General Physics: Mechanics

PHYS-101(en)

Lecture 6a:
Drag, momentum, impulse, and center of mass

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October 14th, 2024

EPFL
Swiss Plasma Center

Announcement

- Next week is "Fall break"
  - There is **<u>no</u>** class Monday nor Tuesday.
  - There are **<u>no</u>** Exercise sessions on Wednesday.
  - Office hours **will take place** as normal.
  - Classes resume Monday, Oct. 28th.

Announcement

- We'll hold a ***mini*** **mock exam** on Tuesday October 29th
  - In-class (SG1) during normal lecture hours (10:15-11:00)
  - Does **not** matter at all for your final grade
  - You can bring a "cheat" sheet containing formulas or all of your notes, as you wish
  - Take to your TA at the exercise session on Wednesday October 30th, to be graded and returned on Wednesday November 6th
  - Exam solutions will be published

Today’s agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. Momentum
   - Conservation of momentum
   - Impulse
3. Center of mass

DEMO (738)

Air drag

Resistive forces, approximately

- Resistive forces oppose the direction of motion of an object

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- A prime example is *viscous drag* with a fluid (e.g. air resistance)

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where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

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Airplane wing

A car driving

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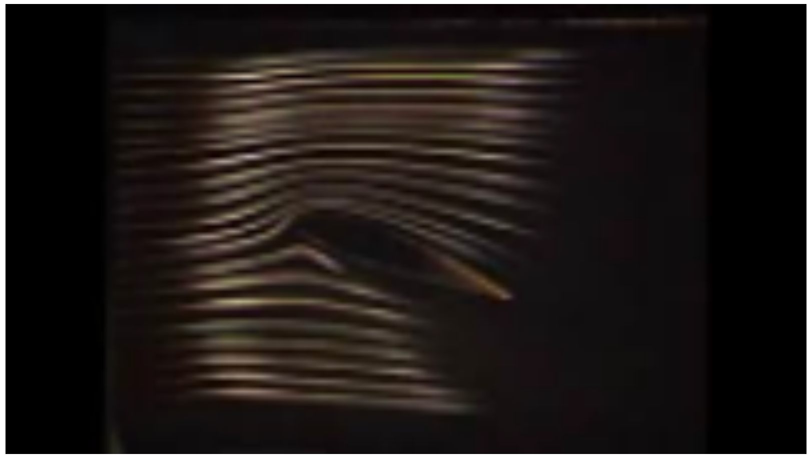
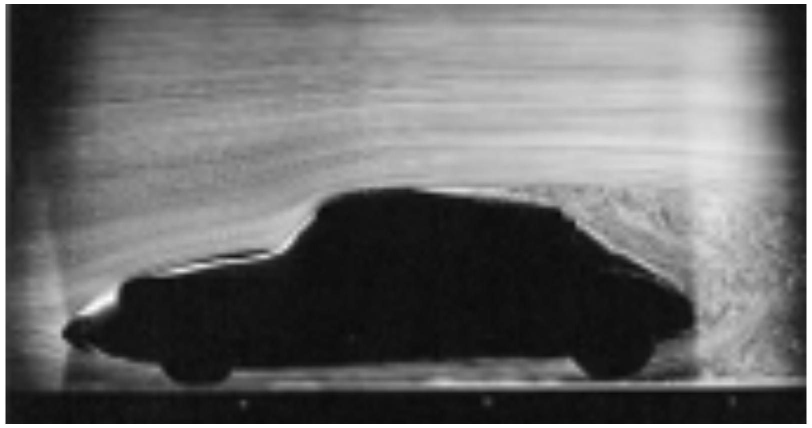
- *Laminar*: for smooth objects at low speeds the flow is steady and $n = 1$
- *Turbulent*: for rough objects at high speeds the flow is chaotic and $n = 2$

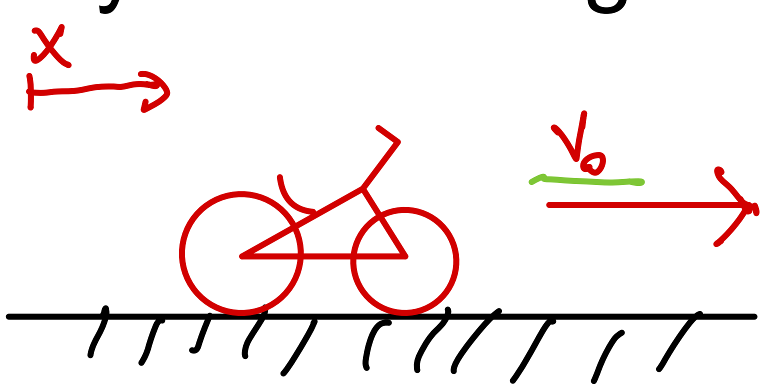

Airplane wing

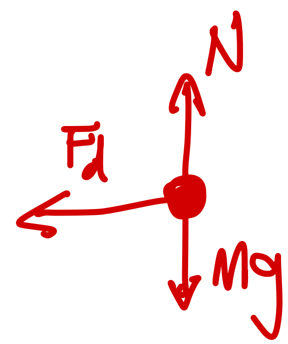

A car driving


Boat wake

Example: Laminar viscous drag

A cyclist is moving at a slow speed v_0 under laminar conditions (i.e. $n = 1$) and has a drag coefficient β. Calculate the bike's speed as a function of time, assuming drag is the only force acting on it.





$$\sum F_x: \; -F_d = m a_x = m \frac{dv_x}{dt}$$

$$-\beta v_x = m \frac{dv_x}{dt}$$

$$\Rightarrow \frac{dv_x}{dt} = -\frac{\beta}{m} v_x$$

$$\frac{dv_x}{dt} = -\frac{\beta}{m} v_x \Rightarrow \overbrace{\int \frac{dv_x}{v_x}}^{\ln(v_x)+C_1} = \underbrace{\int -\frac{\beta}{m}\, dt}_{-\frac{\beta}{m}t + C_2}$$

$$\ln(v_x) = -\frac{\beta}{m} t + \overbrace{(C_2 - C_1)}^{C_3}$$

$$= -\frac{\beta}{m} t + C_3$$

$$v_x = e^{-\frac{\beta}{m}t + C_3} = e^{-\frac{\beta}{m}t} e^{C_3} = C_4 e^{-\frac{\beta}{m}t}$$

$$v_x(t=0) = v_0 = C_4 e^0 = C_4$$

$$\Rightarrow \boxed{v_x(t) = v_0 e^{-\frac{\beta}{m}t}}$$

Example: Turbulent viscous drag

When the cyclist moves at a fast speed v_0 such that they reach the turbulent regime (i.e. $n = 2$), a similar procedure yields the following speed as a function of time:

