

General Physics: Mechanics

PHYS-101(en)

Lecture 6a:

**Drag, momentum, impulse,
and center of mass**

Dr. Marcelo Baquero
marcelo.baquero@epfl.ch
October 14th, 2024



Announcement

- Next week is "Fall break"
 - There is no class Monday nor Tuesday.
 - There are no Exercise sessions on Wednesday.
 - Office hours **will take place** as normal.
 - Classes resume Monday, Oct. 28th.

Announcement

- We'll hold a ***mini mock exam*** on Tuesday October 29th
 - In-class (SG1) during normal lecture hours (10:15-11:00)
 - Does **not** matter at all for your final grade
 - You can bring a “cheat” sheet containing formulas or all of your notes, as you wish
 - Take to your TA at the exercise session on Wednesday October 30th, to be graded and returned on Wednesday November 6th
 - Exam solutions will be published

Today's agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. Momentum
 - Conservation of momentum
 - Impulse
3. Center of mass

DEMO (738)

Air drag

Resistive forces, approximately

- Resistive forces oppose the direction of motion of an object

Resistive forces, approximately

- Resistive forces oppose the direction of motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

$$\vec{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

Resistive forces, approximately

- Resistive forces oppose the direction of motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

$$\vec{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

- *Laminar*: for smooth objects at low speeds the flow is steady and $n = 1$



Airplane wing



A car driving

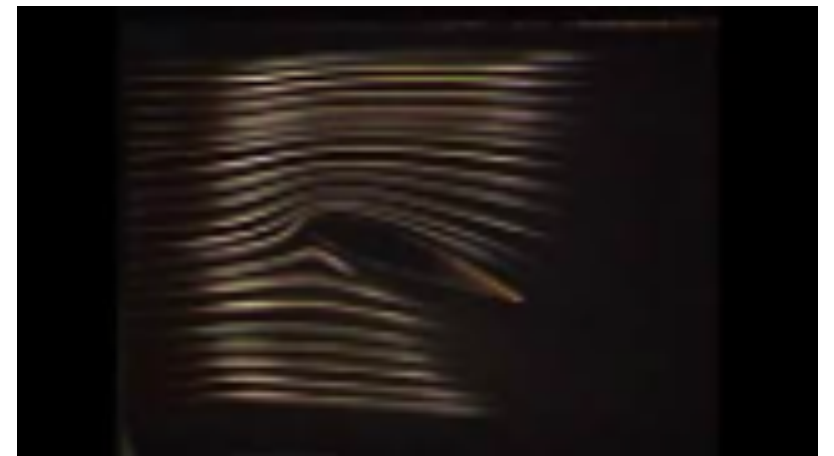
Resistive forces, approximately

- Resistive forces oppose the direction of motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

$$\vec{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

- *Laminar*: for smooth objects at low speeds the flow is steady and $n = 1$
- *Turbulent*: for rough objects at high speeds the flow is chaotic and $n = 2$



Airplane wing



A car driving



Boat wake

Example: Laminar viscous drag

A cyclist is moving at a slow speed v_0 under laminar conditions (i.e. $n = 1$) and has a drag coefficient β . Calculate the bike's speed as a function of time, assuming drag is the only force acting on it.

Example: Turbulent viscous drag

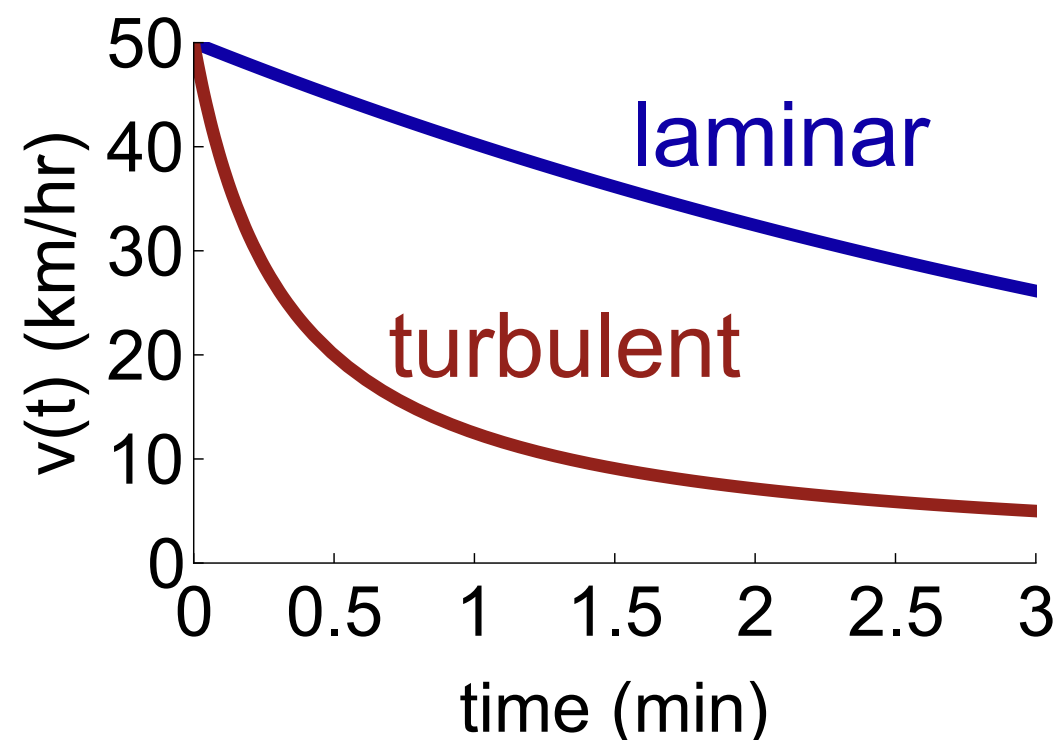
When the cyclist moves at a fast speed v_0 such that they reach the turbulent regime (i.e. $n = 2$), a similar procedure yields the following speed as a function of time:

$$v(t) = \frac{v_0}{1 + \left(\frac{\beta v_0}{m}\right)t}$$

Example: Turbulent viscous drag

When the cyclist moves at a fast speed v_0 such that they reach the turbulent regime (i.e. $n = 2$), a similar procedure yields the following speed as a function of time:

$$v(t) = \frac{v_0}{1 + \left(\frac{\beta v_0}{m}\right)t}$$



Today's agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. **Momentum**
 - **Conservation of momentum**
 - **Impulse**
3. Center of mass

DEMO (86 and 113)

Collisions between two spheres
and
the recoil of a cart

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

- It has units of [kg·m/s] (or equivalently [N·s])

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

- It has units of [kg·m/s] (or equivalently [N·s])
- Newton’s 2nd law:

$$\Sigma \vec{F} = m\vec{a} \quad \Rightarrow \quad \Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

- In English, the time rate of change of an object’s momentum is equal to the net force acting on it

