General Physics: Mechanics

PHYS-101(en)

Lecture 6b: Momentum, impulse, and center of mass

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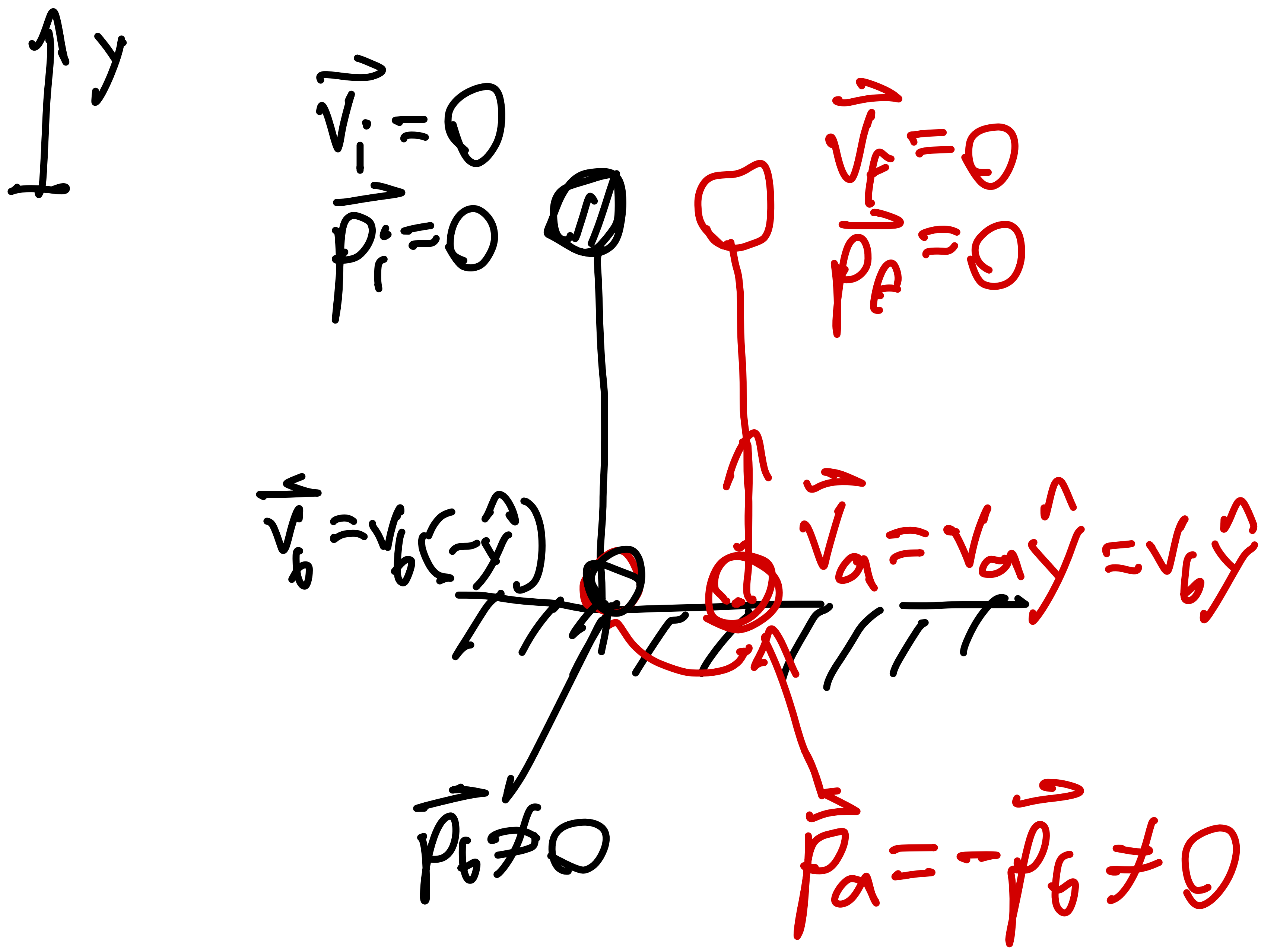
Conceptual question

You drop a ball that bounces off the ground in a perfectly elastic manner. Is the ball's momentum conserved?

