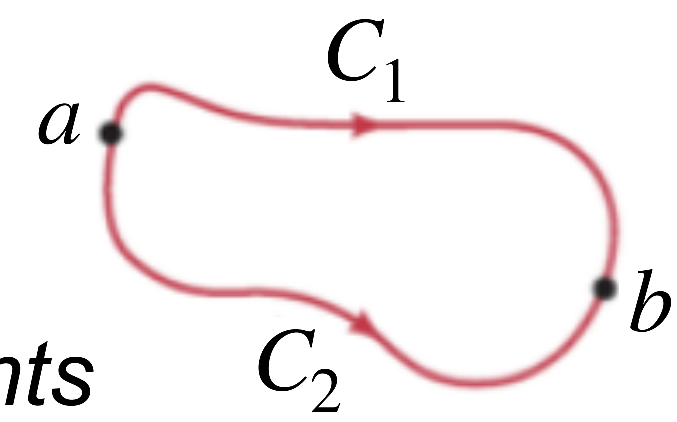


The work done by the force on
a particle moving between any two points
is independent of the path taken by the particle.

or equivalently

The net work done by the force on
a particle moving around any closed path is zero.

From last week — conservative forces



- A force is called *conservative* if

  The work done by the force on a particle moving between any two points is independent of the path taken by the particle.

  or equivalently

  The net work done by the force on a particle moving around any closed path is zero.

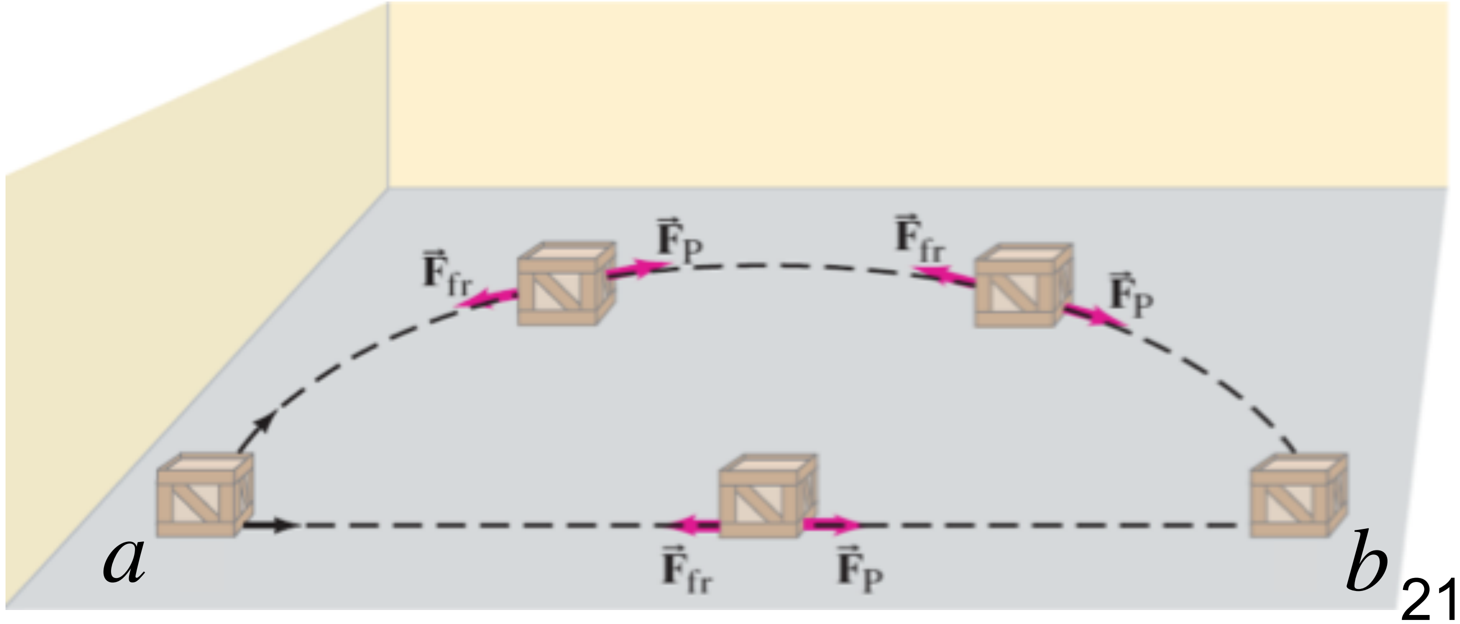
- Otherwise, the force is *non-conservative*
- Work-kinetic energy theorem applies to both

Conservative and nonconservative forces

- Conservative examples:
  - Work by gravity only depends on Δy and work by springs only depends on Δx

Conservative and nonconservative forces

- Conservative examples:
  - Work by gravity only depends on Δy and work by springs only depends on Δx
- Nonconservative example:
  - When friction is present, the work done depends not only on the starting and ending points, but also on the path taken
  - The longer path has more dissipation



Conservative and nonconservative forces

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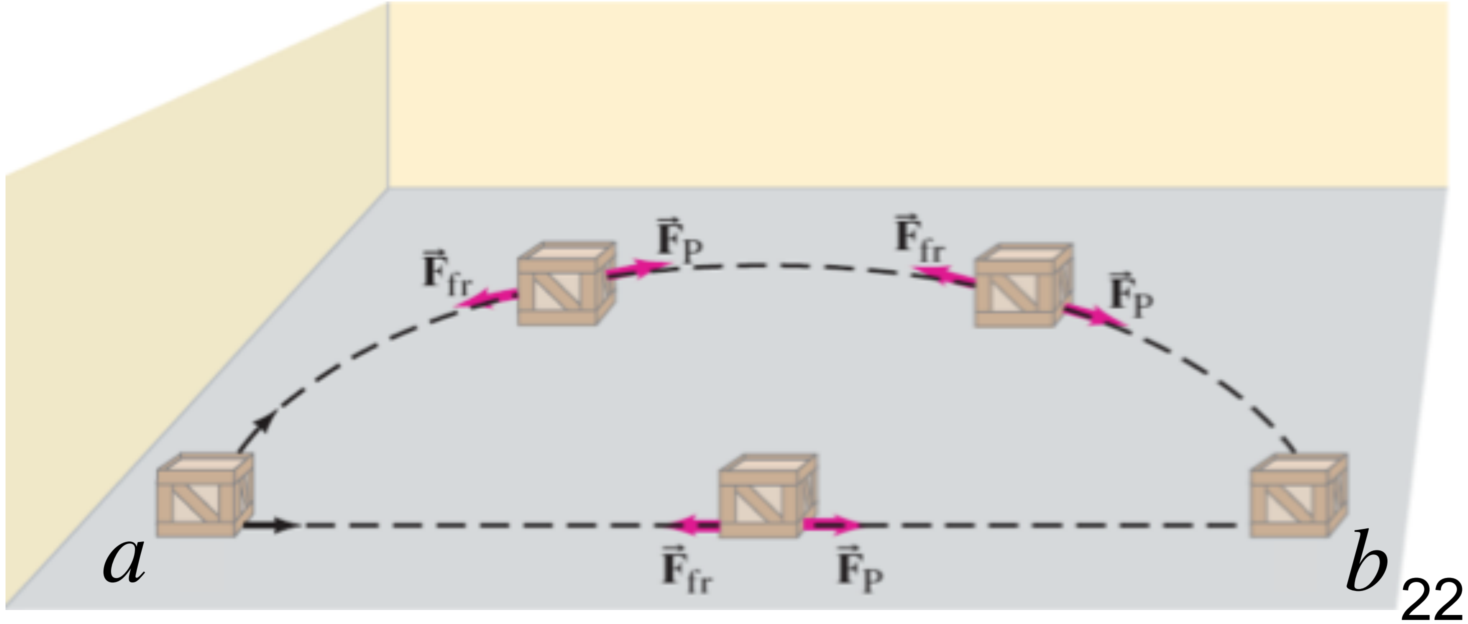
Conservative Force	Nonconservative forces
Gravitational	Friction
Elastic (spring)	Air resistance (drag)
Electric	Tension in rope
	Motor or rocket propulsion
	Push or pull by a person



Potential energy

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- Definition of **potential energy**:

The change in potential energy is the energy stored by an object as the result of work done by a conservative force.