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

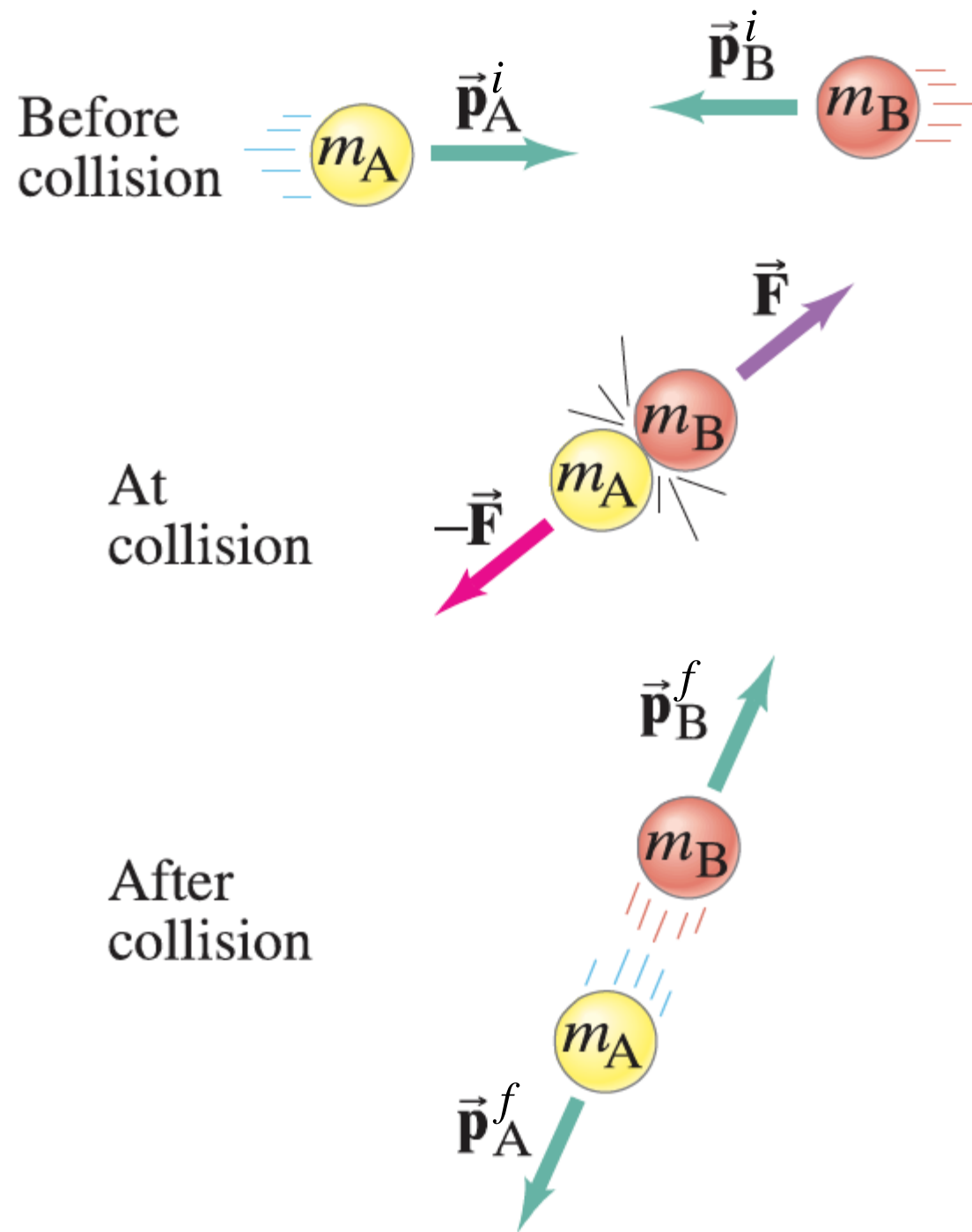
- It has units of [kg·m/s] (or equivalently [N·s])
- Newton’s 2nd law:

Most general form

$$\Sigma \vec{F} = m\vec{a} \Rightarrow \boxed{\Sigma \vec{F} = \frac{d\vec{p}}{dt}}$$

- In English, the time rate of change of an object’s momentum is equal to the net force acting on it

Momentum conservation from Newton's laws



Conservation of momentum

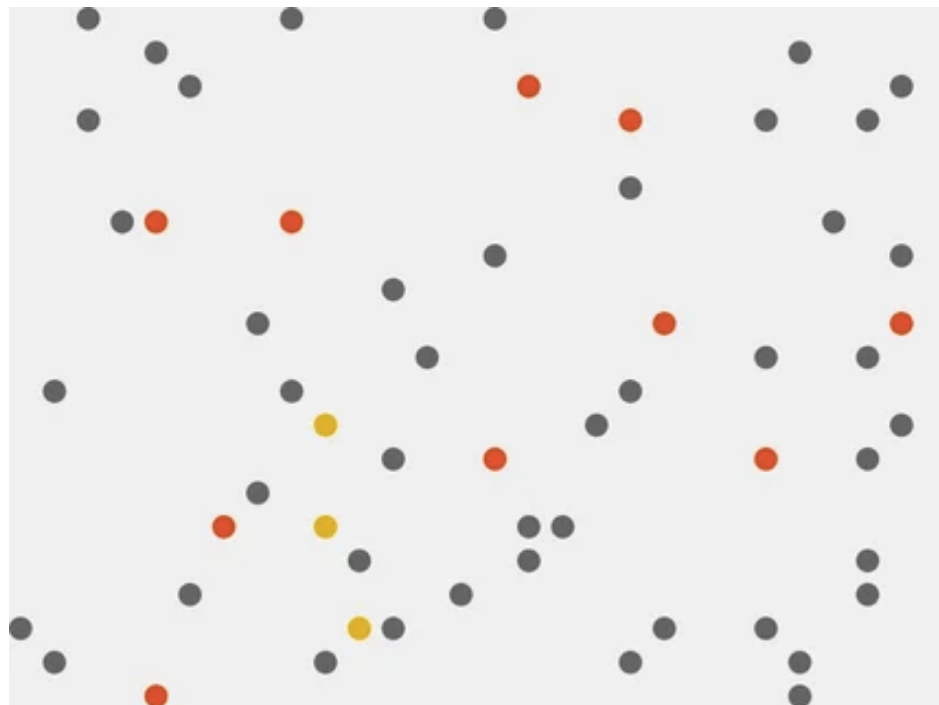
*In a given inertial reference frame,
the total momentum of
an isolated system stays constant.*

Conservation of momentum

*In a given inertial reference frame,
the total momentum of
an isolated system stays constant.*

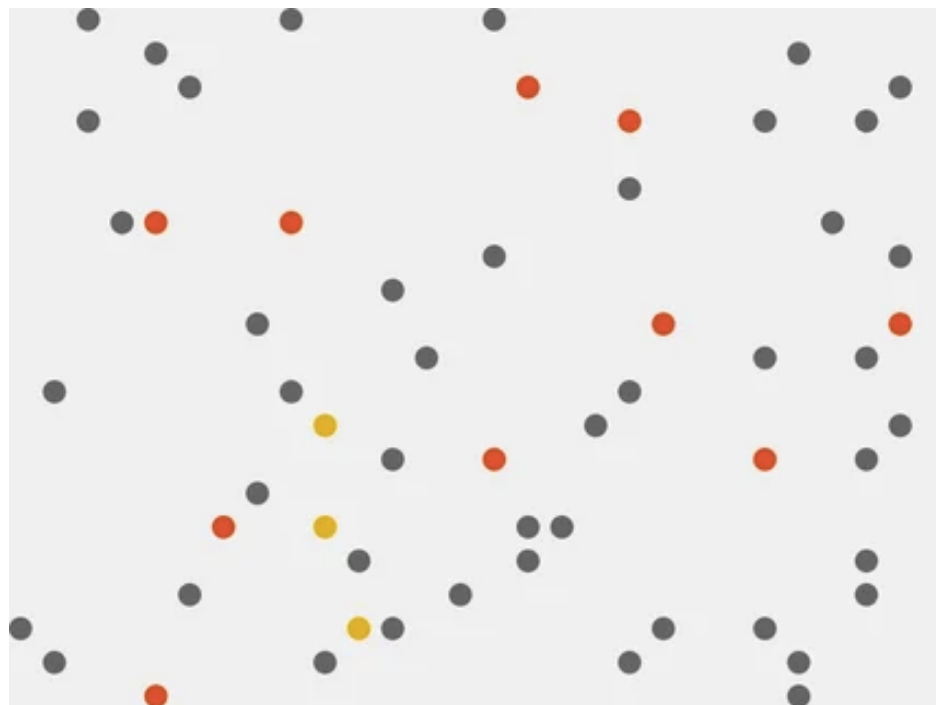
What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want