A. Yes

B. No

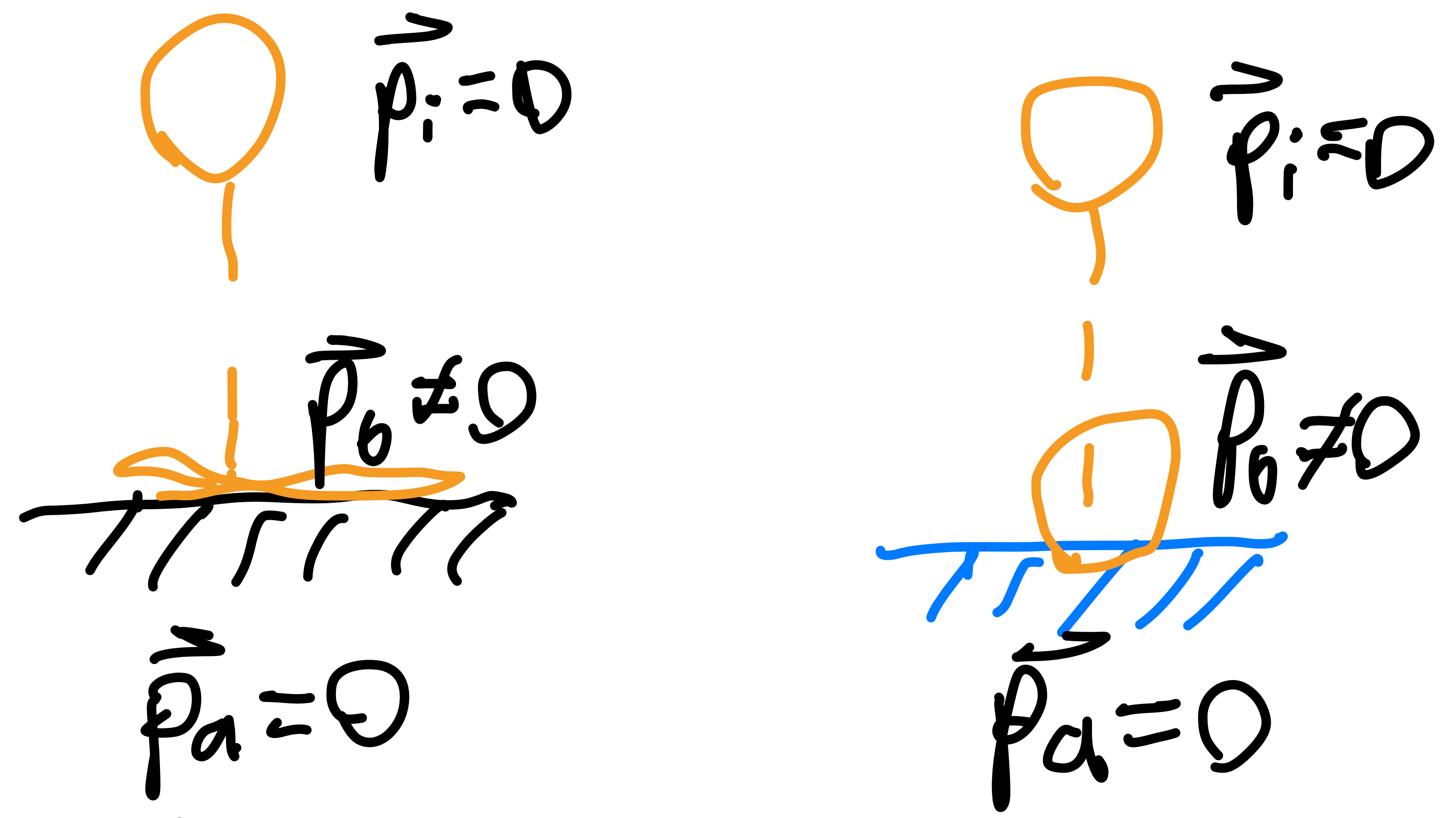


Conceptual question

If you drop an egg on the floor, it will break, but if you drop it (from the same height) on a mattress, it won't. Why?