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- The change in potential energy between points a and b is

$$\Delta U = U_b - U_a = -W = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$

Potential energy

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The change in potential energy is the energy stored by an object as the result of work done by a conservative force.

- The change in potential energy between points a and b is

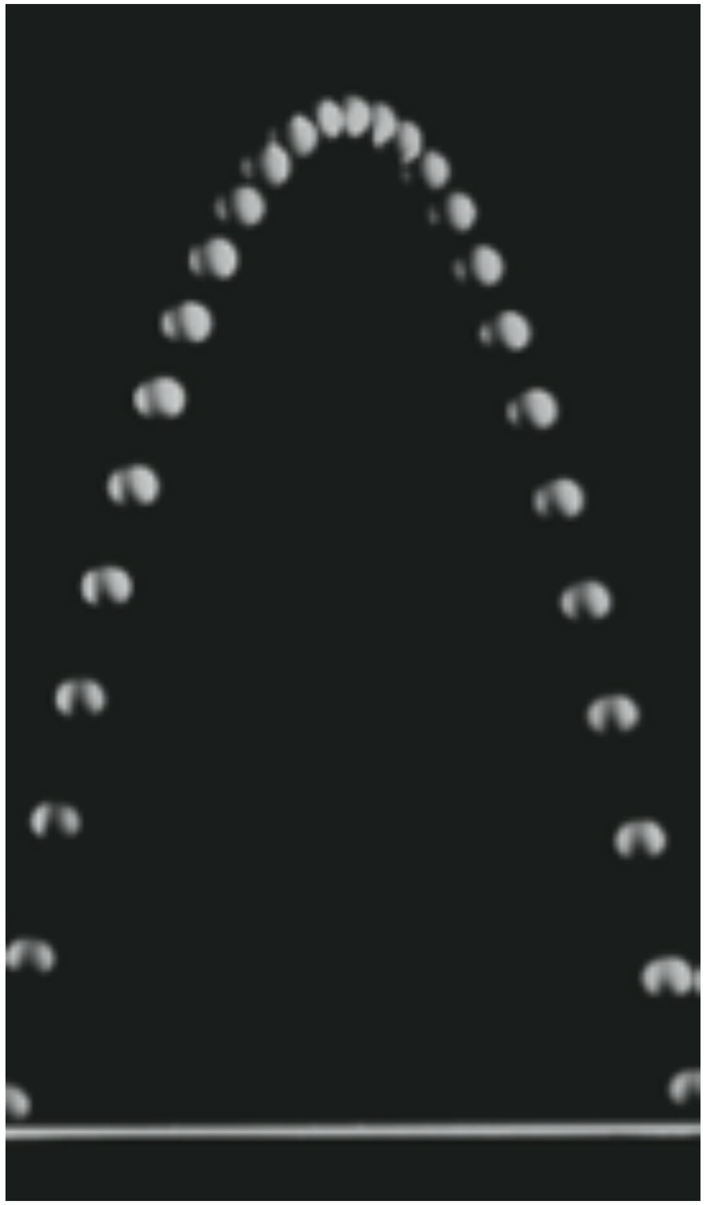
$$\Delta U = U_b - U_a = -W = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

- Thus, there is gravitational potential energy and spring potential energy, but not friction potential energy
- The motivation for this definition is so that energy is conserved (as we will see)

Gravitational potential energy

$$\Delta U = -W = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$

- When a ball is thrown upwards, gravity initially does negative work, meaning the kinetic energy decreases and the potential energy increases



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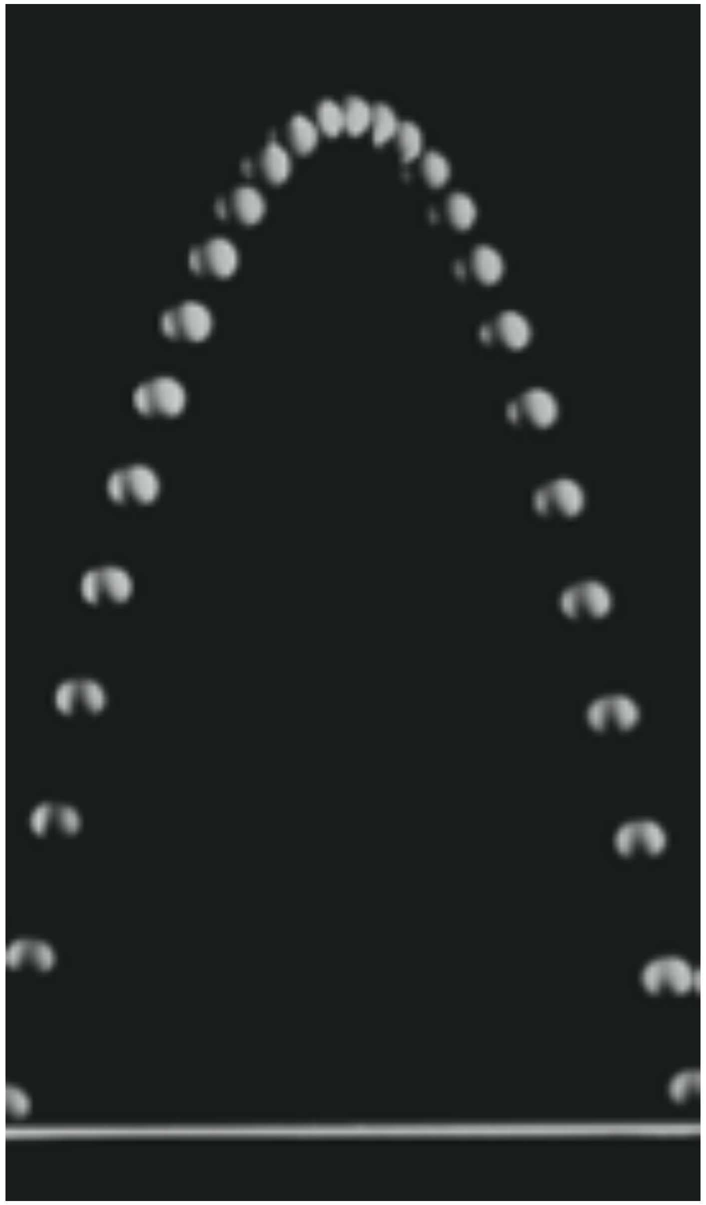
- Kinetic energy decreases until ball reaches peak, where it is minimal

- Then gravity does positive work, meaning the kinetic energy increases and the potential energy decreases



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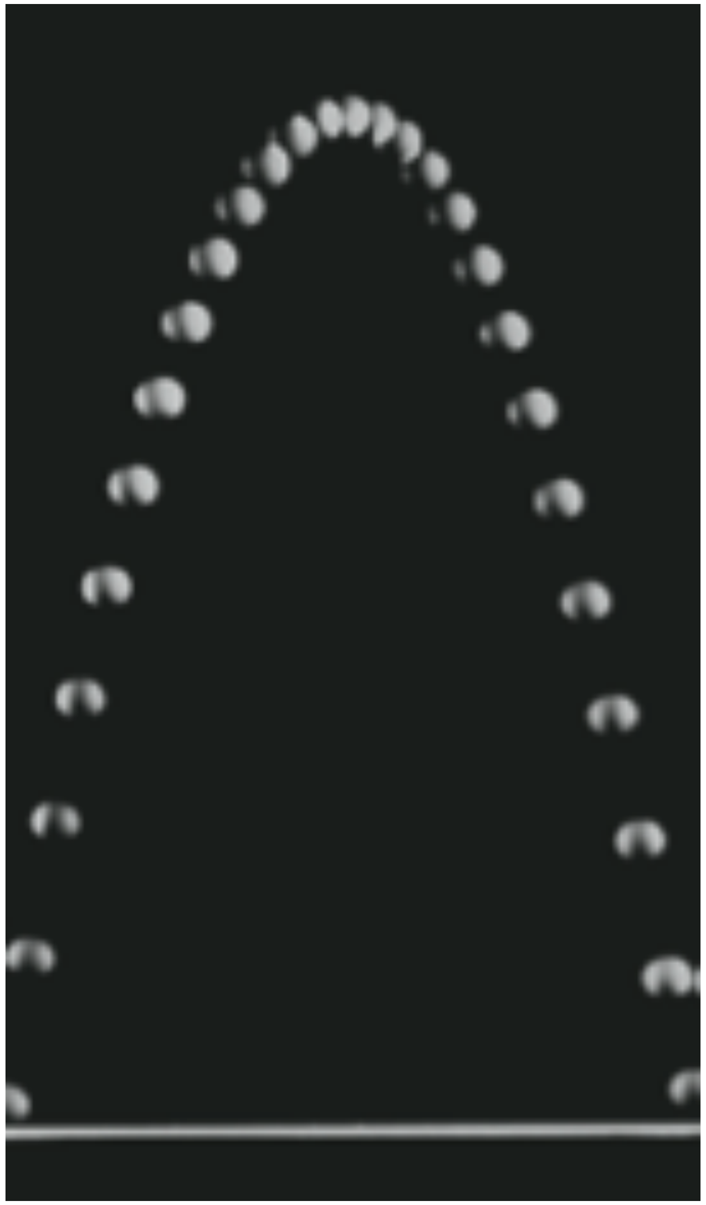