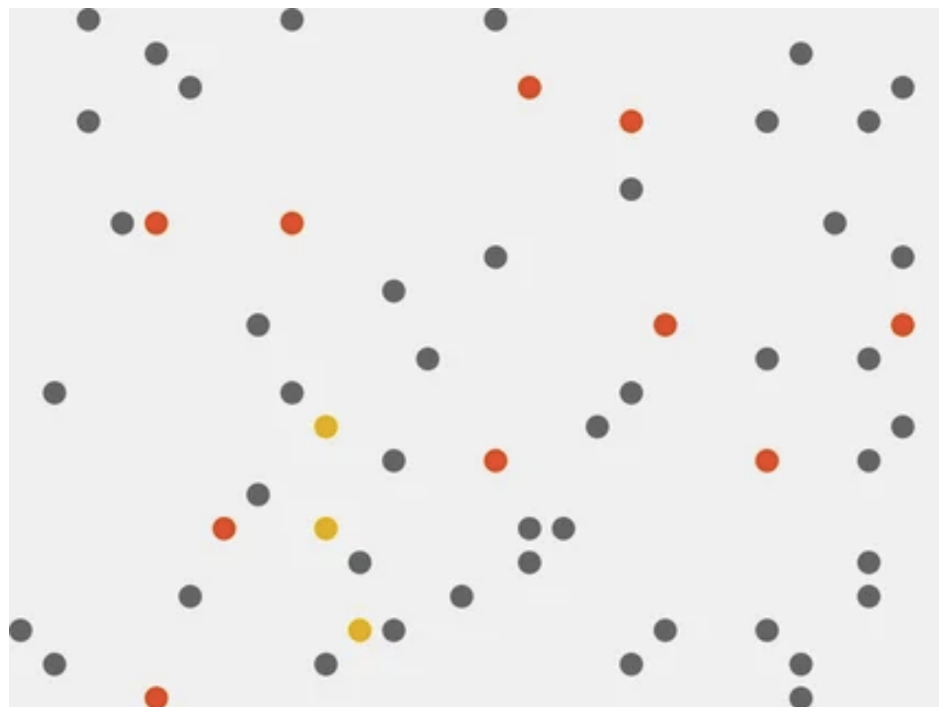
What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want
- If there is no net force from the outside and all the particles remain in the system, then the system is said to be *isolated*



What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want
- If there is no net force from the outside and all the particles remain in the system, then the system is said to be *isolated*



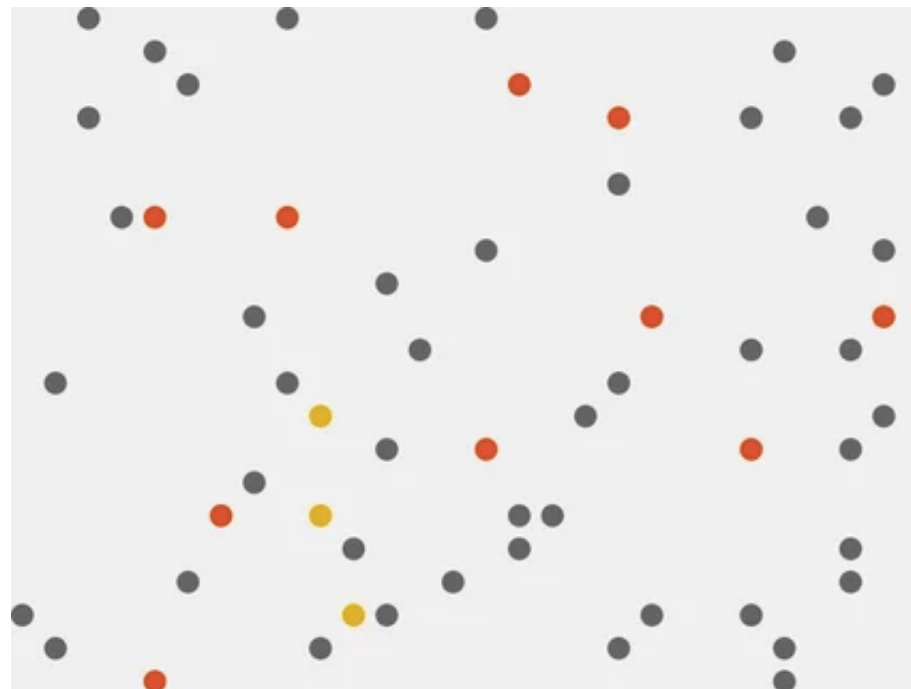
Conservation of momentum

*In a given inertial reference frame,
the total momentum of an isolated
system stays constant.*

What is the total momentum?

- Momentum of a point mass i is $\vec{p}_i = m_i \vec{v}_i$
- Total momentum of a system of N point masses is simply

$$\vec{p}_{sys} = \sum_{i=1}^N \vec{p}_i = \sum_{i=1}^N m_i \vec{v}_i$$



Conservation of momentum

*In a given inertial reference frame,
 the total momentum of
 an isolated system stays constant.*

**The net force on the
 system is zero and
 matter is not exchanged**



**Non-inertial reference
 frames create fictitious
 forces that can change
 the momentum**



Conservation of momentum

*In a given inertial reference frame,
the total momentum of
an isolated system stays constant.*

**The net force on the
system is zero and
matter is not exchanged**

**Non-inertial reference
frames create fictitious
forces that can change
the momentum**

Discussed next lecture

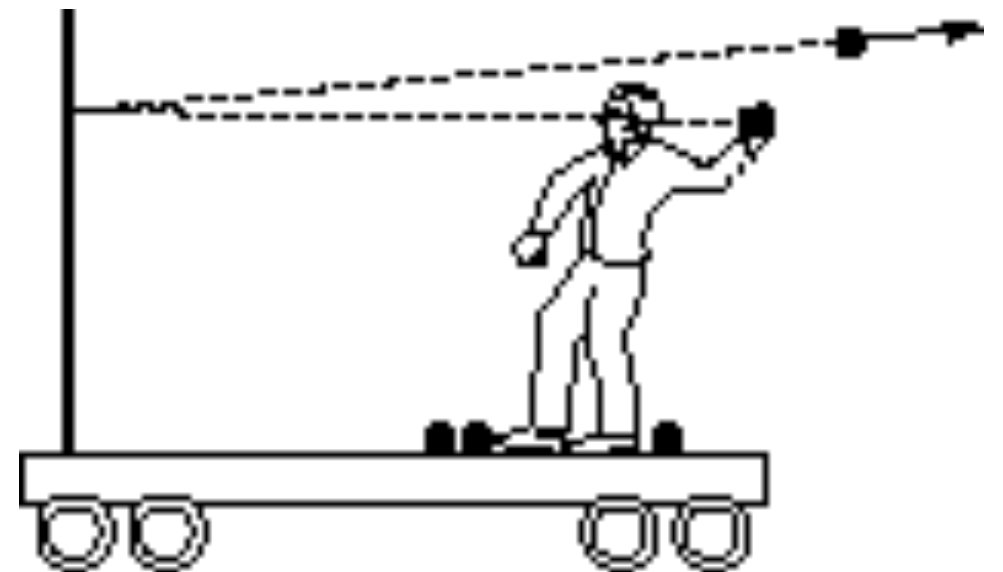
- Veeeeeeeeeeeeerrrrrrrrrrrrrrrrrrrrrrrrrrrry profound



Conceptual question

Suppose you are on a cart, initially at rest. Neglect friction. You throw a ball at a partition that is rigidly mounted on the cart and the ball bounces straight back as shown in the figure. After the ball bounces, is the cart moving?