A. The change in momentum decreases…
B. The average force decreases…
C. The maximum force decreases…
D. The impulse decreases…
E. None of the above.

when using the mattress.



$\Delta\vec{p} = \vec{p}_a - \vec{p}_b = -\vec{p}_b$ $\quad\quad$ $\Delta\vec{p} = \vec{p}_b$

$$\Delta\vec{p} = \vec{I} \approx \int_{t_i}^{t_f} \vec{F}\,dt = F_{avg} \cdot \Delta t$$

Conceptual question

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When Javier Sotomayor broke the high jump world record (going over a 2.45 m high bar), did his center of mass also go over this height?

A. Yes, necessarily.

B. Not necessarily.

Conceptual question

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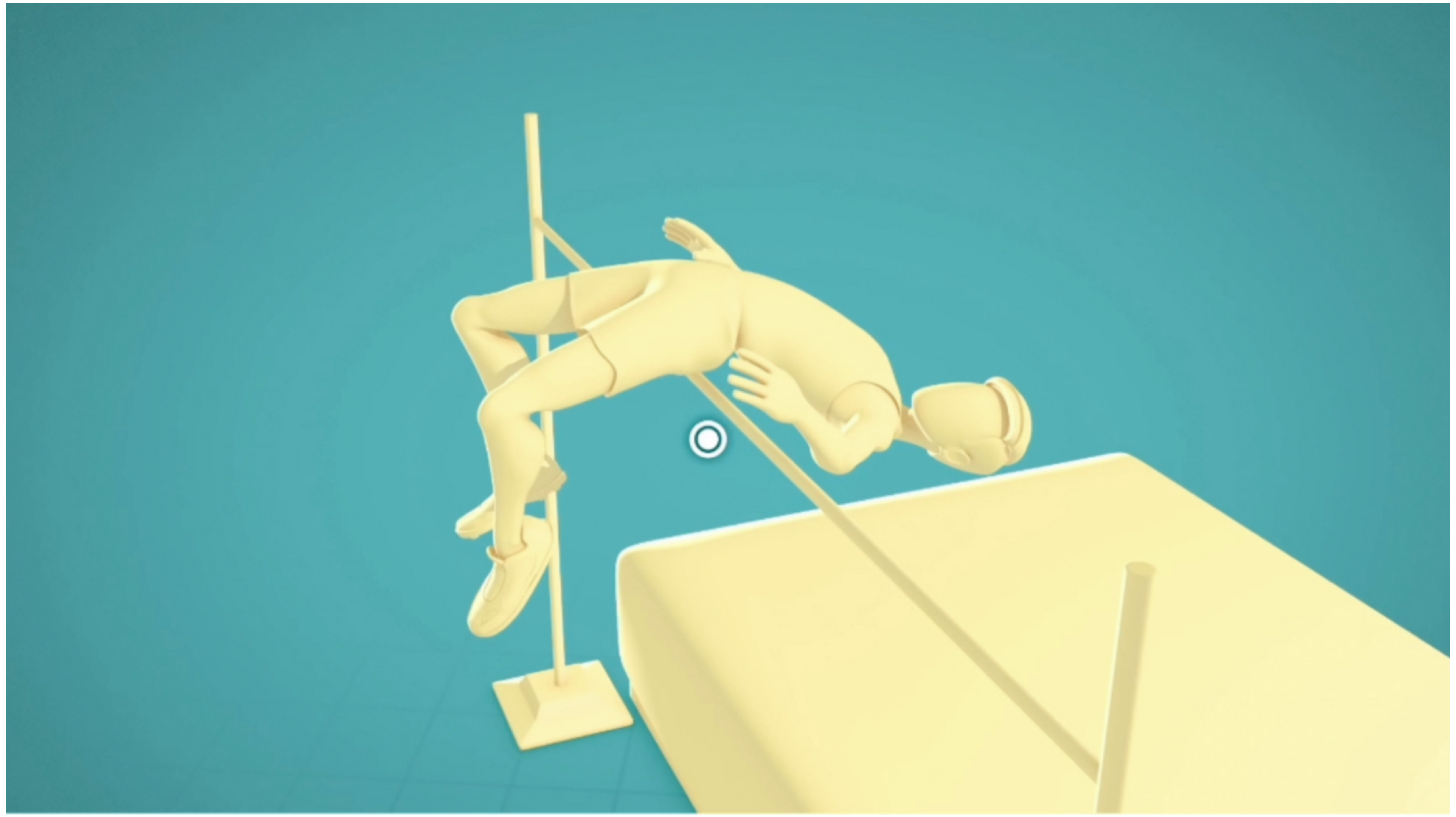