- When a ball is thrown upwards, gravity initially does negative work, meaning the kinetic energy decreases and the potential energy increases
- Kinetic energy decreases until ball reaches peak, where it is minimal
- Then gravity does positive work, meaning the kinetic energy increases and the potential energy decreases
- When the ball returns to its initial height, it will have the same kinetic energy as when it was thrown

Calculating gravitational potential energy

$$\Delta U = -W = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

$\vec{F}_g = mg(-\hat{y}) = -mg\hat{y}$ $\quad d\vec{\ell} = \hat{x}dx + \hat{y}dy + \hat{z}dz$

$\vec{F}_g \cdot d\vec{\ell} = -mg\hat{y} \cdot (\hat{x}dx + \hat{y}dy + \hat{z}dz) = -mg\,dy$

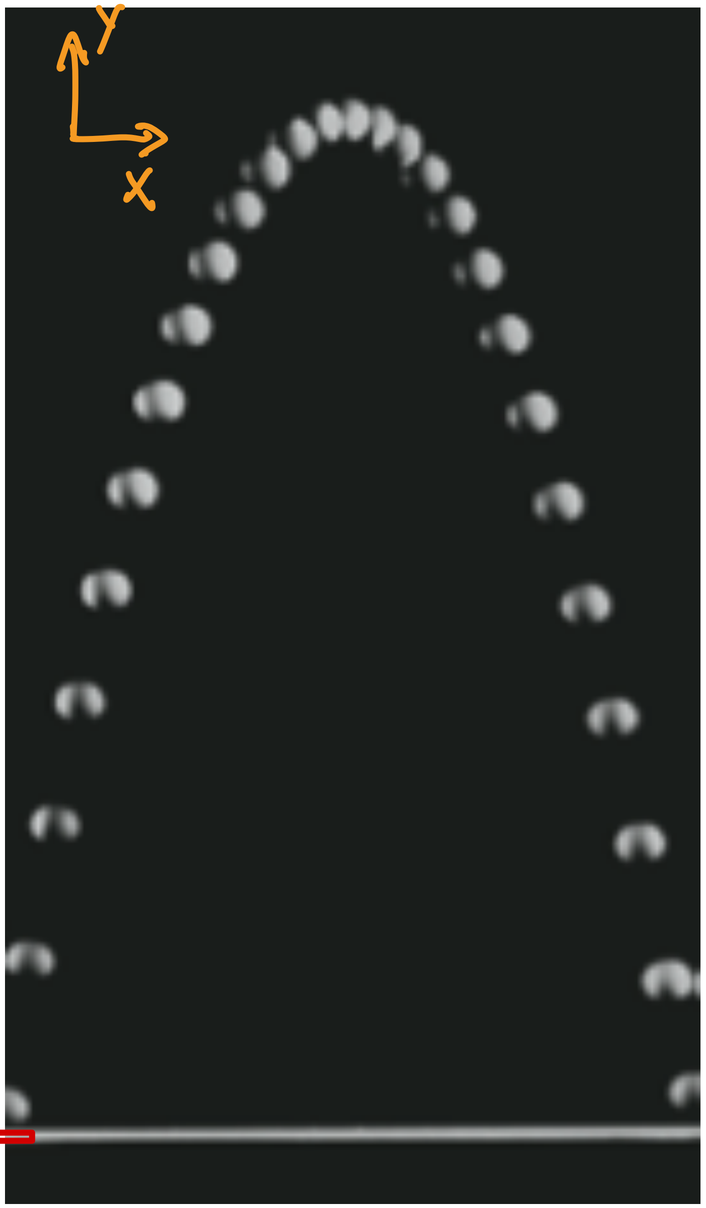
$\Delta U = -W = -\int_a^b \vec{F} \cdot d\vec{\ell}$

$= \cancel{-}\int_{y_a}^{y_b} \cancel{-}mg\,dy = mg\int_{y_a}^{y_b} dy$

$= mg(y_b - y_a)$

$\Delta U = U_{gb} - U_{ga} = mgy_b - mgy_a$

$$\boxed{U_g = mgy + C}$$



$y=0$

$U_g = C = 0$

Force and potential energy

$$\Delta U = U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}$$

- Given the potential energy, how do we find the force?
- In 1D, it is straightforward — just take the derivative

For example system only in y: $\vec{F} = F_y \hat{y}$ $\quad d\vec{\ell} = \hat{y}\, dy$

$$\Delta U = U(y) - U(y_{ref}) = -\int_{y_{ref}}^{y} \vec{F} \cdot d\vec{\ell} = -\int_{y_{ref}}^{y} F_y \, dy$$

$$\frac{d}{dy}\left[U(y) - U(y_{ref})\right] = \frac{d}{dy}\left[-\int_{y_{ref}}^{y} F_y \, dy\right] = -F_y \quad \Rightarrow \quad \boxed{F_y = -\frac{dU}{dy}}$$

For gravity, $U_g(y) = mgy + C \Rightarrow F_y = -\frac{d}{dy}\left[mgy + C\right] = -mg$

Force and potential energy

$$\Delta U = U_b - U_a = -\int_a^b \overrightarrow{F} \cdot d\overrightarrow{\ell}$$

- Given the potential energy, how do we find the force?
- In 3D, we must use the *gradient* $\overrightarrow{\nabla}$

$$\overrightarrow{F} = -\overrightarrow{\nabla} U = -\left(\hat{x}\frac{\partial U}{\partial x} + \hat{y}\frac{\partial U}{\partial y} + \hat{z}\frac{\partial U}{\partial z}\right)$$

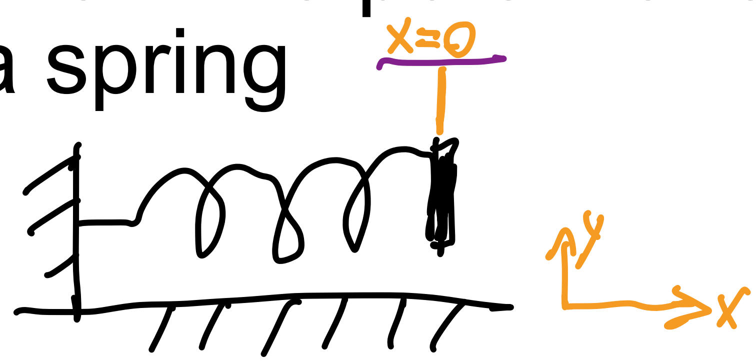
For example, $U(x,y,z) = Kxy$

$$\vec{F} = -\vec{\nabla} U = -\hat{x}\frac{\partial}{\partial x}[Kxy] - \hat{y}\frac{\partial}{\partial y}[Kxy] - \hat{z}\frac{\partial}{\partial z}[Kxy]$$

$$= -\hat{x}Ky - \hat{y}Kx$$

Potential energy diagrams

- A *potential energy diagram* is just a plot of potential energy versus position
- Draw the potential energy diagram for a spring