- A. Yes, it moves to the right.
- B. Yes, it moves to the left.
- C. No, it remains in place.
- D. Not enough information is given to decide



Conceptual question

A compact car and a large truck collide head on and stick together. Consider them to be an isolated system. Which undergoes the larger magnitude momentum change?

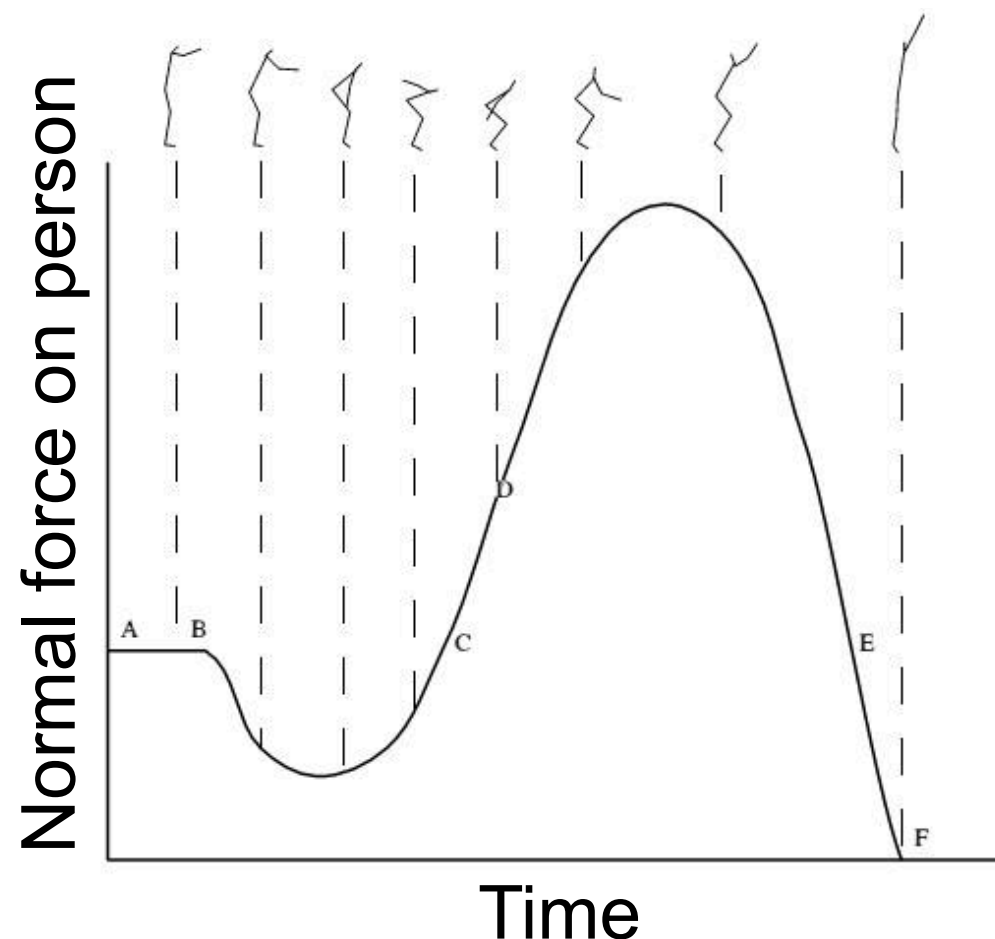
- A. The car.
- B. The truck.
- C. They are the same.
- D. Not enough information is given to decide

How do we move (i.e. change momentum)?

- If momentum is conserved, how do we move? We push off of things, usually the ground

How do we move (i.e. change momentum)?

- If momentum is conserved, how do we move? We push off of things, usually the ground
- Consider a “system” to include only your body, then exert a net force on an external object
- We change momentum by applying force for a time interval



Impulse and momentum

- Defined as the integral of a net force over a time period:

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}_{net} dt$$

- Using Newton's 2nd law $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$$\vec{I} = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt \Rightarrow \vec{I} = \vec{p}(t_f) - \vec{p}(t_i) = \Delta\vec{p}$$

- It is simply a change of momentum in time
- It has the same units as momentum of [kg·m/s] (or equivalently [N·s])

Neglecting the details

- Often we don't care about the details of the force (e.g. when it's very short)