$$v(t) = \frac{v_0}{1 + \left(\frac{\beta v_0}{m}\right) t}$$

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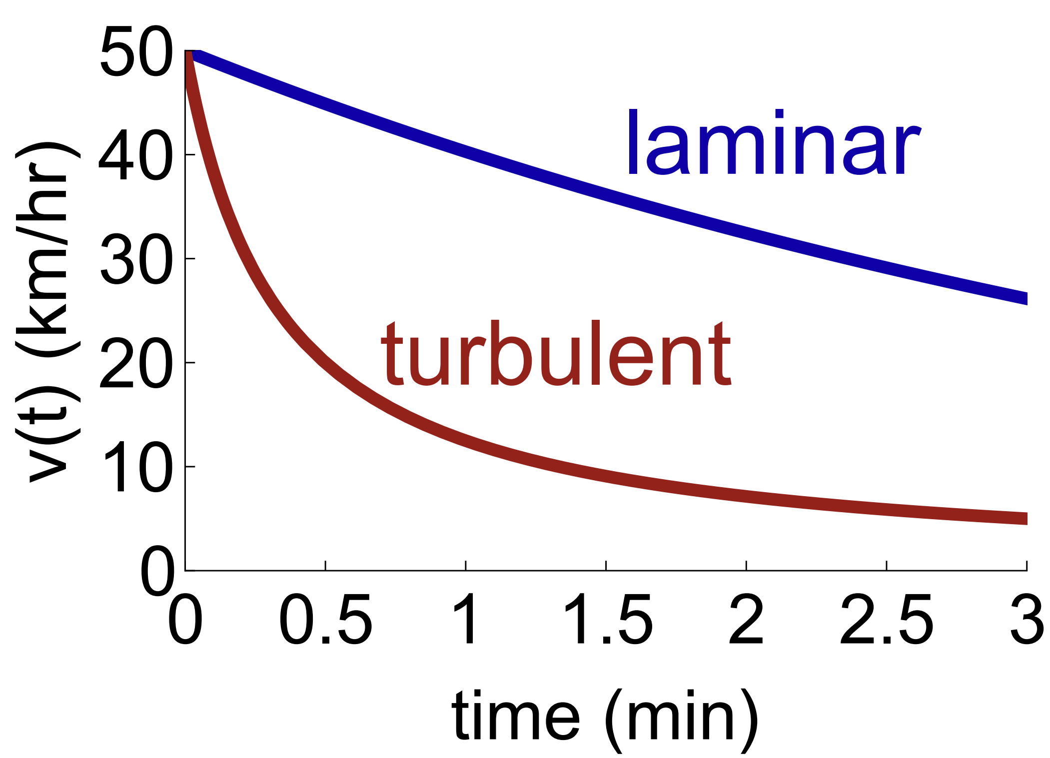

Today's agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. **Momentum**
   - **Conservation of momentum**
   - **Impulse**
3. Center of mass

DEMO (86 and 113)

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Collisions between two spheres

and

the recoil of a cart

Definition of momentum

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- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

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- Newton's 2nd law:

$m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$

$$\Sigma\overrightarrow{F} = m\vec{a} \quad \Rightarrow \quad \Sigma\overrightarrow{F} = \frac{d\vec{p}}{dt}$$

- In English, the time rate of change of an object's momentum is equal to the net force acting on it

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Most general form

$$\Sigma\overrightarrow{F} = m\vec{a} \quad \Rightarrow \quad \boxed{\Sigma\overrightarrow{F} = \frac{d\vec{p}}{dt}}$$

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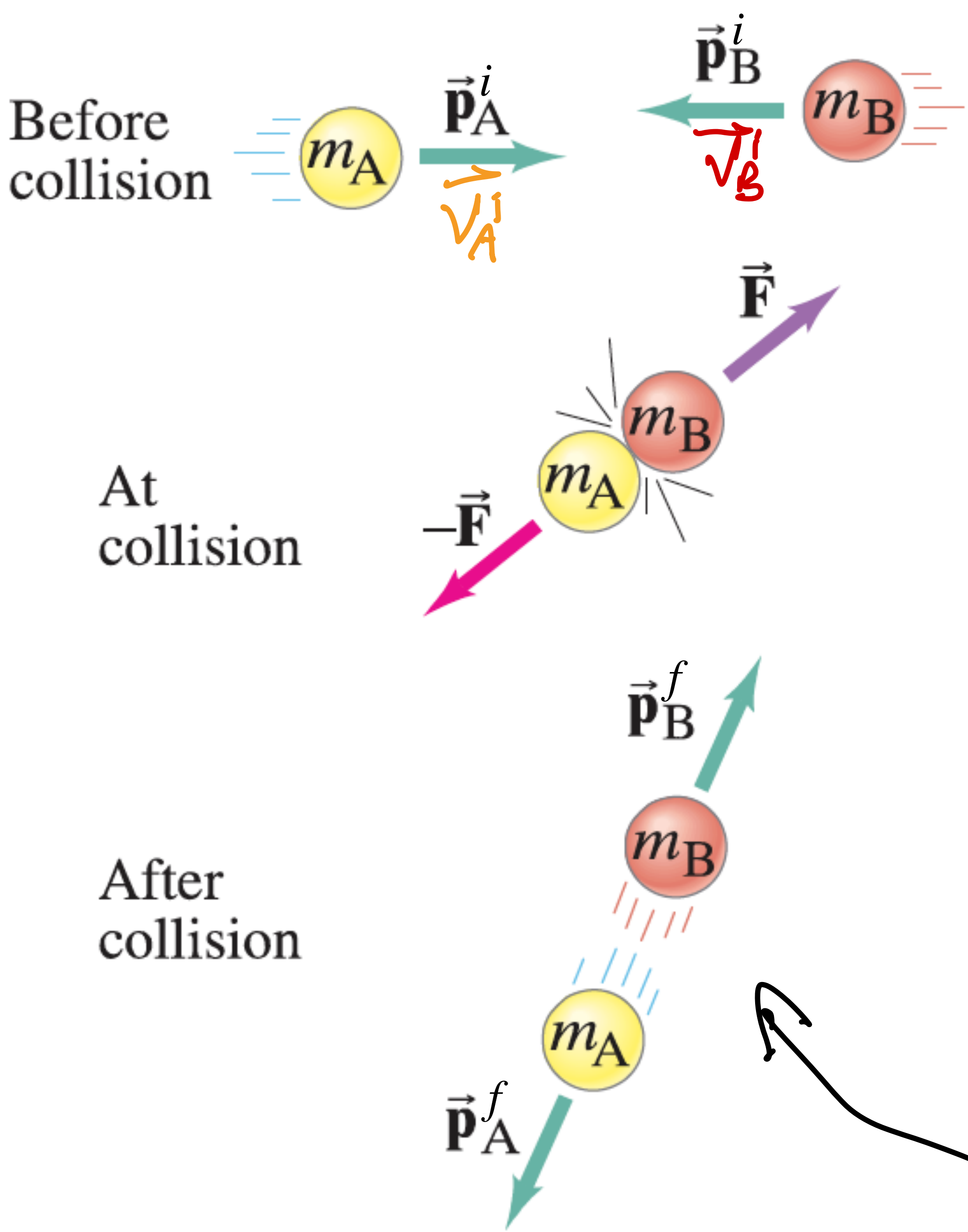
Momentum conservation from Newton's laws



Newton's first law:

$\vec{p}_A^{\,i} = m_A \vec{v}_A^{\,i}$ $\quad$ $\vec{p}_B^{\,i} = m_B \vec{v}_B^{\,i}$

$\vec{p}_{net}^{\,i} = \vec{p}_A^{\,i} + \vec{p}_B^{\,i}$ = const vec.