Example: Center of mass Sun-Earth

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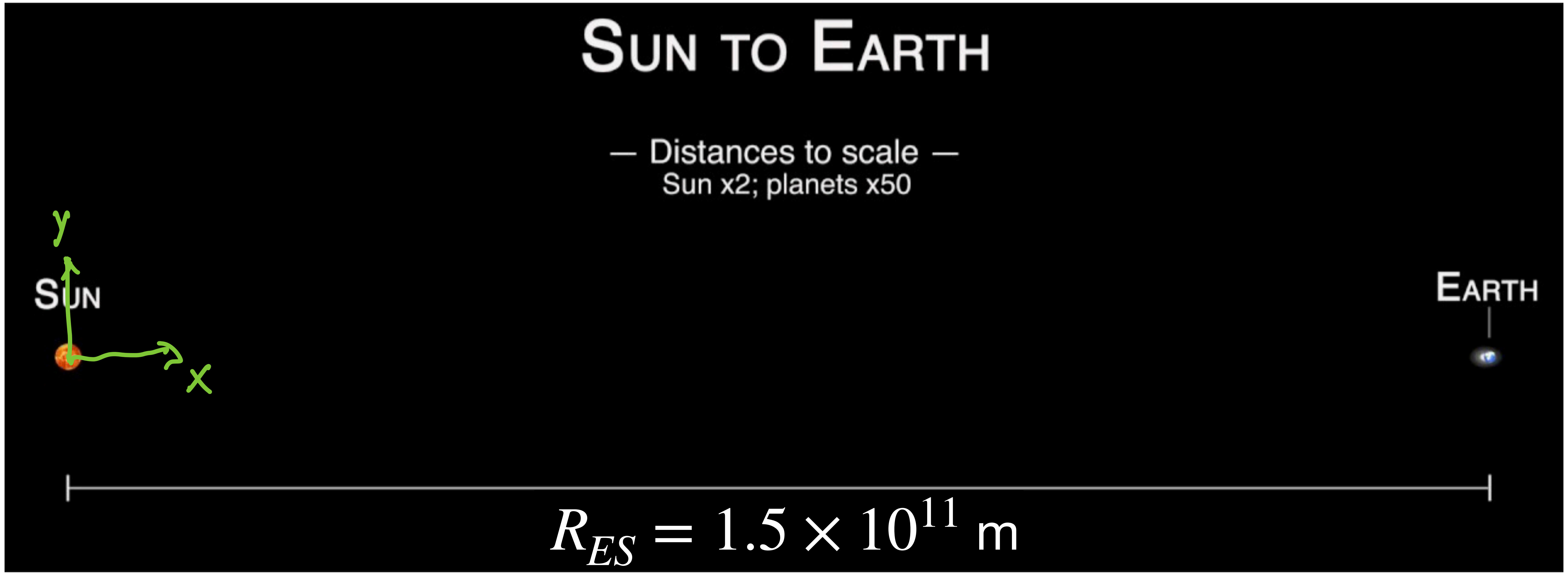
The mean distance from the Earth to the Sun is $R_{ES} = 1.5 \times 10^{11}$ m. The mass of the Earth is $m_E = 6.0 \times 10^{24}$ kg and the mass of the Sun is $m_S = 2.0 \times 10^{30}$ kg. The mean radius of the Earth is $r_E = 6.4 \times 10^{6}$ m. The mean radius of the Sun is $r_S = 6.4 \times 10^{8}$ m. Where is the location of the center of mass of the Earth-Sun system?