Neglecting the details

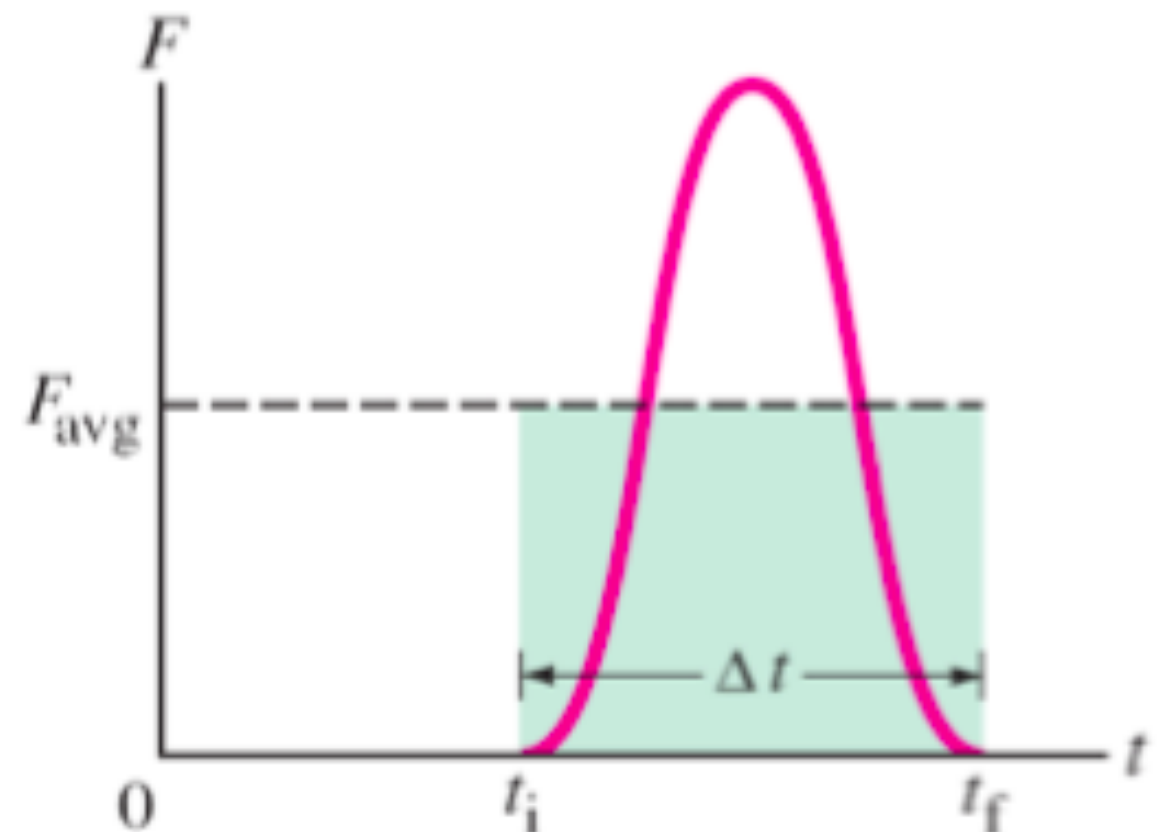
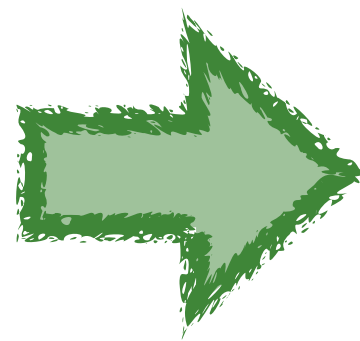
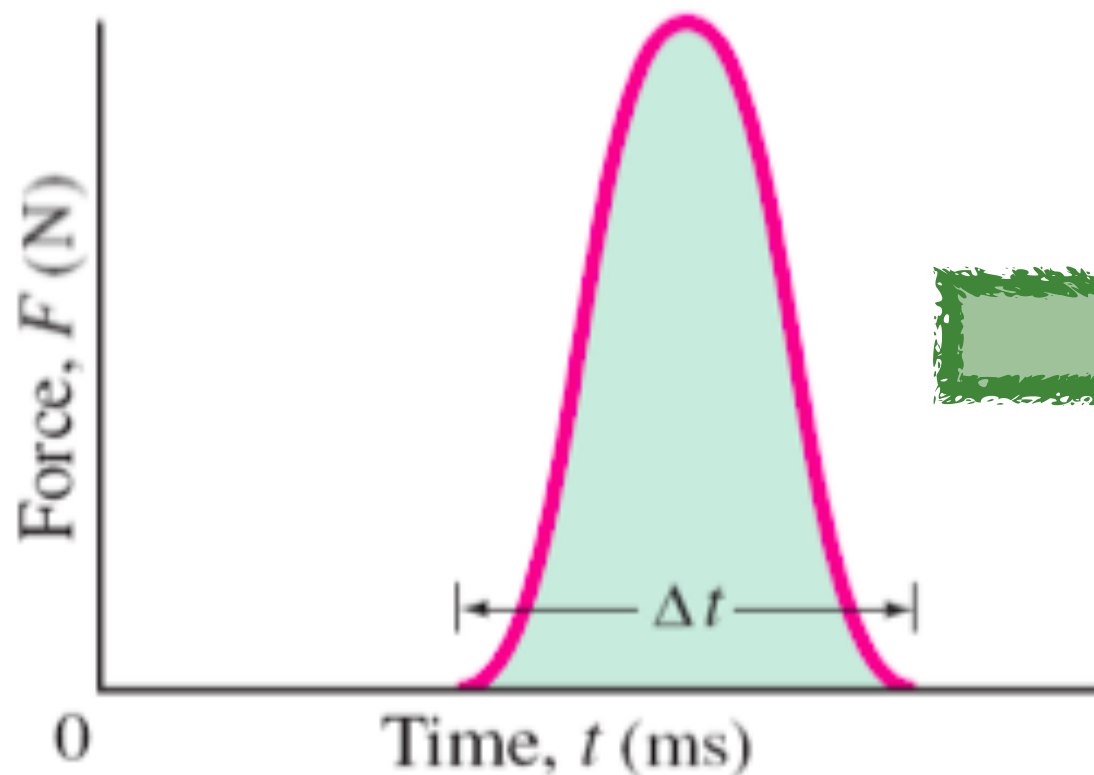
- Often we don't care about the details of the force (e.g. when it's very short)
- Model the impulse as an average force applied over the same time interval

$$\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \vec{F}_{avg} \Delta t$$

Neglecting the details

- Often we don't care about the details of the force (e.g. when it's very short)
- Model the impulse as an average force applied over the same time interval

$$\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \vec{F}_{avg} \Delta t$$



Duration of a collision

Impulse approximation

- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision

Impulse approximation

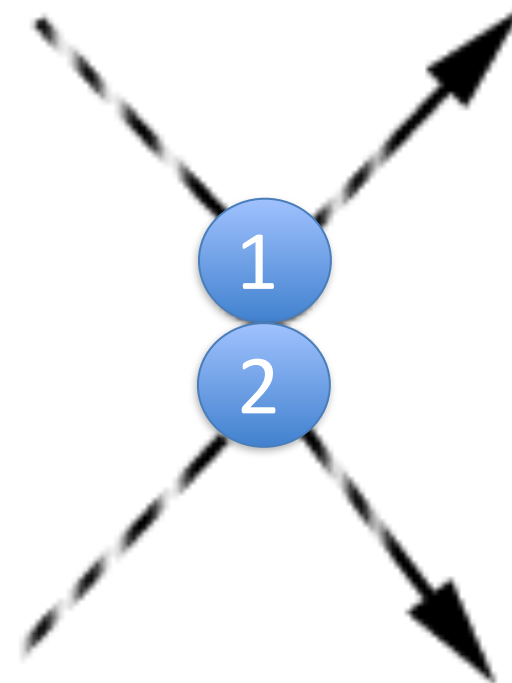
- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision
- Example: A tennis ball is coming at you at $\vec{v}_1$. You hit it with a tennis racket and it departs leaving at $\vec{v}_2$.
 - What is the change in momentum?
 - What is the impulse?
 - What is the force and how long was it applied?
 - Impulse approximation: don't consider the effect of gravity during the collision with the tennis racket



Conceptual question

The figure below depicts the paths of two colliding blue circles, 1 and 2. Which of the following arrows best represents the direction of the impulse applied to circle 2 by circle 1 during the collision?

- A. 
- B. 
- C. 
- D. 
- E. 