$\vec{F}_{AB} = \frac{d\vec{p}_B}{dt}$ $\quad$ $\vec{F}_{BA} = \frac{d\vec{p}_A}{dt}$

$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \vec{F}_{AB} + \vec{F}_{BA} = 0$

Newton's 3rd law: $\vec{F}_{BA} = -\vec{F}_{AB}$

$\frac{d}{dt}(\vec{p}_A^{\,c} + \vec{p}_B^{\,c}) = 0$

$\vec{p}_A^{\,c} + \vec{p}_B^{\,c}$ = const vec.

$\vec{p}_{net}^{\,f}$ = const vec

Conservation of momentum

In a given inertial reference frame,
the total momentum of
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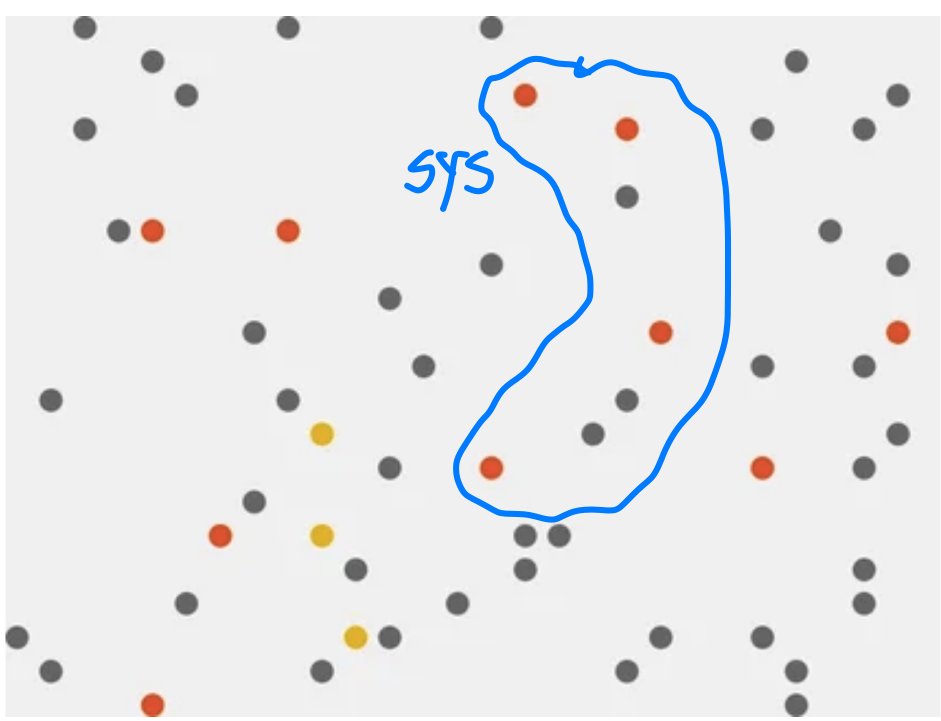
What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want



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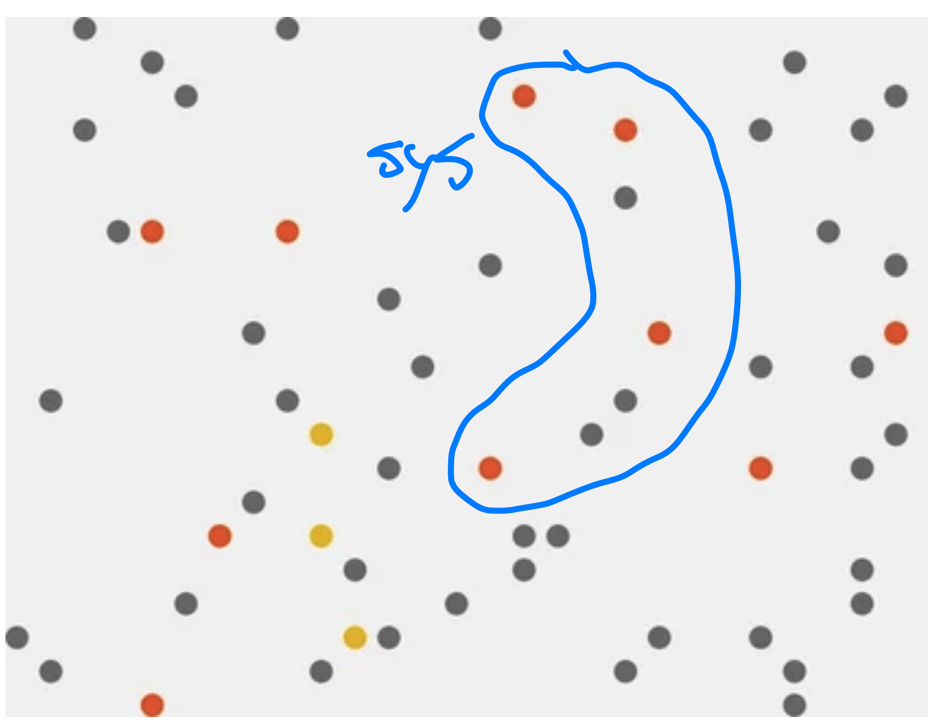
- If there is no net force from the outside and all the particles remain in the system, then the system is said to be *isolated*