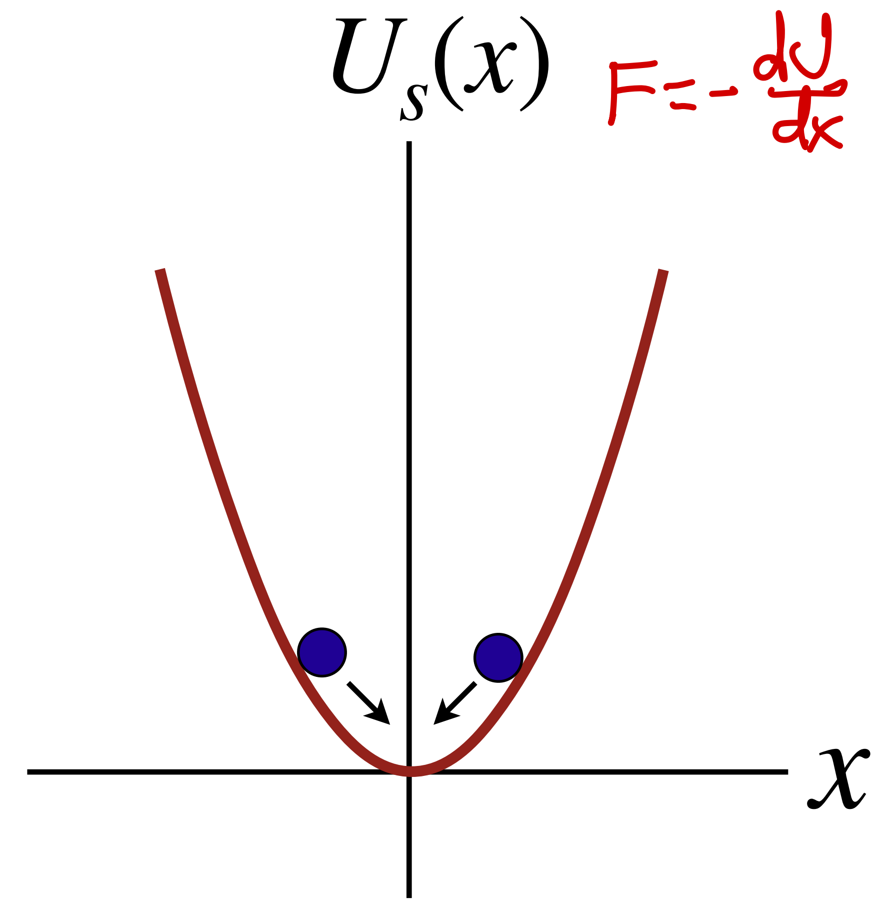


$$\vec{F}_s = -k(x - x_0)\hat{x}, \quad x_0 = 0$$
$$= -kx\hat{x}$$
$$d\vec{l} = \hat{x}\,dx$$

$$\Delta U = -W = -\int_{x_a}^{x_b} \vec{F}_s \cdot d\vec{l} = +\int_{x_a}^{x_b} (+kx)\,dx$$
$$= k\int_{x_a}^{x_b} x\,dx = \frac{1}{2}k\left(x_b^2 - x_a^2\right)$$
$$U_{sb} - U_{sa} = \frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2$$

$$\boxed{U_s = \frac{1}{2}kx^2 + C}$$

$F = -\frac{dU}{dx}$



Potential energy diagram for a spring

Potential energy diagrams

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- Equilibrium points:

$$F = 0$$
$$\frac{dU}{dx} = 0$$

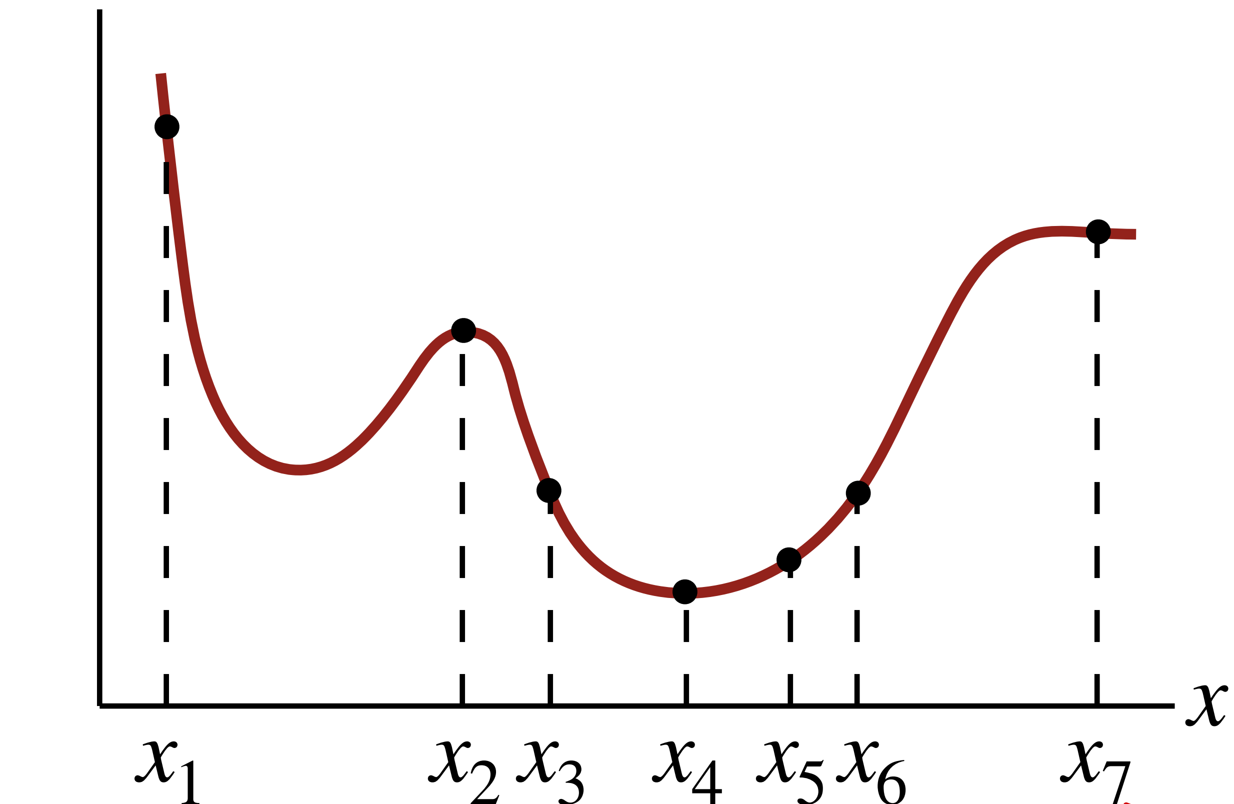
- Stable equilibrium:

$$\frac{dU}{dx} = 0$$
$$\frac{dF}{dx} < 0 \Rightarrow \frac{d^2U}{dx^2} > 0$$

- Unstable equilibrium:

$$\frac{dF}{dx} > 0 \Rightarrow \frac{d^2U}{dx^2} < 0$$

$$F = -\frac{dU}{dx}$$



Potential energy diagram for a particle moving under the influence of a conservative force

$$\frac{dF}{dx} = 0$$ Neutrally stable

Conceptual question

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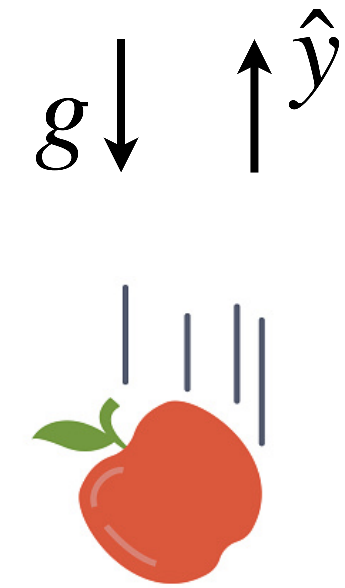
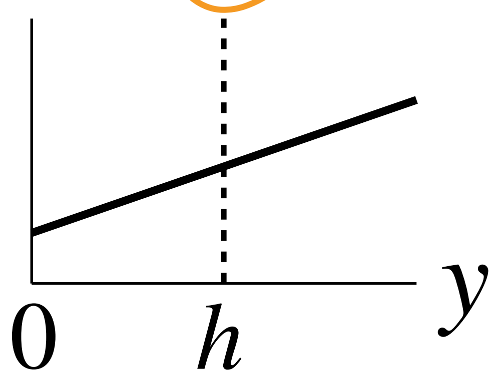
An apple at height h falls from a tree. What is the potential energy diagram $U(y)$ for this situation?