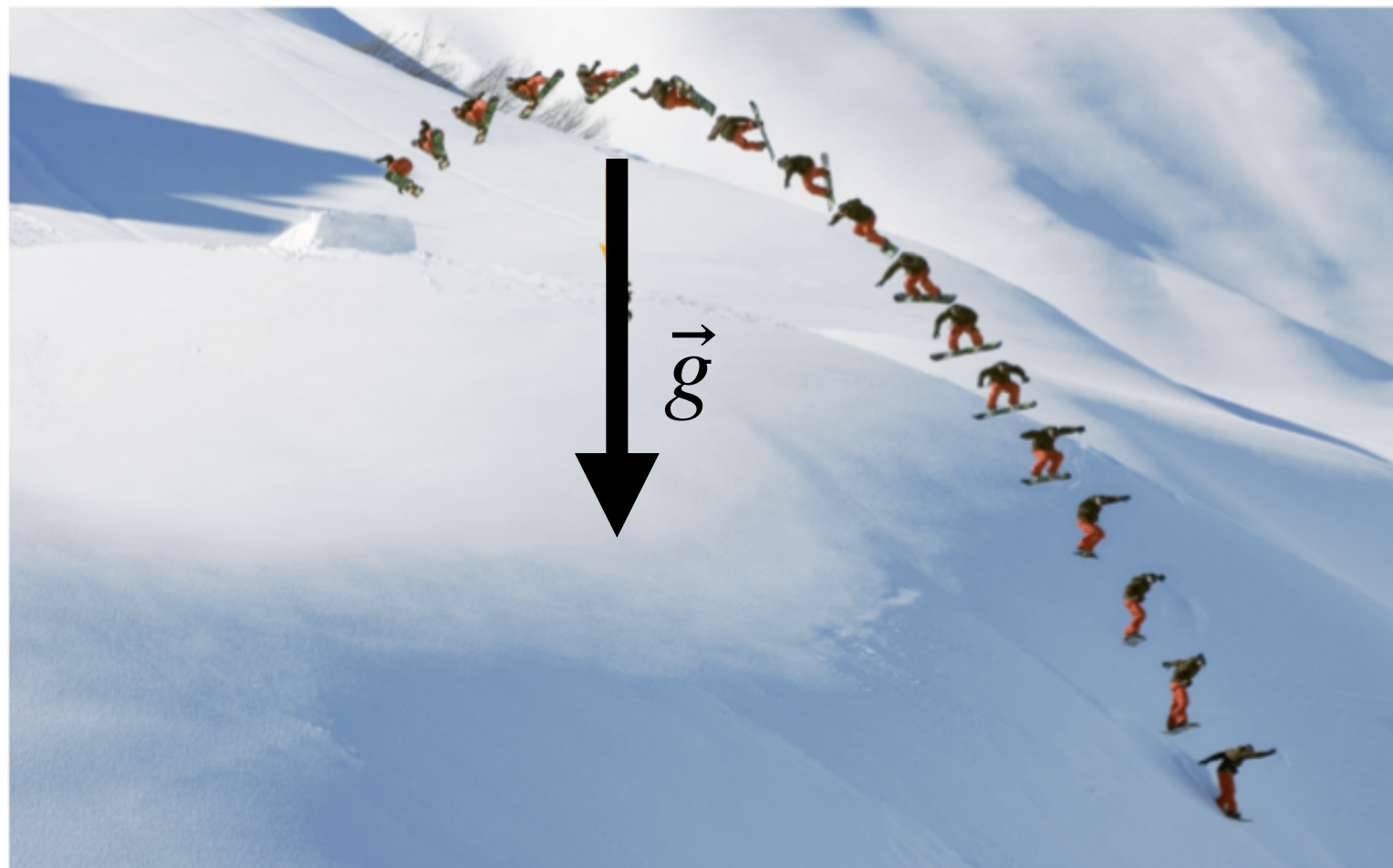


Today's agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. Momentum
 - Conservation of momentum
 - Impulse
3. **Center of mass**

Center of mass

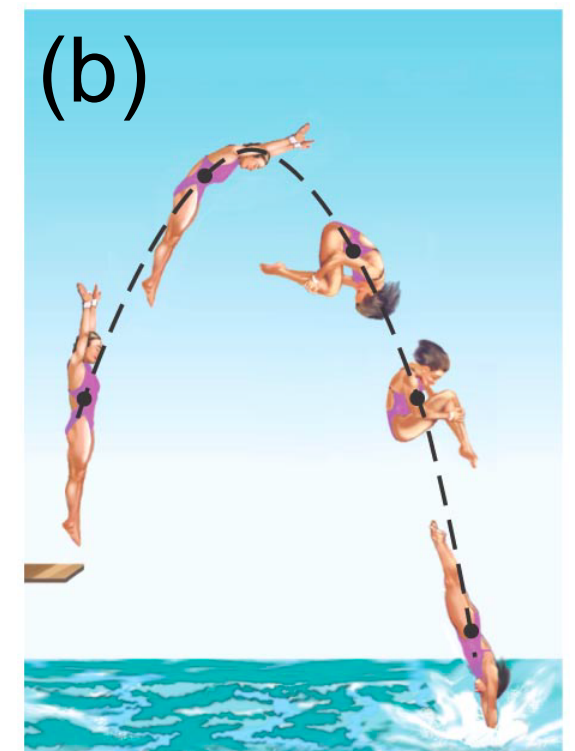
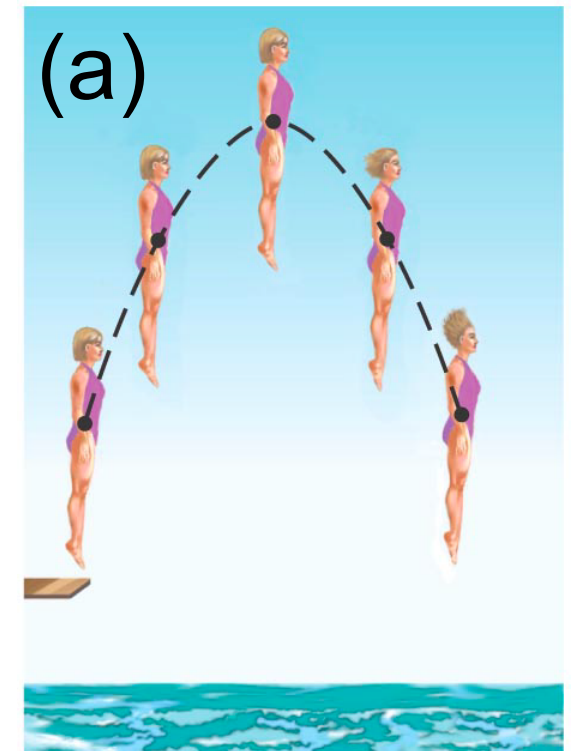
- We've used the point mass approximation a lot, but the appropriate point isn't always obvious



- If we want to summarize this guy's trajectory, which point should we take?

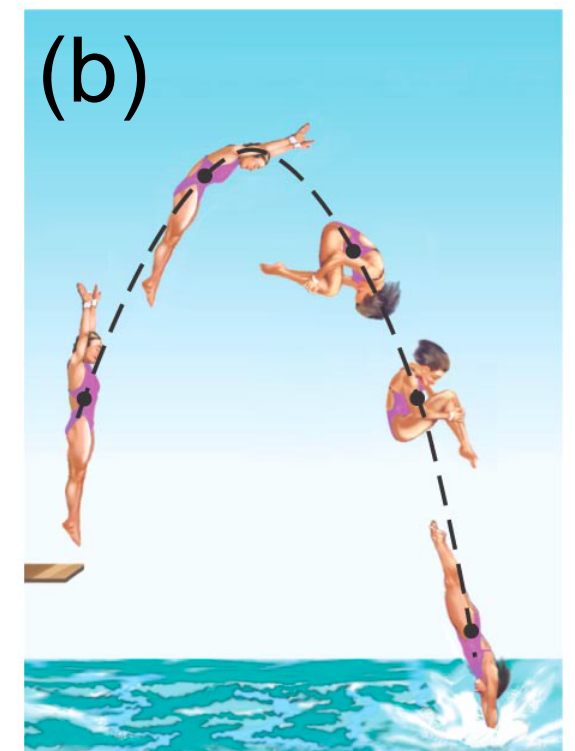
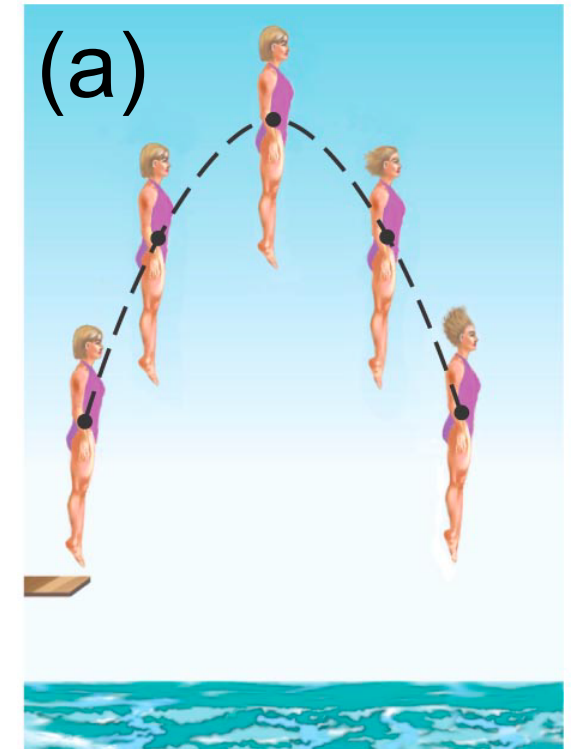
Center of mass

- In (a), the diver's motion is pure translation
- In (b), the motion is translation plus rotation



Center of mass

- In (a), the diver's motion is pure translation
- In (b), the motion is translation plus rotation
- One point, called the *Center of Mass* (CM), that moves the same in both (i.e. it's the point the diver rotates around)
- It's the one point that would move in the same path as a point mass subjected to the same net force