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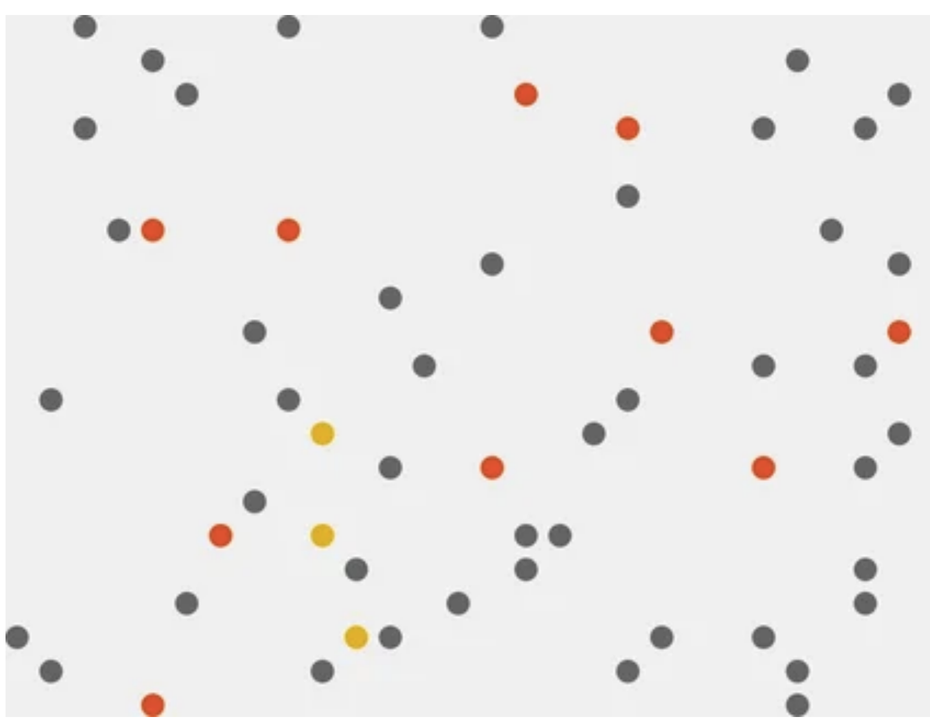


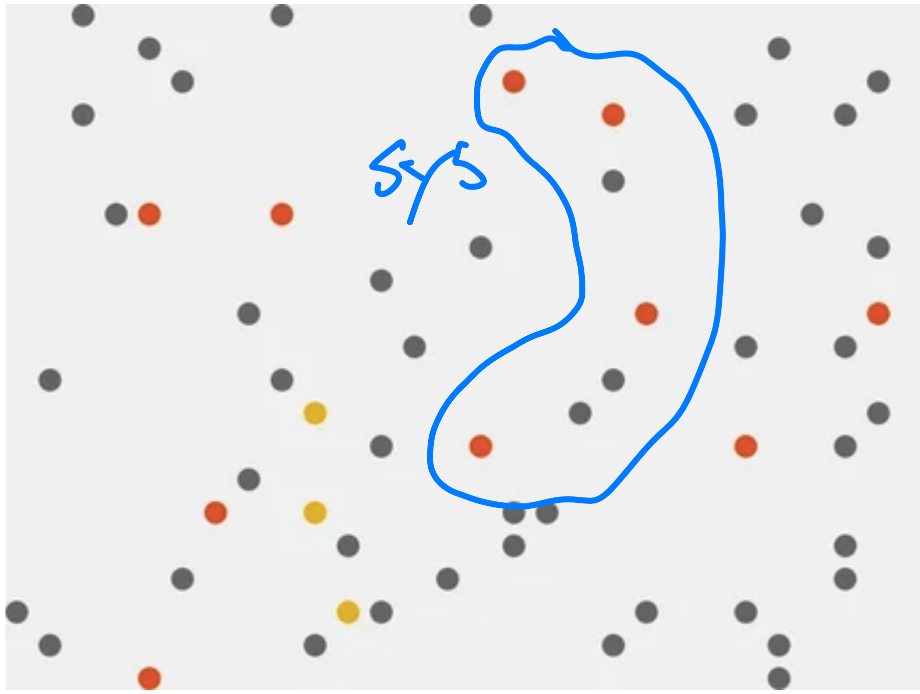
Conservation of momentum

In a given inertial reference frame, the total momentum of an isolated system stays constant.

What is the total momentum?

- Momentum of a point mass i is $\vec{p}_i = m_i \vec{v}_i$
- Total momentum of a system of N point masses is simply

$$\vec{p}_{sys} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$



Conservation of momentum

*In a given <u>inertial reference frame</u>,
the total momentum of
an <u>isolated</u> <u>system</u> stays constant.*

The net force on the system is zero and matter is not exchanged

Non-inertial reference frames create fictitious forces that can change the momentum

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Discussed next lecture

Conservation of momentum

In a given inertial reference frame,
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- Veeeeeeeeeeeeerrrrrrrrrrrrrrrrrrry profound

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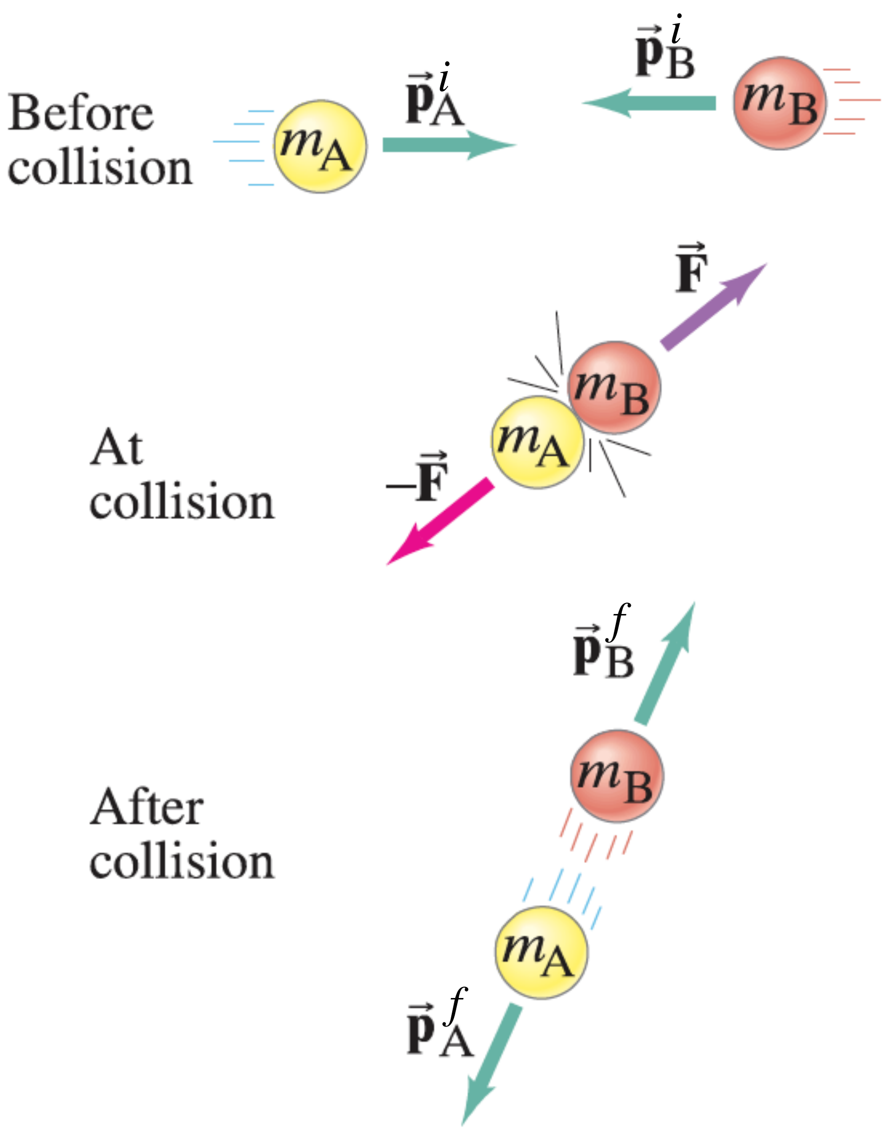


