General Physics: Mechanics

PHYS-101(en)

Lecture 8b: Kinetic energy and work

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November 5th, 2024

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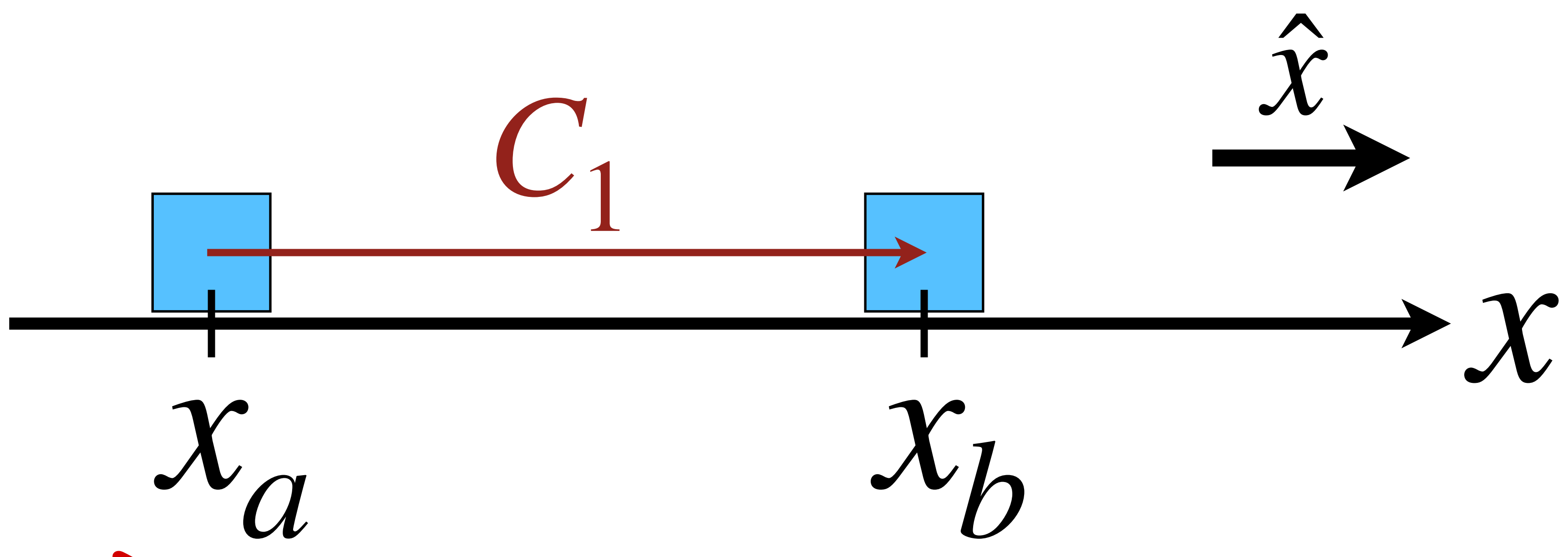
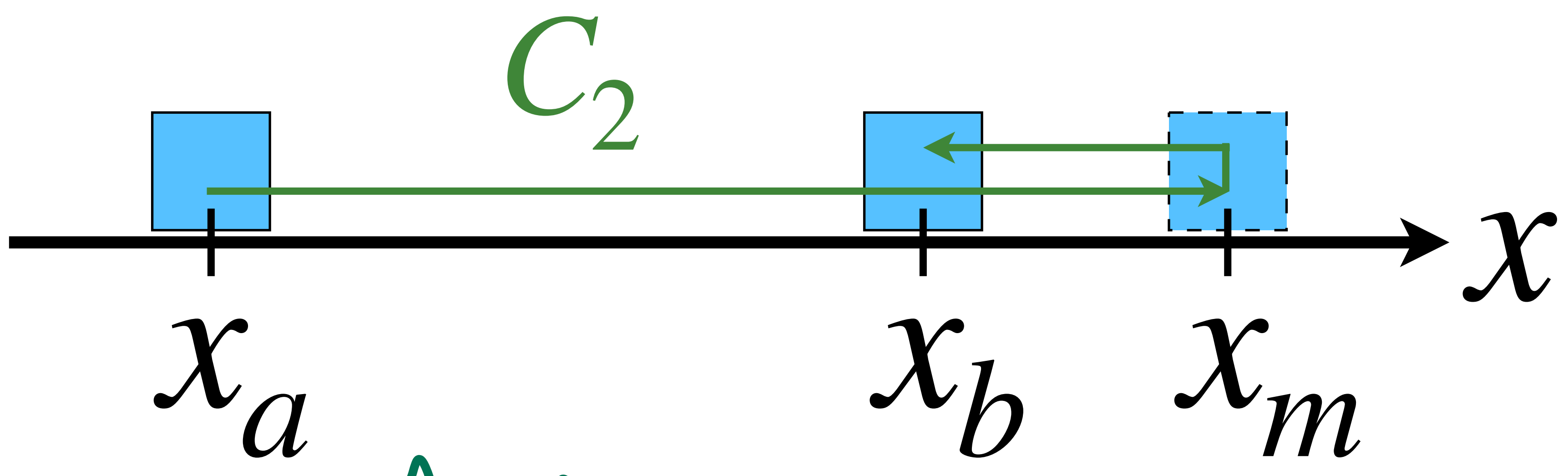
Path dependence of kinetic friction