A



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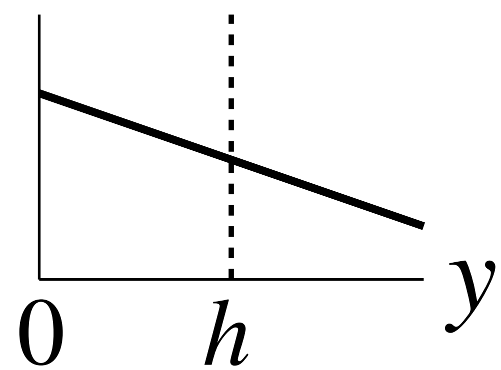

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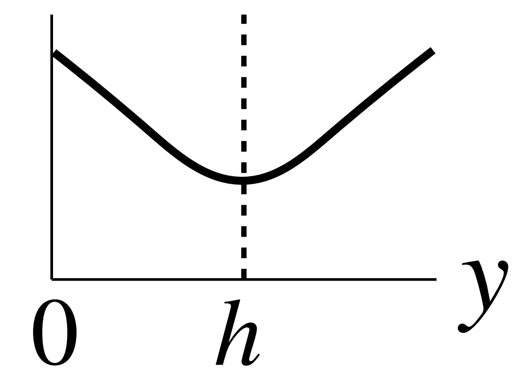

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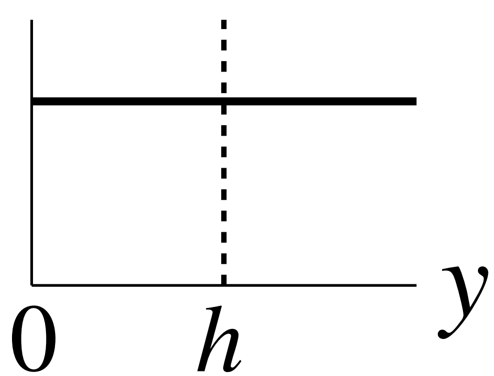
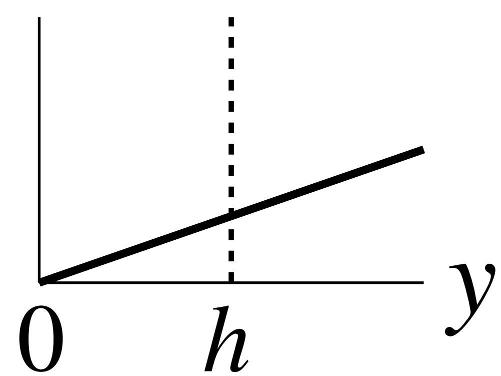
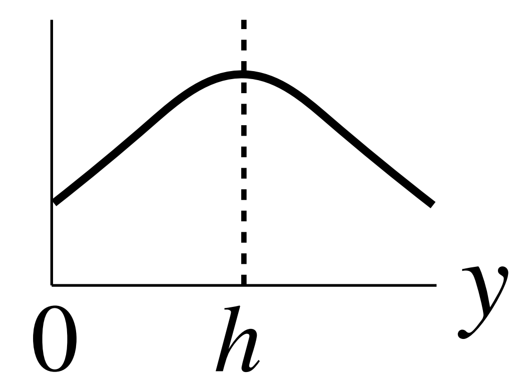


E



F



$U_g = mgy + C$

Conceptual question

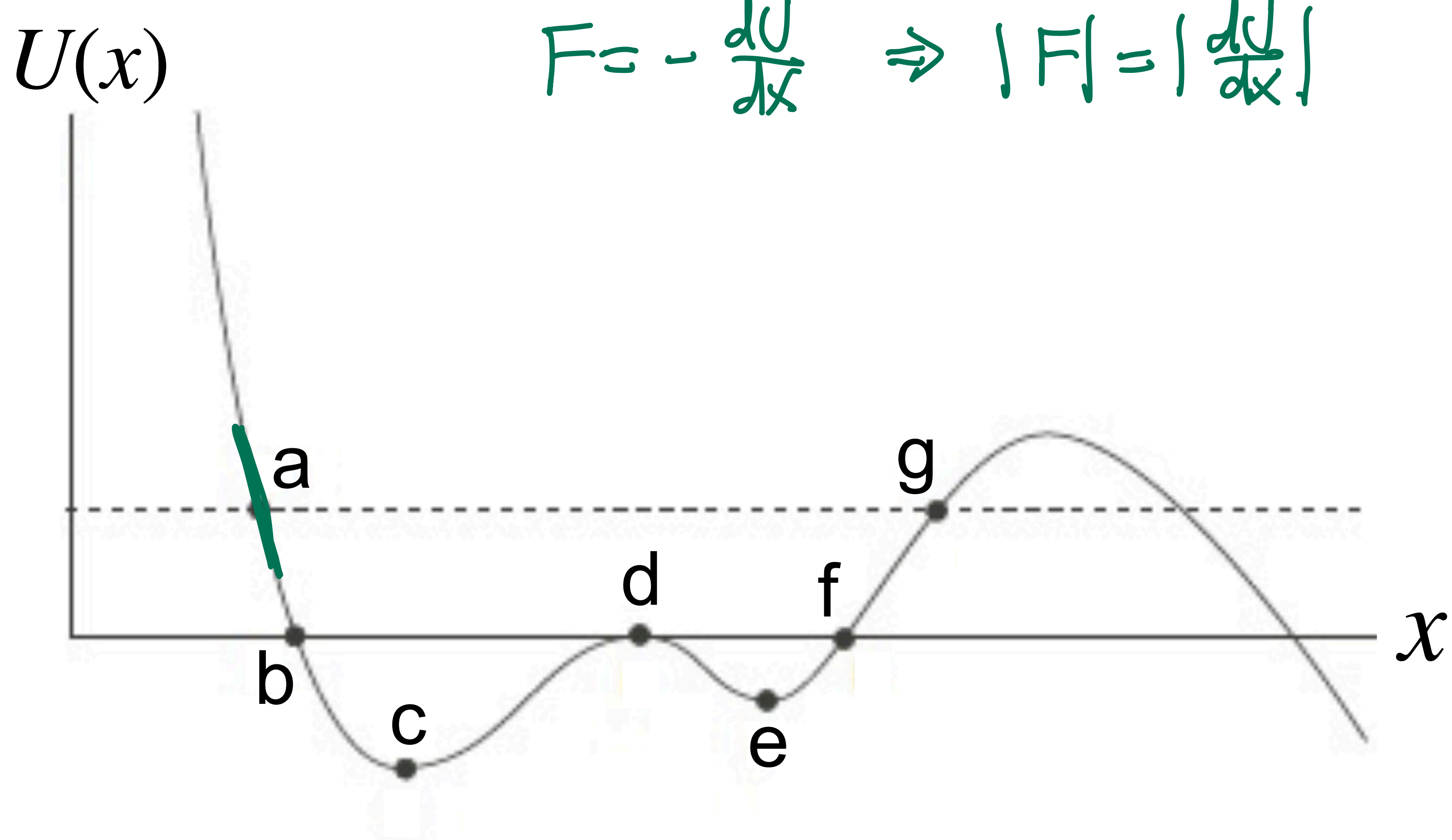
responseware.eu
Session ID: epflphys101en

The figure below shows the potential energy diagram for a particle executing one dimensional motion between points "a" and "g".

At which point will the magnitude of the force be the largest?