$$\boxed{\vec{p}_A^{\,i} + \vec{p}_B^{\,i} = \vec{p}_A^{\,f} + \vec{p}_B^{\,f}}$$

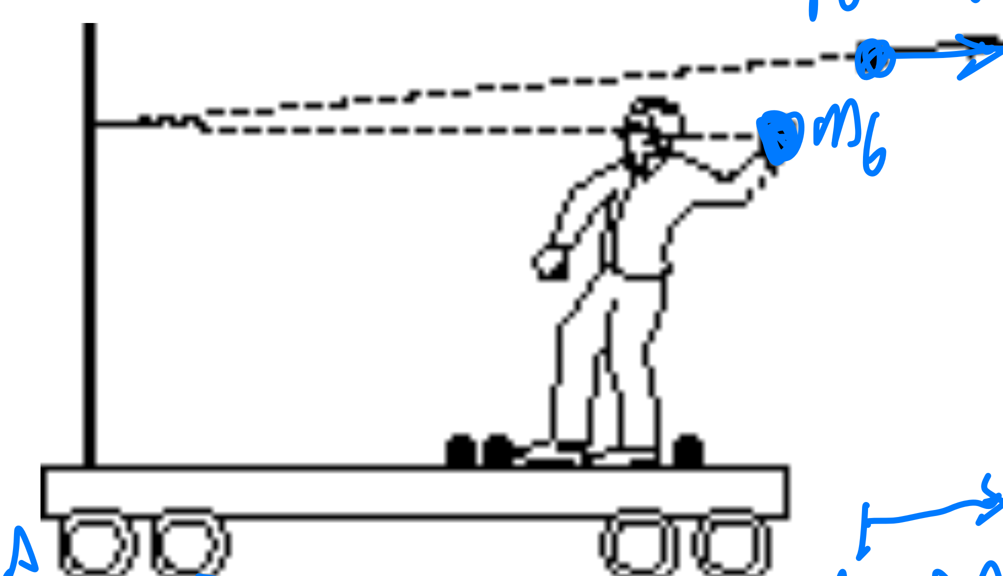
Conceptual question

Suppose you are on a cart, initially at rest. Neglect friction. You throw a ball at a partition that is rigidly mounted on the cart and the ball bounces straight back as shown in the figure. After the ball bounces, is the cart moving?

A. Yes, it moves to the right.
B. Yes, it moves to the left.
C. No, it remains in place.
D. Not enough information is given to decide

$\vec{v}_M = -\frac{m_b}{M} v_b^f \hat{x}$

$\vec{p}_b^f = m_b v_b^f \hat{x}$



System has two objects:
1. Ball of m_b
2. Cart, dude, wall of M

$\vec{p}_M^i = 0 \quad \vec{p}_b^i = 0 \Rightarrow \vec{p}_{tot}^i = 0$

$\vec{p}_b^f = m_b v_b^f \hat{x} \quad \vec{p}_M^f = M\vec{v}_M \Rightarrow \vec{p}_{tot}^f = M\vec{v}_M + m_b v_b^f \hat{x} = \vec{p}_{tot}^i = 0 \Rightarrow \vec{v}_M = -\frac{m_b}{M} v_b^f \hat{x}$

Conceptual question

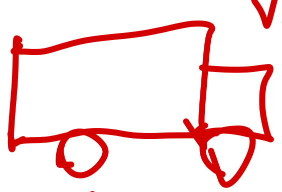
responseware.eu
Session ID: epflphys101en

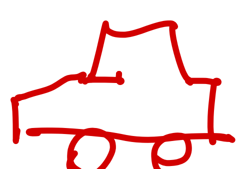
A compact car and a large truck collide head on and stick together. Consider them to be an isolated system. Which undergoes the larger magnitude momentum change?

A. The car.
B. The truck.
C. They are the same.
D. Not enough information is given to decide

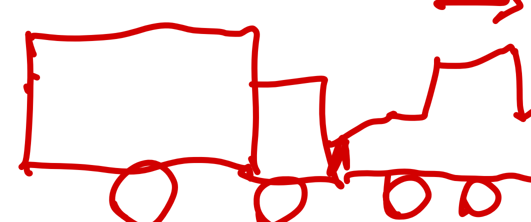
$\hat{x}$

$\vec{v}_T^i = v_T^i \hat{x}$ $\quad$ $\vec{v}_C^i = -v_T^i \hat{x}$





$\vec{v}_T^f$ $\quad$ $\vec{v}_C^f$



$\vec{p}_T^i = m_T \vec{v}_T^i$

$\vec{p}_C^i = m_C \vec{v}_C^i = -m_C v_T^i \hat{x}$

$\vec{p}_T^f = m_T \vec{v}_T^f$ $\quad$ $\vec{p}_C^f = m_C \vec{v}_C^f$

$$\vec{p}_T^i + \vec{p}_C^i = \vec{p}_T^f + \vec{p}_C^f \Rightarrow \vec{p}_T^i - \vec{p}_T^f = \vec{p}_C^f - \vec{p}_C^i \Rightarrow -\Delta\vec{p}_T = \Delta\vec{p}_C$$

$$\boxed{|\Delta\vec{p}_T| = |\Delta\vec{p}_C|}$$

How do we move (i.e. change momentum)?