Show that the kinetic friction force $\vec{F}_k = \pm\,\mu_k N\,\hat{x}$ is not conservative





$\vec{F}_K = -\mu_K N \hat{x}$ $\qquad \vec{dl} = \hat{x}\, dx$

$W_K = \int_{C_1} \vec{F}_K \cdot \vec{dl} = \int_a^b (-\mu_K N \hat{x}) \cdot (\hat{x}\, dx)$

$= \int_{x_a}^{x_b} (-\mu_K N)\, dx = -\mu_K N \int_{x_a}^{x_b} dx$

$= -\mu_K N (x_b - x_a)$

$\neq$

$\vec{F}_K = \begin{cases} -\mu_K N \hat{x} & \text{from } x_a \text{ to } x_m \\ \mu_K N \hat{x} & \text{from } x_m \text{ to } x_b \end{cases}$

$W_K = \int_{C_2} \vec{F}_K \cdot \vec{dl} = \int_{x_a}^{x_m} (-\mu_K N \hat{x}) \cdot (\hat{x}\, dx) + \int_{x_m}^{x_b} \mu_K N\, dx$

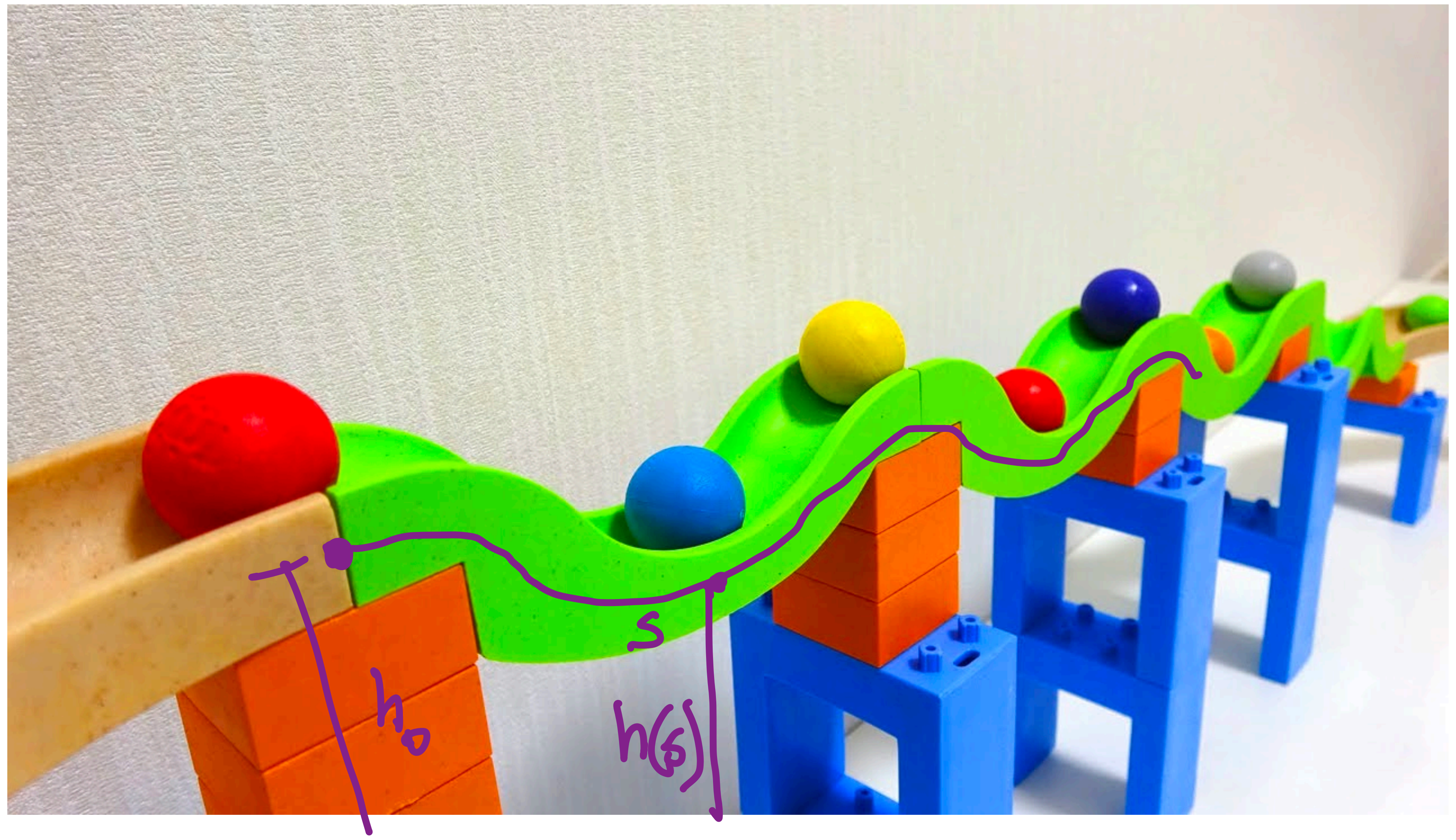
$= -\mu_K N \int_{x_a}^{x_m} dx + \mu_K N \int_{x_m}^{x_b} dx$

$= -\mu_K N (x_m - x_a) + \mu_K N (x_b - x_m)$

$= -\mu_K N (2x_m - x_a - x_b)$

Example: Marbula One racing

You're designing a *marble racing* track! The course is straight (i.e. no left or right turns) and the marbles are released from rest at a height h_0. Given the height $h(s)$ as a function of the distance s along the track, you need to calculate the marble's speed $v(s)$ to ensure they can make it to the bottom. Ignore friction and drag.