Center of mass

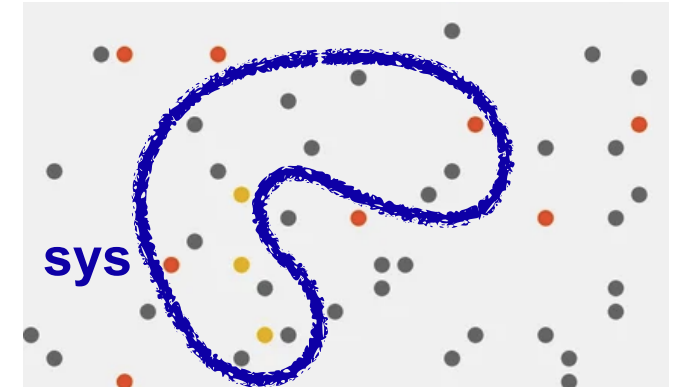
- In (a), the diver's motion is pure translation
- In (b), the motion is translation plus rotation
- One point, called the *Center of Mass* (CM), that moves the same in both (i.e. it's the point the diver rotates around)
- It's the one point that would move in the same path as a point mass subjected to the same net force
- The motion of an object can always be decomposed into translational motion of the center of mass, plus rotation, deformation, ...
- How do we find it, you ask...



Calculating the center of mass

- It is the “average” position of the system, weighted by mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{\sum_{i=1}^N m_i}$$



where $M = \sum_{i=1}^N m_i$ is the total mass

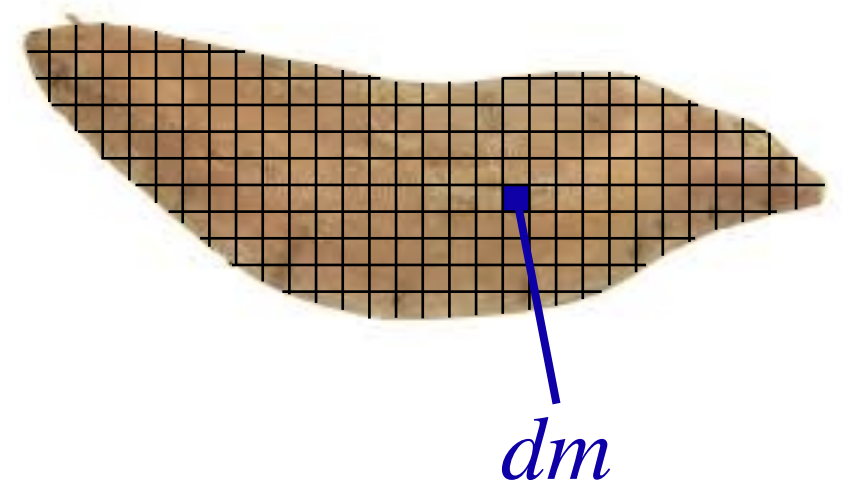
- For just two particles in 1D, the center of mass lies closer to the one with more mass

$$\begin{aligned} X_{CM} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ &= \frac{m_1}{M} x_1 + \frac{m_2}{M} x_2 \end{aligned}$$

Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds

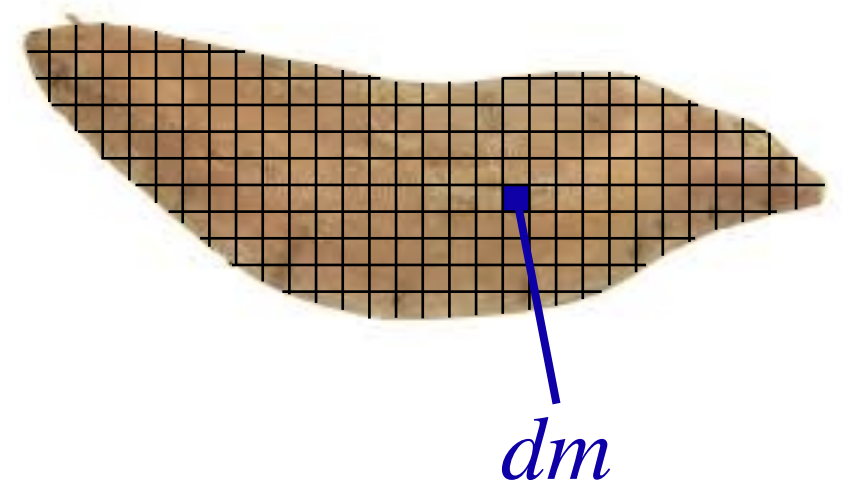
$$\vec{R}_{CM} = \frac{\sum_{i=1}^N \vec{r}_i m_i}{\sum_{i=1}^N m_i}$$



Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds

$$\vec{R}_{CM} = \frac{\sum_{i=1}^N \vec{r}_i m_i}{\sum_{i=1}^N m_i}$$

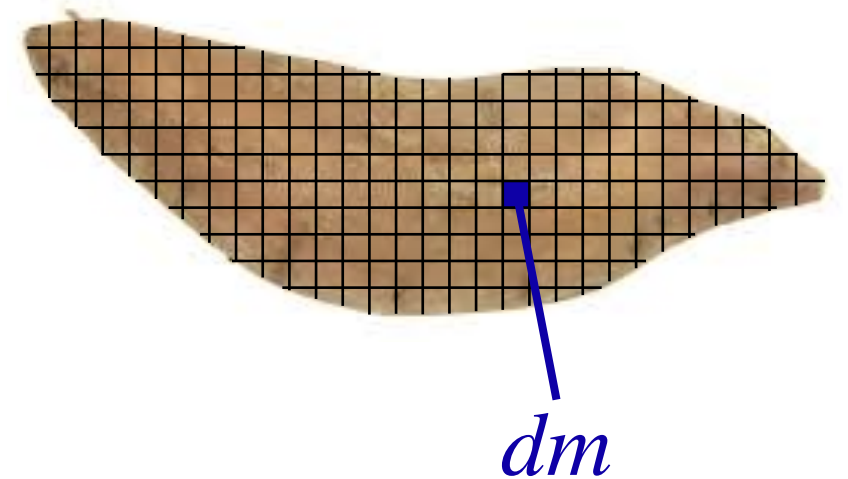


- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\vec{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r} dm}{M}$$

Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds



$$\vec{R}_{CM} = \frac{\sum_{i=1}^N \vec{r}_i m_i}{\sum_{i=1}^N m_i}$$

- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\vec{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r} dm}{M}$$

- Use density to convert mass to spatial integral

Finding the center of mass:

Hanging

Kinematics of the center of mass

- Given a system with constant masses, take the derivative of

$$\vec{R}_{CM} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{M} \quad \text{or} \quad \vec{R}_{CM} = \frac{\int_M \vec{r} dm}{M}$$

to find

$$\vec{V}_{CM} = \frac{\sum_{i=1}^N m_i \vec{v}_i}{M} \quad \text{or} \quad \vec{V}_{CM} = \frac{\int_M \vec{v} dm}{M}$$