A. a
B. b
C. c
D. d
E. e
F. f
G. g

$F = -\frac{dU}{dx} \Rightarrow |F| = \left|\frac{dU}{dx}\right|$



Conceptual question

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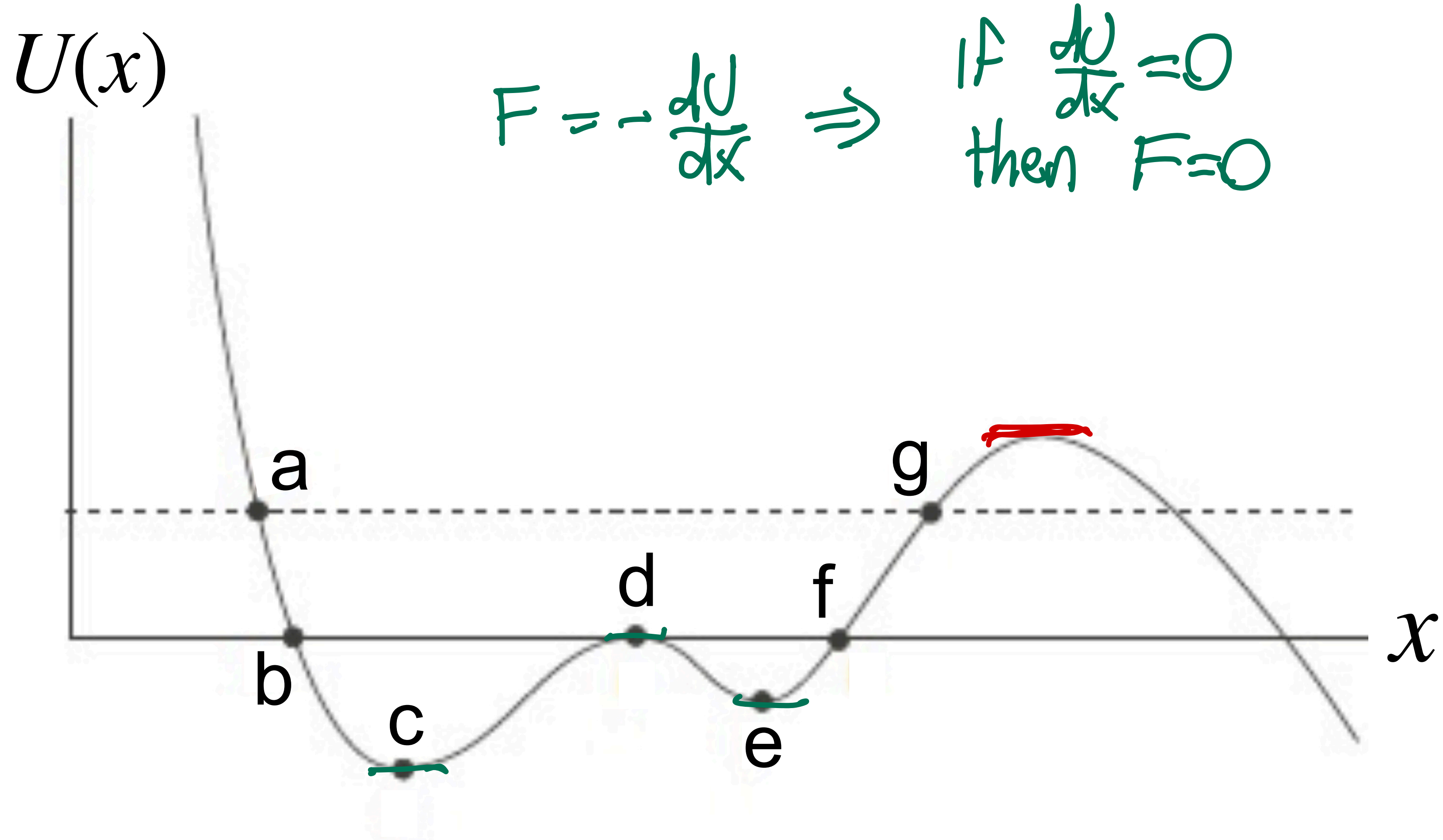
The figure below shows the potential energy diagram for a particle executing one dimensional motion between points “a” and “g”.

At how many of the points will the force be zero?

A. 0
B. 1
C. 2
D. 3
E. 4

$F = -\frac{dU}{dx} \Rightarrow$ If $\frac{dU}{dx} = 0$ then $F = 0$



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1. A few words on Mock exam 1
2. Power
3. **Energy and work**
   - Potential energy
   - **Conservation of mechanical energy**

Conservation of energy

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- Energy is conserved! Always.

The total energy is neither increased nor decreased in any process.

Conservation of energy

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- Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant

Conservation of energy

- Energy is conserved! Always.

The total energy is neither increased nor decreased in any process.

- Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant

- Whenever it seems that energy is disappearing, we always find that it is actually just hiding in a different form

- No known exceptions

Energy has many different forms

- Mechanical
  - Kinetic
    - Translational

Energy has many different forms

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- Mechanical
  - Kinetic
    - Translational
    - Rotational

Energy has many different forms

- Mechanical
  - Kinetic
    - Translational
    - Rotational
  - Potential
    - Gravitational
    - Elastic (spring)

Energy has many different forms

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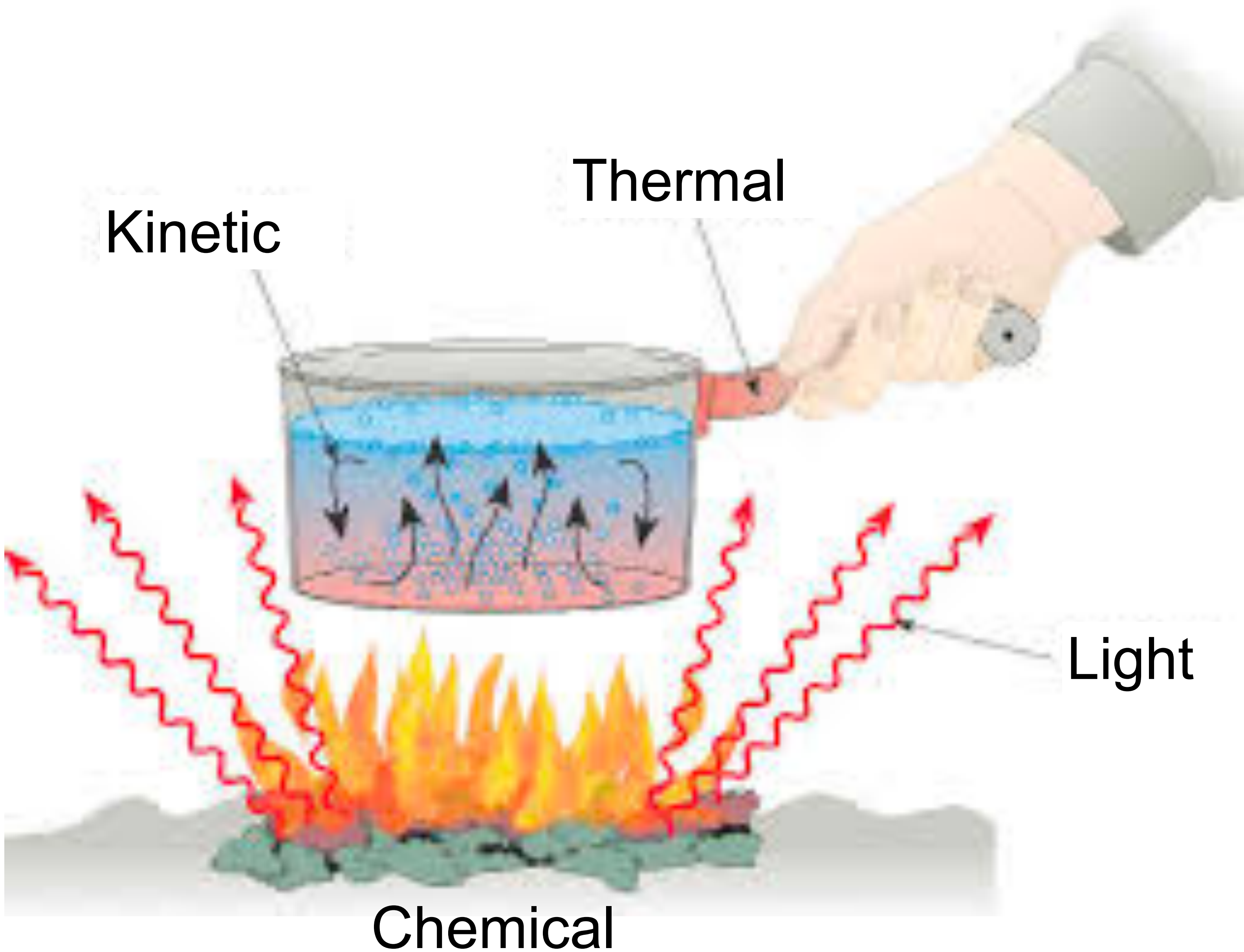
- Mechanical
  - Kinetic
    - Translational
    - Rotational
  - Potential
    - Gravitational
    - Elastic (spring)
- Thermal
- Chemical
- Nuclear
- Electrical
- Light (electromagnetic)
- and more!