$$\Delta K = W_{net}$$

$$\Delta K = \frac{1}{2}mv^2(s) - \cancel{\frac{1}{2}mv^2(0)} \to 0$$

$$\frac{1}{2}mv^2(s) = W_{net}$$

Example: Marbula One racing

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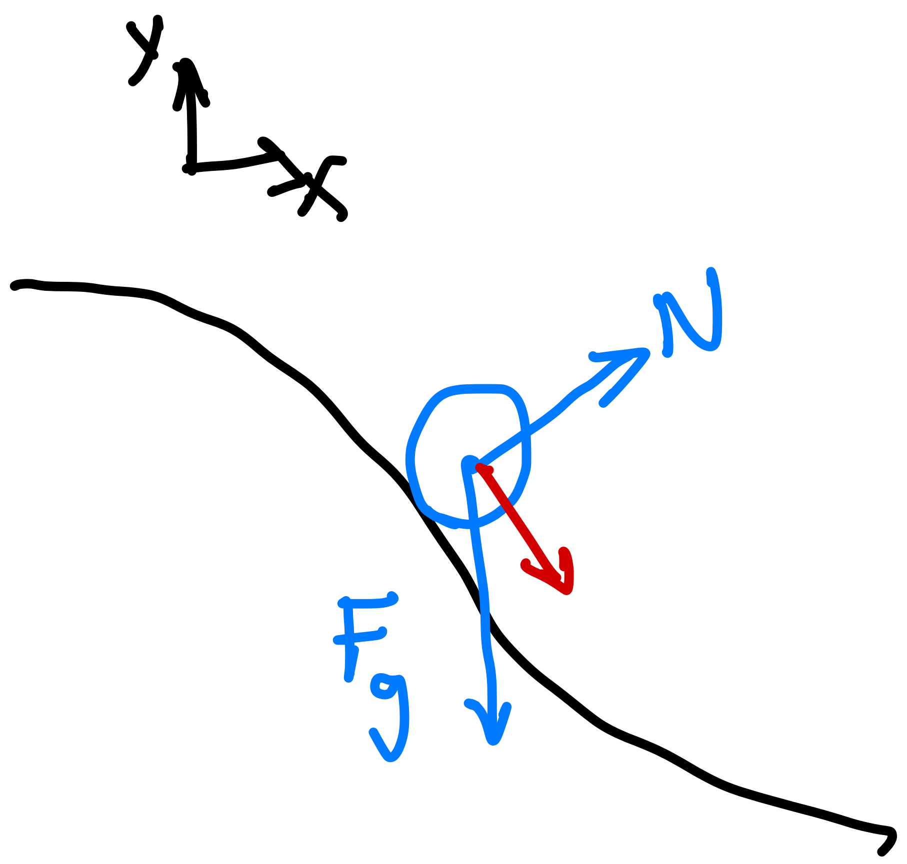


$$W_{net} = \int_C \vec F_{net} \cdot d\vec l = \int_C (\vec F_g + \vec N)\cdot d\vec l = \int_C \vec F_g \cdot d\vec l + \cancelto{0}{\int_C \vec N \cdot d\vec l}$$

$$= \int_C \vec F_g \cdot d\vec l = \int_C (-mg\hat y)\cdot(\hat x dx + \hat y dy) = \int_C -mg\, dy$$