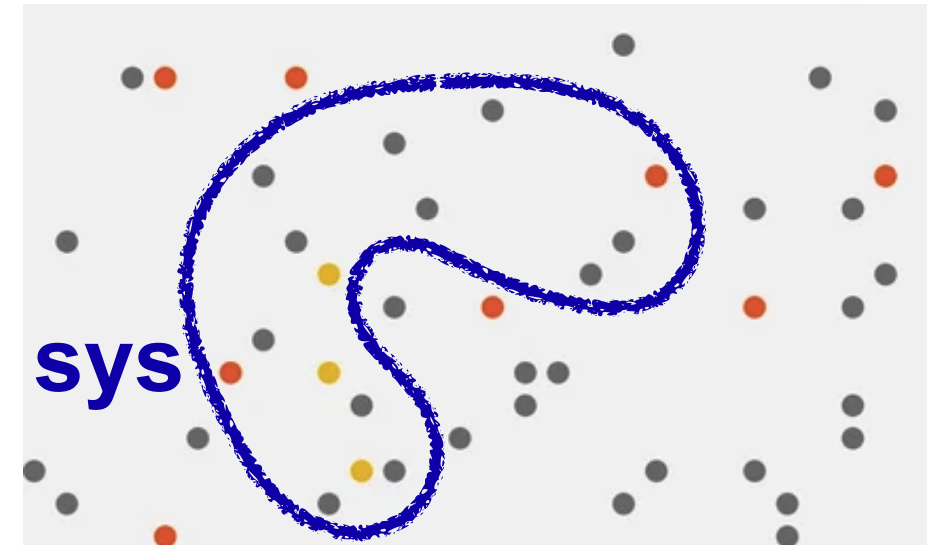
and again to find

$$\vec{A}_{CM} = \frac{\sum_{i=1}^N m_i \vec{a}_i}{M} \quad \text{or} \quad \vec{A}_{CM} = \frac{\int_M \vec{a} dm}{M}$$

Forces are applied at the center of mass

- The center of mass can prove it's own usefulness

$$\vec{A}_{CM} = \frac{\sum_{i=1}^N m_i \vec{a}_i}{M} = \frac{\sum_{i \in \text{sys}} m_i \vec{a}_i}{M}$$



Forces are applied at the center of mass

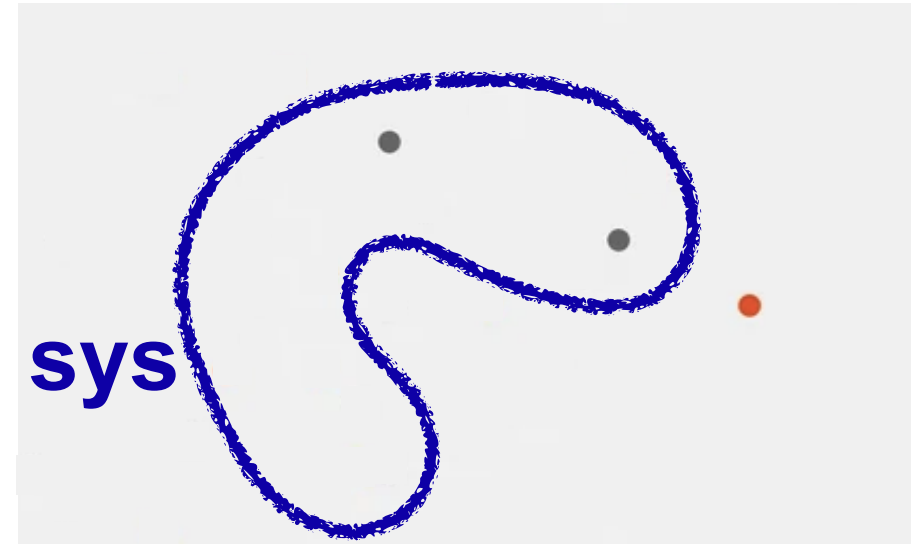
$$M\vec{A}_{CM} = \vec{F}_{net}^{ext}$$

- Thus, we can pretend the entire system is located at the center of mass

Forces are applied at the center of mass

- Separate **internal** and **external** forces

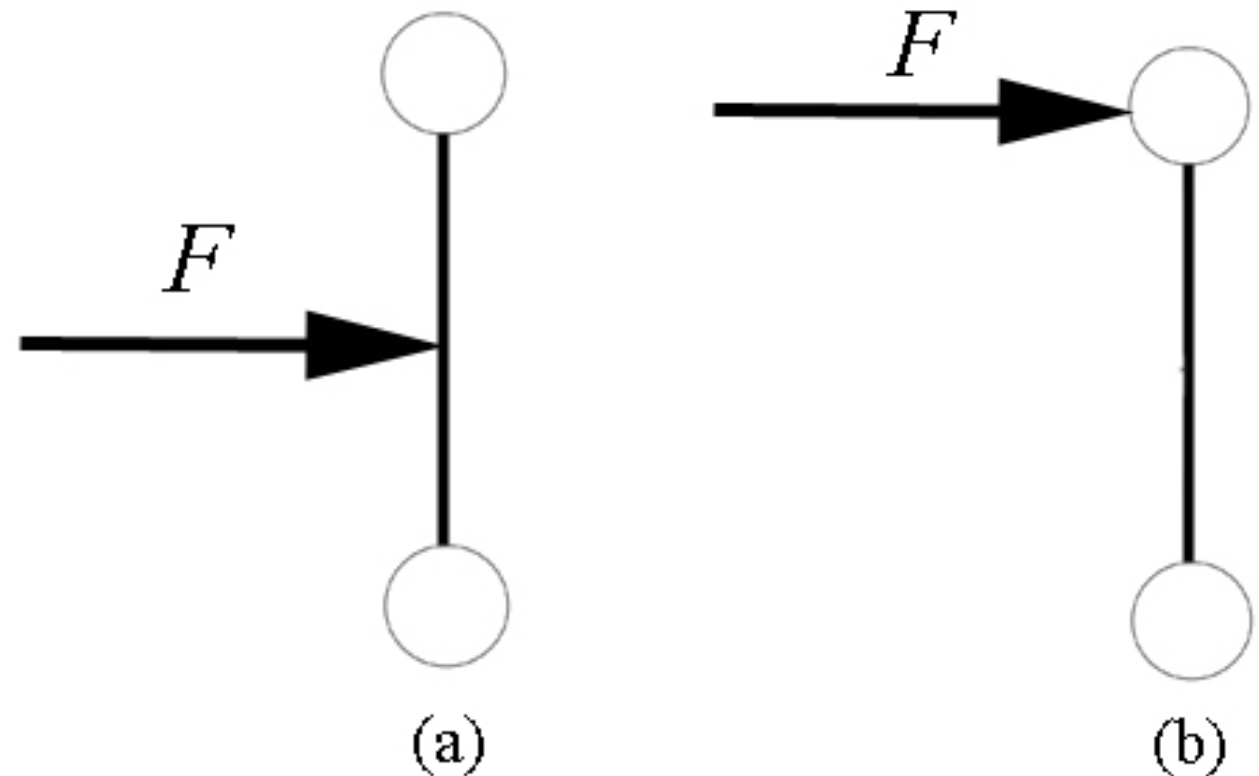
$$M\vec{A}_{CM} = \sum_{i \in \text{sys}} \left(\sum_{j \in \text{sys}} \vec{F}_{ji} + \sum_{j \notin \text{sys}} \vec{F}_{ji} \right)$$



Conceptual question

A force of magnitude F is applied for a short time to a dumbbell, either as in (a) or as in (b). In which case does the dumbbell gain more **momentum**?

- A. (a)
- B. (b)
- C. They are the same



See you tomorrow!

Conceptual question

A compact car and a large truck collide head on and stick together. Consider them to be an isolated system. Which undergoes the larger magnitude acceleration?

- A. The car.
- B. The truck.
- C. They are the same.
- D. Not enough information is given to decide

Conceptual question

Consider two carts, of mass m and $2m$, at rest on an air track. If you push first cart for 3 s and then the other for the same length of time, exerting equal force on each, the **momentum** of the light cart is...

- A. 4 times...
- B. 2 times...
- C. equal to...
- D. half...
- E. one-fourth...

the momentum of the heavy cart.