Energy has many different forms

- Mechanical
  - Kinetic
    - Translational
    - Rotational
  - Potential
    - Gravitational
    - Elastic (spring)
- Thermal
- Chemical
- Nuclear
- Electrical
- Light (electromagnetic)
- and more!



DEMO (483, 86, 764 and 38)

Conservation of energy

Conservation of mechanical energy

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- *Mechanical energy* refers to the energy of an object's motion (i.e. kinetic) and the energy that is stored in an object by its position (i.e. potential)

$$E_m = K + U$$

- If all forces doing work on a system are *conservative*, then its mechanical energy is conserved

- Can be proved from the work-kinetic energy theorem applied to conservative forces

Conservation of mechanical energy

$$\Delta K = W_{net} = \underline{W_{cons}} + W_{non} \qquad \rightarrow W_{cons} = \underline{\Delta U}$$

$$= -\Delta U + W_{non}$$

$$\Rightarrow W_{non} = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i) = \underbrace{(K_f + U_f)}_{= E_{mf}} - \underbrace{(K_i + U_i)}_{= E_{mi}}$$

$$\Rightarrow \boxed{W_{non} = E_{mf} - E_{mi} = \Delta E_m}$$

if $W_{non} = 0$, then $\Delta E_m = 0$

DEMO (701)

Tension in a pendulum

Conceptual question

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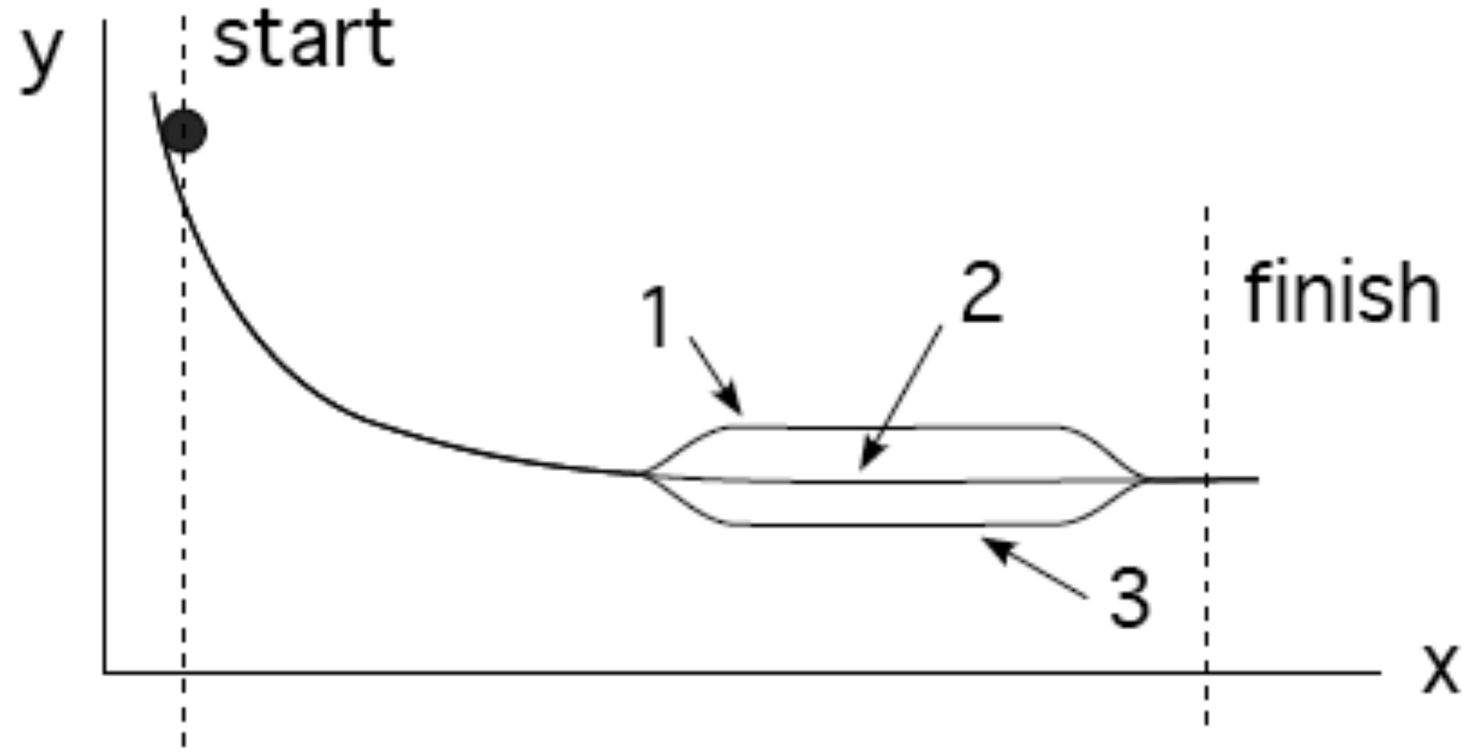
An object starts from rest and slides down a frictionless hill without air drag. Which path leads to the highest speed at the finish?

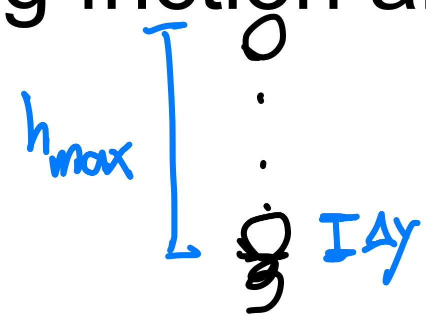


A. 1
B. 2
C. 3
D. All are the same.

Conceptual question

A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height of h_{max}. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far up does the dart go this time, neglecting friction and assuming an ideal spring?

A. $h_{max}/4$
B. $h_{max}/2$
C. h_{max}
D. $2h_{max}$
E. $4h_{max}$



$\Delta E_m = 0$ because $W_{net} = 0$

$E_i = \cancel{K_i} + U_{si} + \cancel{U_{gi}}$

$E_f = \cancel{K_f} + \cancel{U_{sf}} + U_{gf}$

$\Rightarrow U_{si} = U_{gf}$

$\frac{1}{2}K(\Delta y)^2 = mgh_{max}$

$\Rightarrow h_{max} = \frac{K}{2mg}(\Delta y)^2$

If now $\Delta y_s = \frac{1}{2}\Delta y \Rightarrow (\Delta y_s)^2 = \frac{1}{4}(\Delta y)^2$

$h_s = \frac{K}{2mg}(\Delta y_s)^2 = \frac{1}{4}\frac{K}{2mg}(\Delta y)^2 = \frac{1}{4}h_{max}$

Conceptual question

A stone is launched directly upwards into the air. In addition to the force of gravity, the stone is subject to a drag force due to air resistance. The time the stone takes to reach the top of its trajectory is…