$$= \int_{h_0}^{h(s)} -mg\, dy = -mg(h(s)-h_0) = mg(h_0 - h(s))$$

$\vec N \cdot d\vec l = 0$

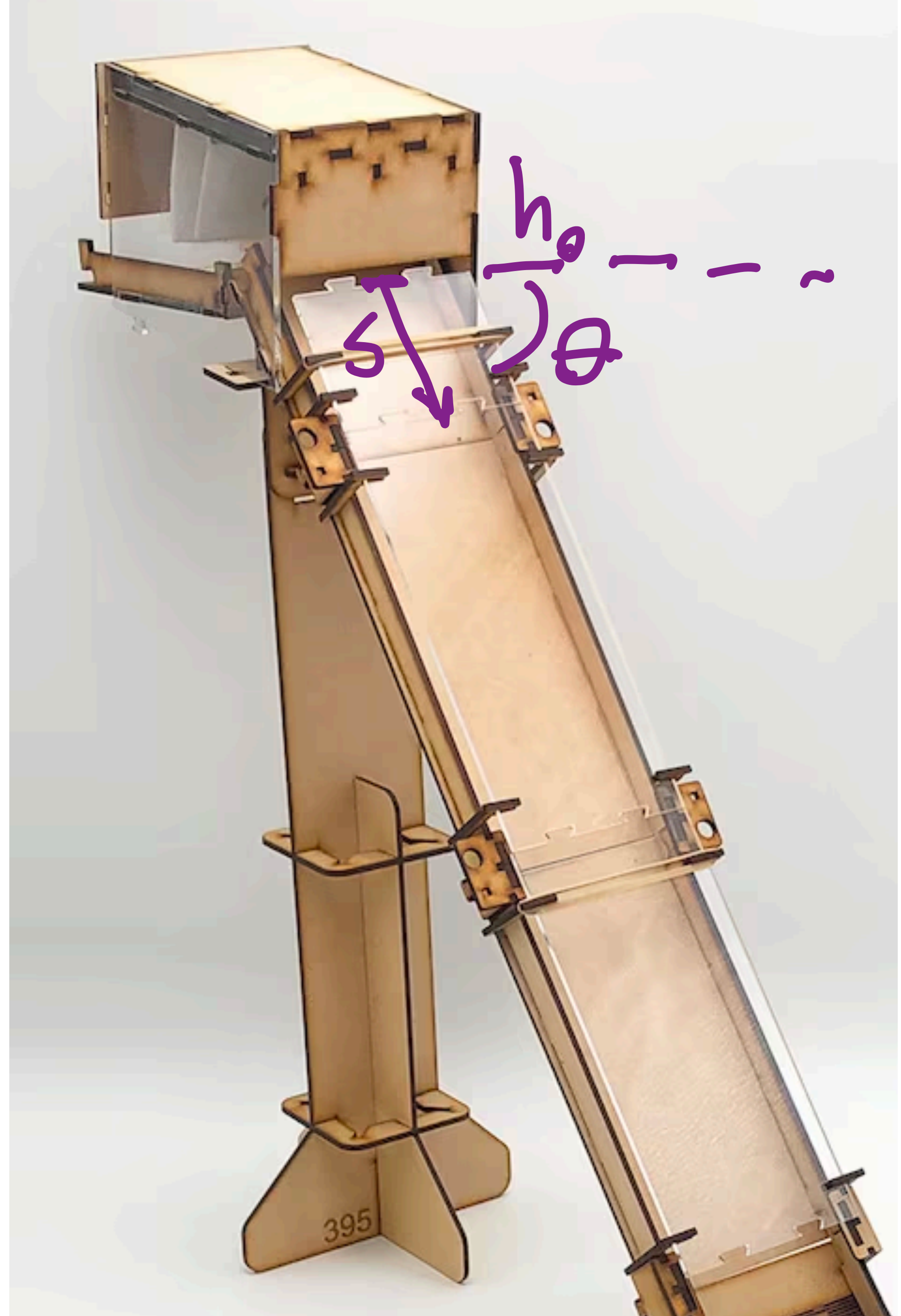
$$\frac{1}{2}\cancel{m}v^2(s) = \cancel{m}g(h_0 - h(s)) \Rightarrow v^2(s) = 2g(h_0 - h(s))$$

$$\Rightarrow v(s) = \sqrt{2g(h_0 - h(s))}$$

We require $h_0 > h(s)$

Example: Marbula One racing (w/ friction)

You're designing a *marble racing* track! The course is straight (i.e. no left or right turns) and the marbles are released from rest