- If momentum is conserved, how do we move? We push off of things, usually the ground

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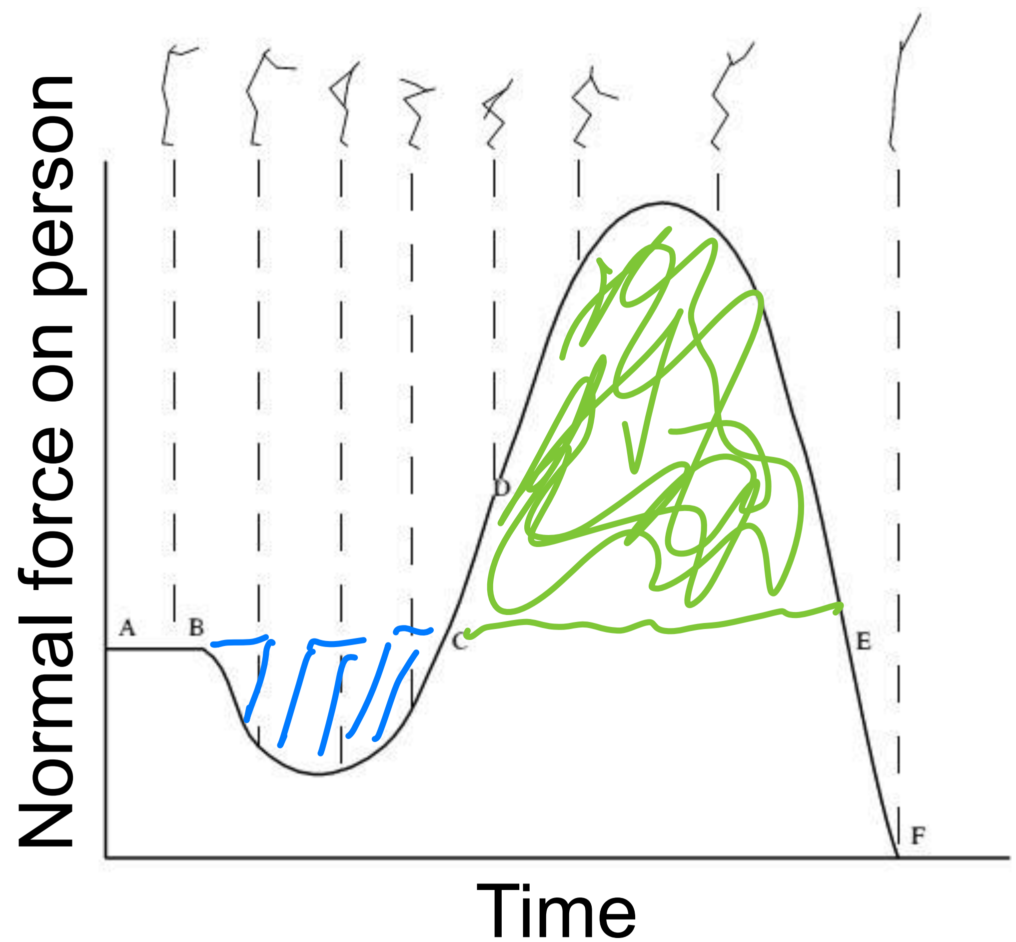
- If momentum is conserved, how do we move? We push off of things, usually the ground
- Consider a "system" to include only your body, then exert a net force on an external object
- We change momentum by applying force for a time interval



Impulse and momentum

- Defined as the integral of a net force over a time period:

$$\vec{I} = \int_{t_i}^{t_f} \overrightarrow{F}_{net}\, dt$$

- Using Newton's 2nd law $\overrightarrow{F}_{net} = \dfrac{d\vec{p}}{dt}$

$$\vec{I} = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt \quad \Rightarrow \quad \vec{I} = \vec{p}(t_f) - \vec{p}(t_i) = \Delta\vec{p}$$

- It is simply a change of momentum in time
- It has the same units as momentum of [kg·m/s] (or equivalently [N·s])

Neglecting the details

- Often we don’t care about the details of the force (e.g. when it’s very short)

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- Model the impulse as an average force applied over the same time interval

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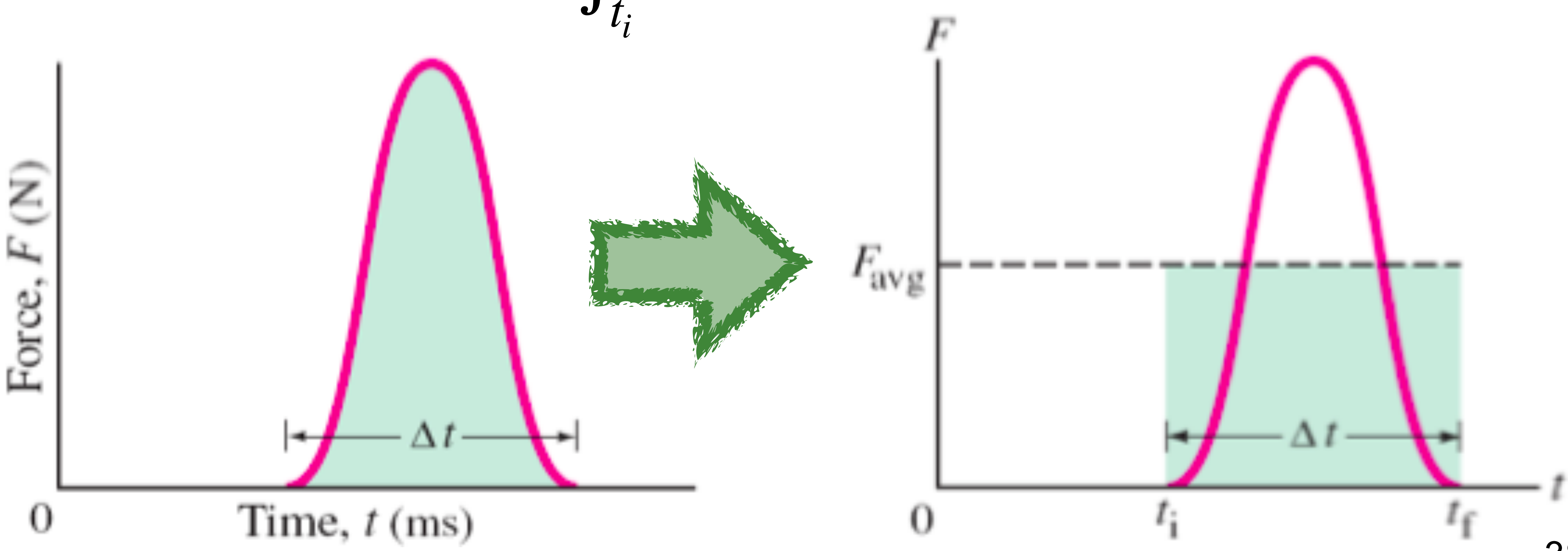


DEMO (84)

Duration of a collision

Impulse approximation

- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision

Impulse approximation

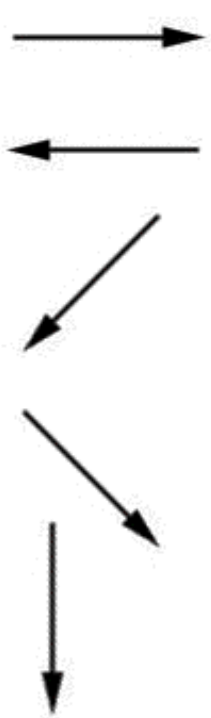
- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision
- Example: A tennis ball is coming at you at $\vec{v}_1$. You hit it with a tennis racket and it departs leaving at $\vec{v}_2$.
  - What is the change in momentum? $\Delta\vec{p} = m_b\vec{v}_2 - m_b\vec{v}_1$
  - What is the impulse? $\vec{I} = \Delta\vec{p}$
  - What is the force and how long was it applied? 
  - Impulse approximation: don't consider the effect of gravity *during* the collision with the tennis racket

Conceptual question

The figure below depicts the paths of two colliding blue circles, 1 and 2. Which of the following arrows best represents the direction of the impulse applied to circle 2 by circle 1 during the collision?