A. greater than…
B. equal to…
C. less than…

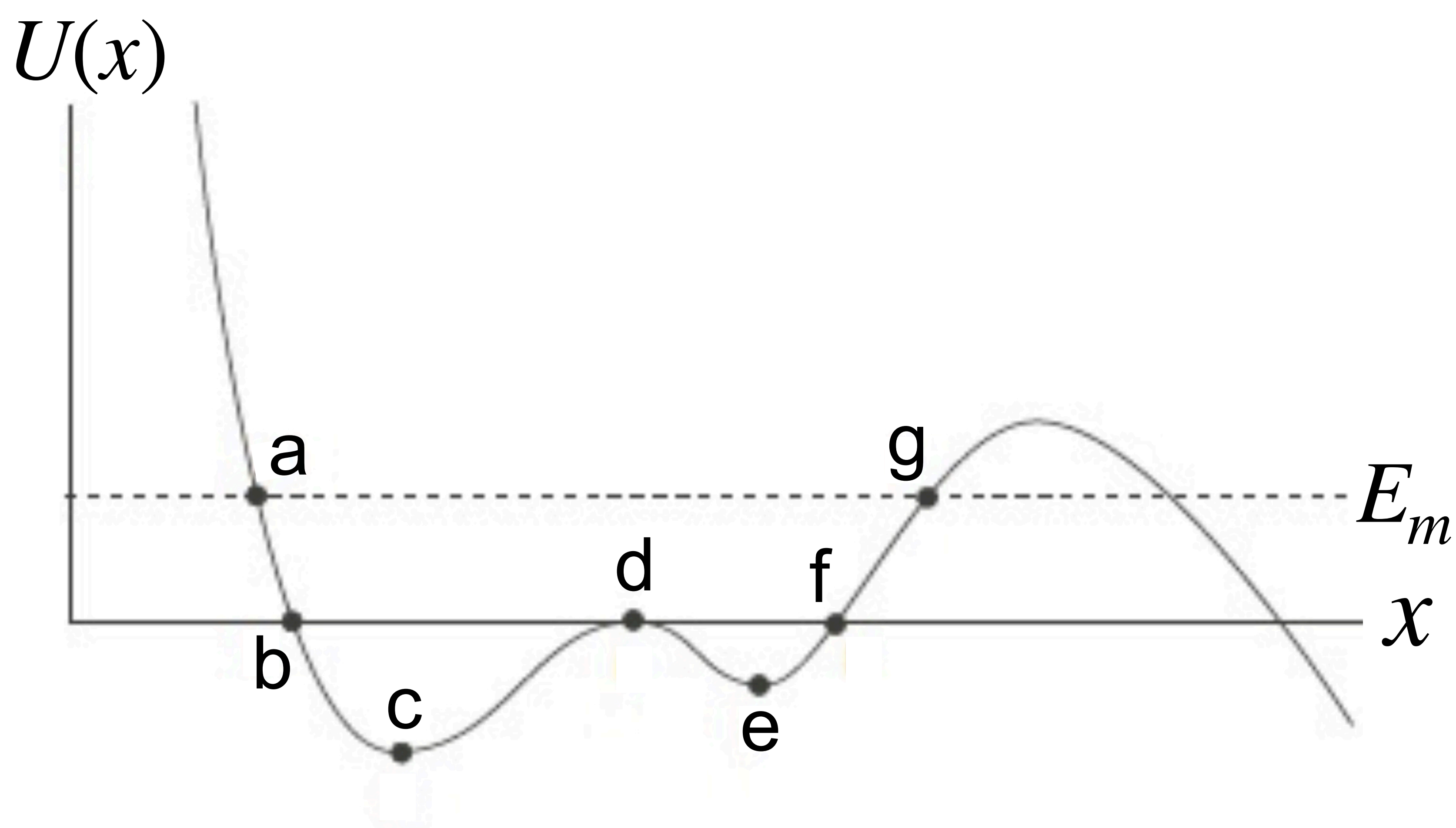
the time it takes to return from the top to its original position.

Conceptual question

The figure below shows the potential energy diagram for a particle executing one dimensional motion between points “a” and “g”. The total mechanical energy of the system is indicated by the dashed line.

At which point will the kinetic energy be the largest?

A. a
B. b
C. c
D. d
E. e
F. f
G. g



Conceptual question

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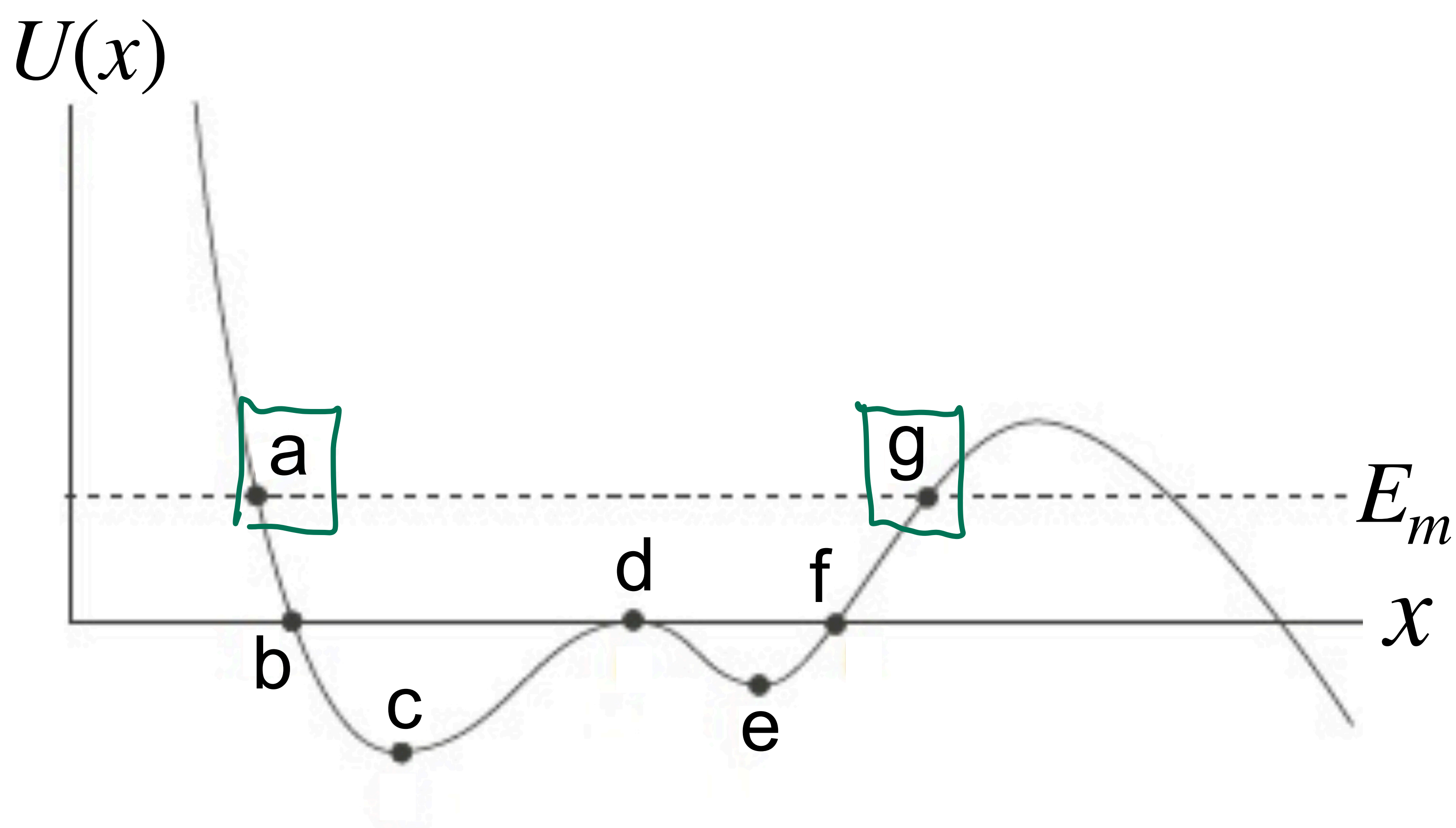
Swiss Plasma Center

The figure below shows the potential energy diagram for a particle executing one dimensional motion between points "a" and "g". The total mechanical energy of the system is indicated by the dashed line.

At how many of the labeled points will the velocity be zero?

A. 0
B. 1
C. 2
D. 3
E. 4



Summary

- Power is rate at which work is done (or energy is transferred/ converted)
- The potential energy, which is only associated with conservative forces, is given by

$$\Delta U = -W, \quad \text{e.g.} \quad U_g = mgy \quad \text{and} \quad U_s = kx^2/2$$

- Force related to the derivative (or gradient) of the potential
- **Total energy, considering all forms, is *always* conserved**
- Total mechanical energy is the sum of kinetic energy and potential energy
- If all forces doing work are conservative, then mechanical energy is conserved

See you tomorrow for cliff jumping