at the top. Given a **linear height profile that makes an angle θ *below* the horizon,** $h(s) = -\sin\theta\ s + h_0$, you need to calculate the marble's speed $v(s)$ to ensure they can make it to the bottom. Let the friction coefficient be μ_k and ignore drag.



$\Delta K = W_{net}$ $\quad \Delta K = \frac{1}{2} m v^2(s) - \frac{1}{2} m \cancel{v^2(0)}^{0} = \frac{1}{2} m v^2(s)$

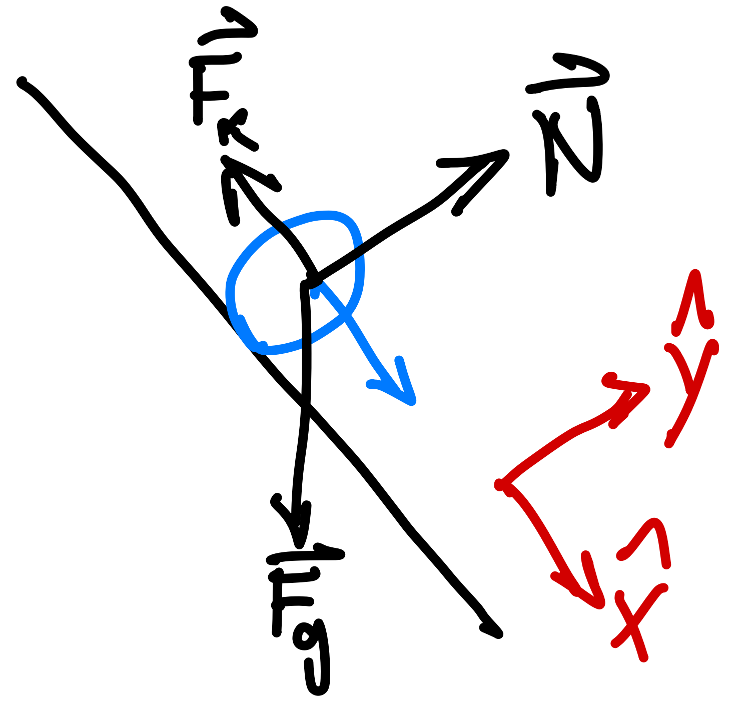
$\vec{F}_{net} = \vec{F}_g + \vec{N} + \vec{F}_K$