$\vec{R}_E = R_{ES}\hat{x}$

$R_S = 0$

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$$\vec{R}_{CM} = \frac{m_S \cancel{\vec{R}_S} + m_E \vec{R}_E}{m_S + m_E} = \frac{m_E}{m_S + m_E} \vec{R}_E = \frac{m_E}{\underline{m_S + m_E}} R_{ES}\,\hat{x} \approx \frac{m_E}{m_S} R_{ES}\,\hat{x} \approx \frac{6 \cdot 10^{24}\,\cancel{kg}}{2 \cdot 10^{30}\,\cancel{kg}} \cdot 1.5 \cdot 10^{11}\,m\,\hat{x}$$

$\vec{R}_S = 0$

$m_S \gg m_E \Rightarrow m_S + m_E \approx m_S$

$$\vec{R}_{cm} = 3 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}\,m\,\hat{x} = \underbrace{4.5 \cdot 10^5\,m}_{\ll R_S}\,\hat{x}$$

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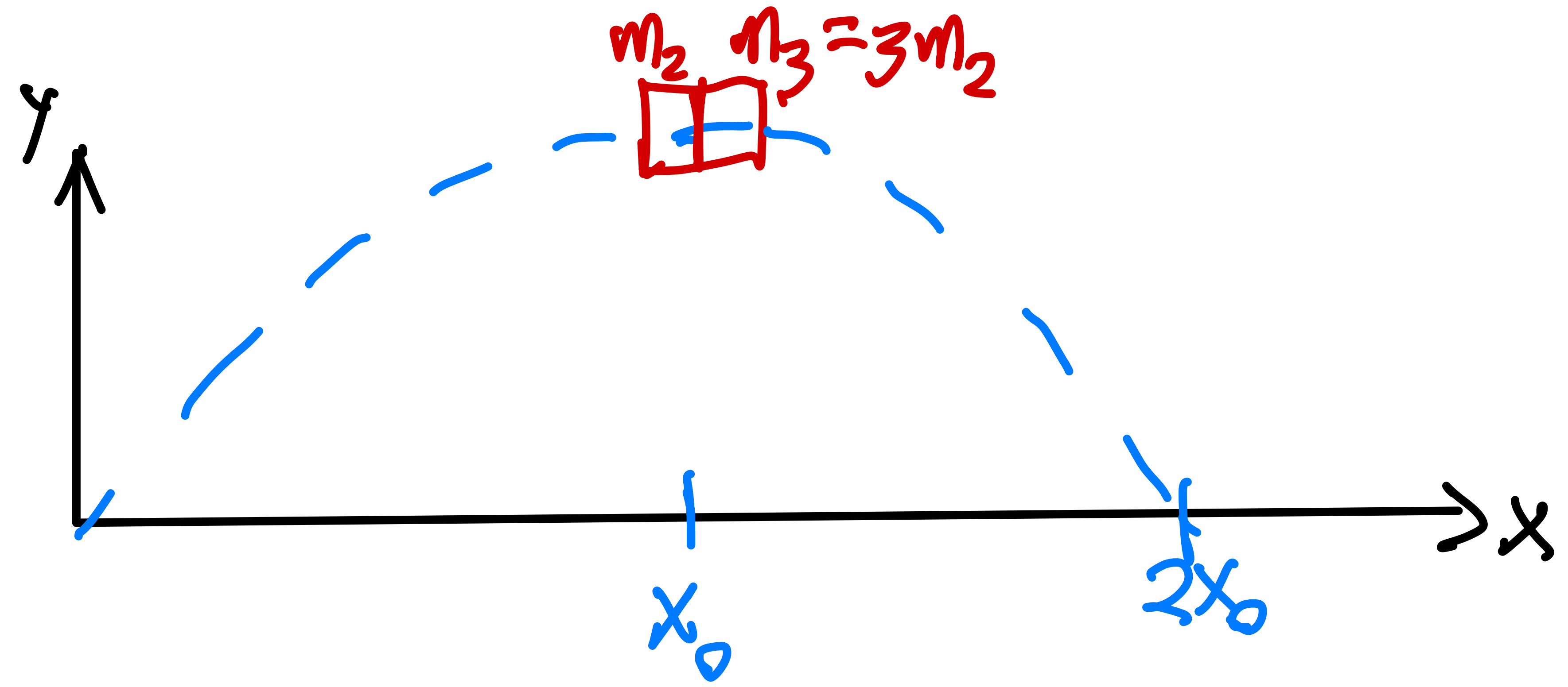
Example: Exploding projectile