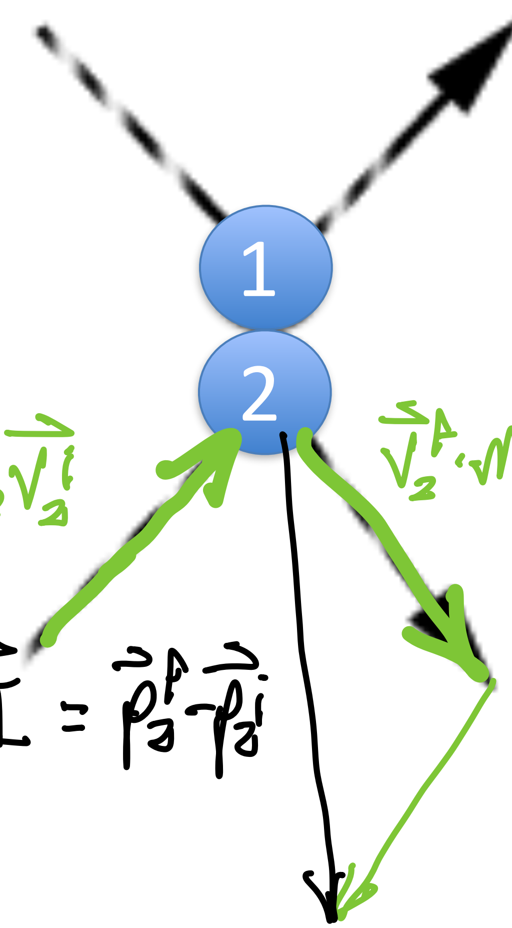
$\vec{I}_2 = \Delta\vec{p}_2 = \vec{p}_2^{\,f} - \vec{p}_2^{\,i} = m_2\vec{v}_2^{\,f} - m_2\vec{v}_2^{\,i}$

A.
B.
C.
D.
E.



$\vec{p}_2^{\,i} = m_2\vec{v}_2^{\,i}$

$\vec{v}_2^{\,f} \cdot m_2 = \vec{p}_2^{\,f}$



$\vec{I} = \vec{p}_2^{\,f} - \vec{p}_2^{\,i}$

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2. Momentum
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3. **Center of mass**

Center of mass

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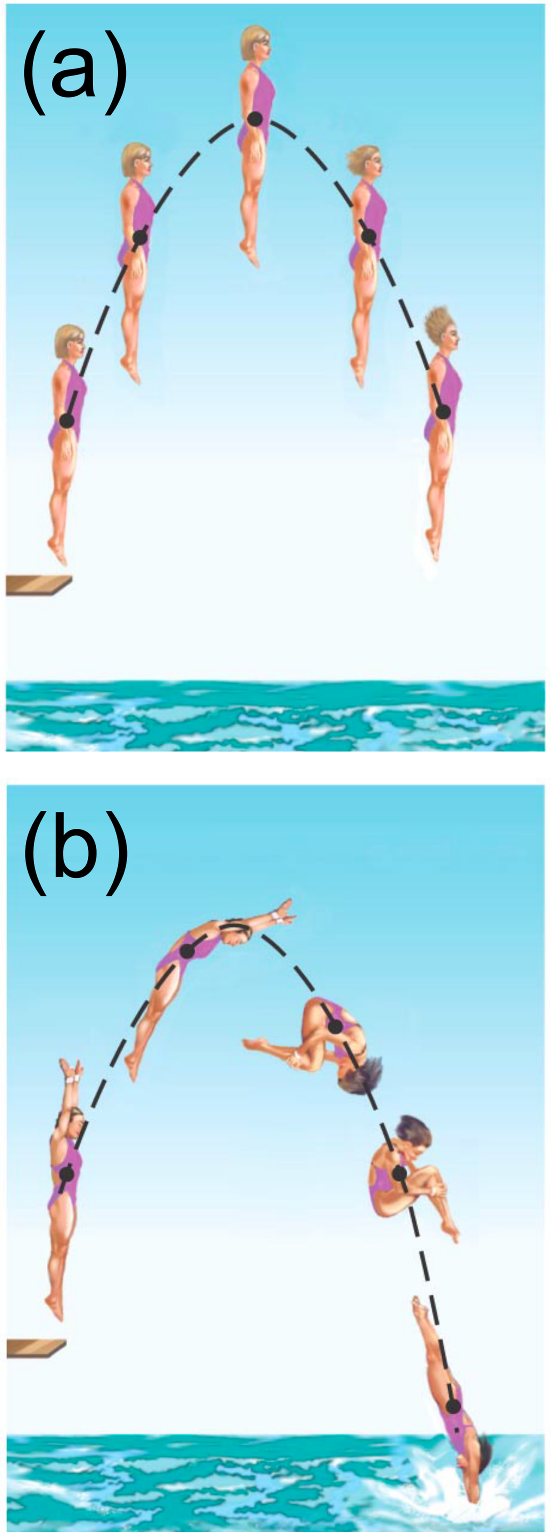
- We've used the point mass approximation a lot, but the appropriate point isn't always obvious

- If we want to summarize this guy's trajectory, which point should we take?