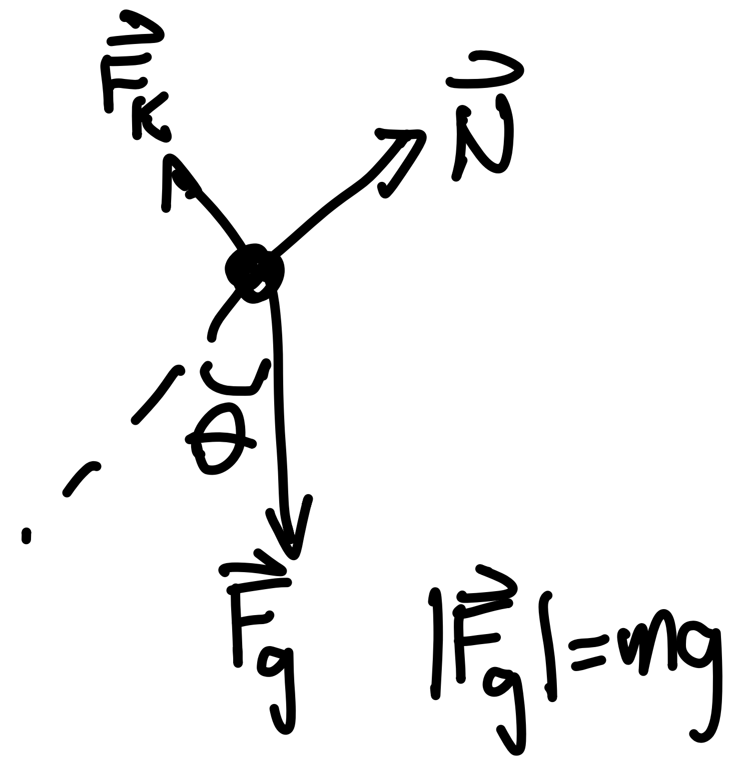
$W_{net} = \int \vec{F}_{net} \cdot d\vec{l} = \int (\vec{F}_g + \vec{N} + \vec{F}_K) \cdot d\vec{l}$

$= \underline{\int_C \vec{F}_g \cdot d\vec{l}} + \int_C \vec{F}_K \cdot d\vec{l}$

$= mg(h_0 - h(s)) = mg[\cancel{h_0} + \sin(\theta) \cdot s - \cancel{h_0}] = mgs \cdot \sin(\theta)$

Example: Marbula One racing (w/ friction)





$$\vec{F}_K = \mu_K N(-\hat{x}) = -\mu_K N\hat{x}$$

$$\Sigma F_y: \; N - mg\cdot\cos(\theta) = 0 \Rightarrow N = mg\cdot\cos(\theta)$$

$$W_K = \int_C \vec{F}_K \cdot d\vec{l} = \int_C -\mu_K mg\cos(\theta)\hat{x}\cdot\hat{x}\,dx$$

$$= \int_0^s -\mu_K mg\cos(\theta)\,dx$$

$$= -\mu_K mgs\cos(\theta)$$

$$\frac{1}{2}\cancel{m}v^2(s) = W_{net} = \cancel{m}gs\cdot\sin(\theta) - \mu_K\cancel{m}gs\cdot\cos(\theta)$$

$$\Rightarrow v(s) = \sqrt{2gs[\sin(\theta) - \mu_K\cos(\theta)]}$$

condition: $\sin(\theta) > \mu_K\cos(\theta)$

$$\Rightarrow \tan(\theta) > \mu_K$$