An instrument-carrying projectile of mass m_1 accidentally explodes at the top of its trajectory. The horizontal distance between launch point and the explosion is x_0. The projectile breaks into two pieces that fly apart horizontally. The larger piece, m_3, has three times the mass of the smaller piece, m_2. To the surprise of the scientist in charge, the smaller piece returns to earth at the launching station. Neglect air resistance and effects due to the earth's curvature. $m_1 = m_2 + m_3$

How far away, x_3^f, from the original launching position does the larger piece land?



$$\vec{R}_{CM} = \frac{m_2\vec{r}_2 + m_3\vec{r}_3}{m_2+m_3} \Rightarrow X_{CM} = \frac{m_2 x_2 + m_3 x_3}{m_2+m_3}$$

$$=2x_0 \leftarrow X_{CM}^f = \frac{m_2 \cancel{x_2^f}^{\,0} + m_3 x_3^f}{m_2+m_3}$$

$$2x_0 = \frac{m_3}{m_2+m_3} x_3^f \Rightarrow x_3^f = 2\left(\frac{m_2+m_3}{m_3}\right)x_0$$

$$\Rightarrow x_3^f = 2\left(\frac{m_2}{m_3}+1\right)x_0 = 2\left(\frac{m_2}{3m_2}+1\right)x_0 = \frac{8}{3}x_0$$

Different approach: Conservation of momentum along x

Example: Exploding projectile

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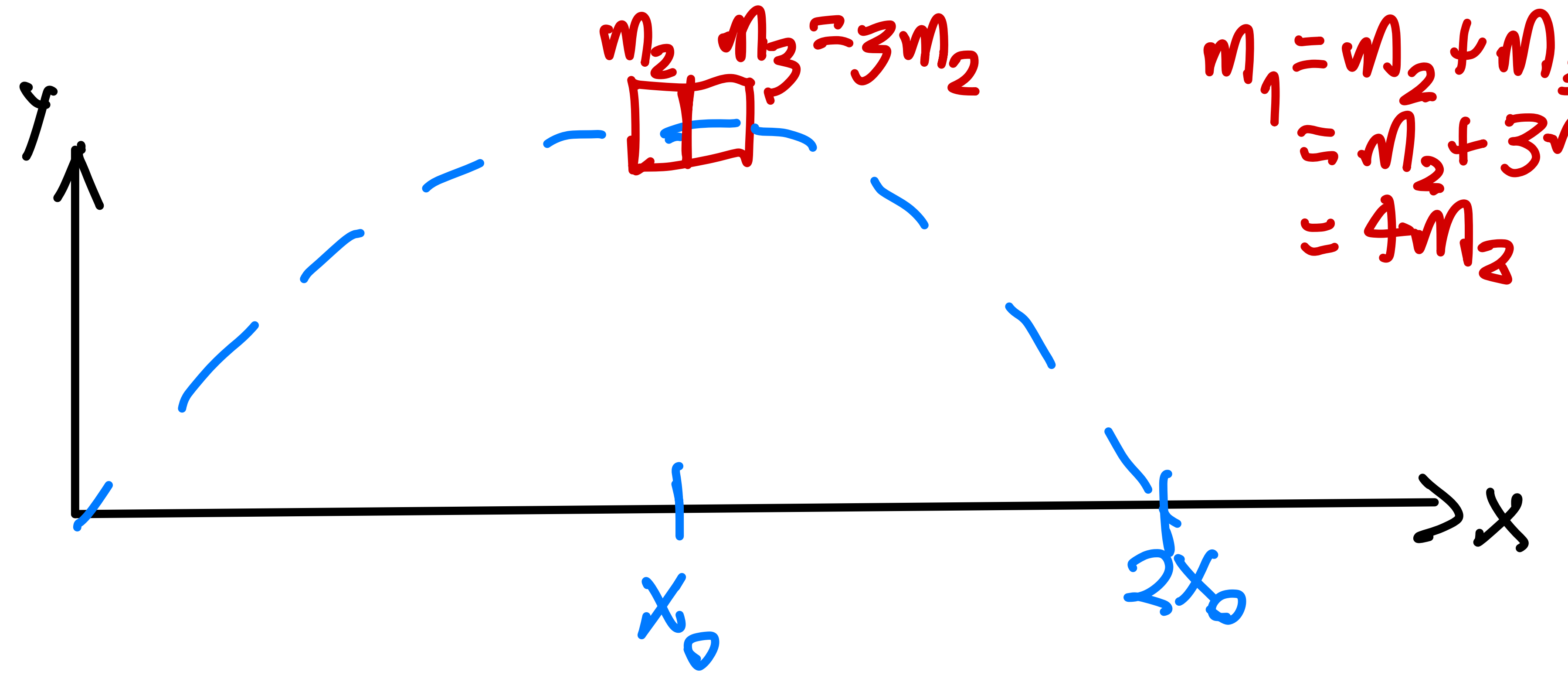

$m_1 = m_2 + m_3$
$= m_2 + 3m_2$
$= 4m_2$

- Velocity of m_1 along x: $v_1 \hat{x}$
- Total momentum along x:

Before explosion: $p_{1x} = m_1 v_1 = p^b_{tot,x}$

After explosion: $p^a_{tot,x} = m_2 v_{2x} + m_3 v_{3x}$
$= -m_2 v_1 + m_3 v_{3x}$

Total momentum along x is conserved:

$$p^a_{tot,x} = p^b_{tot,x} \Rightarrow m_1 v_1 = -m_2 v_1 + m_3 v_{3x}$$
$$\Rightarrow (m_1 + m_2) v_1 = m_3 v_{3x} \Rightarrow v_{3x} = \frac{m_1 + m_2}{m_3} v_1$$
$$\Rightarrow v_{3x} = \frac{4m_2 + m_2}{3m_2} v_1 = \frac{5}{3} v_1$$

Now define $T = \frac{x_0}{v_1}$. This is the time it takes m_1 to reach the top of the trajectory.

Then $v_1 T = x_0$. Also, $v_{3x} T = (x_3^f - x_0)$ because T is also the time it takes for particle m_3 to fall from the top back to the ground. Therefore:

$$v_{3x} T = \frac{5}{3} v_1 T \Rightarrow x_3^f - x_0 = \frac{5}{3} x_0$$
$$\Rightarrow \boxed{x_3^f = \frac{5}{3} x_0 + x_0 = \frac{8}{3} x_0}$$