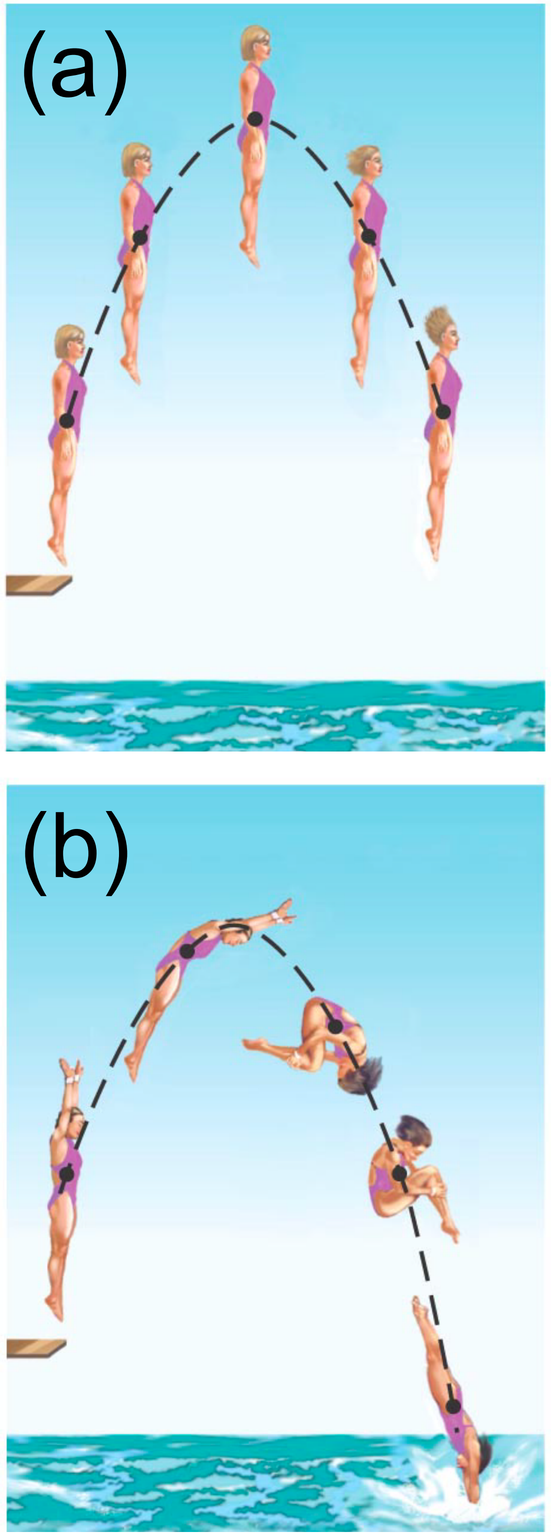
Center of mass

- In (a), the diver's motion is pure translation
- In (b), the motion is translation plus rotation



Center of mass

- In (a), the diver’s motion is pure translation
- In (b), the motion is translation plus rotation
- One point, called the *Center of Mass* (CM), that moves the same in both (i.e. it’s the point the diver rotates around)
- It’s the one point that would move in the same path as a point mass subjected to the same net force



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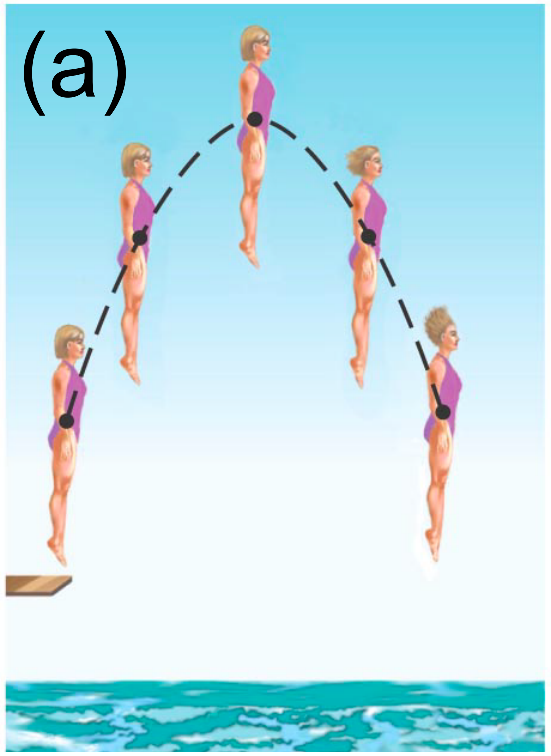
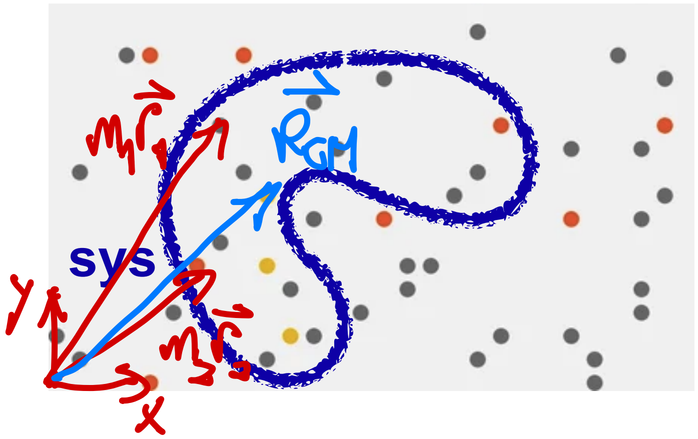
- It's the one point that would move in the same path as a point mass subjected to the same net force
- The motion of an object can always be decomposed into translational motion of the center of mass, plus rotation, deformation, …
- How do we find it, you ask…



Calculating the center of mass

- It is the "average" position of the system, weighted by mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$



where $M = \sum_{i=1}^{N} m_i$ is the total mass

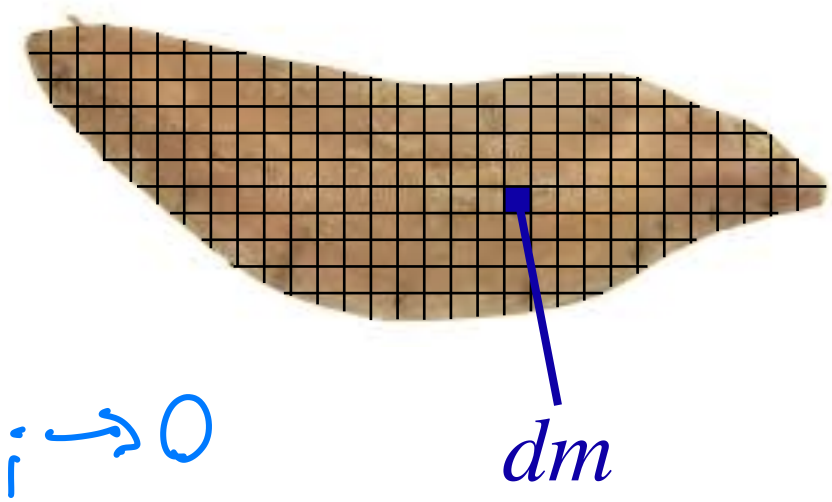
- For just two particles in 1D, the center of mass lies closer to the one with more mass

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{m_1}{M} x_1 + \frac{m_2}{M} x_2$$

Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds



$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_i \vec{r}_i \, dm_i}{\sum_i dm_i}$$

$dm_i \to 0$

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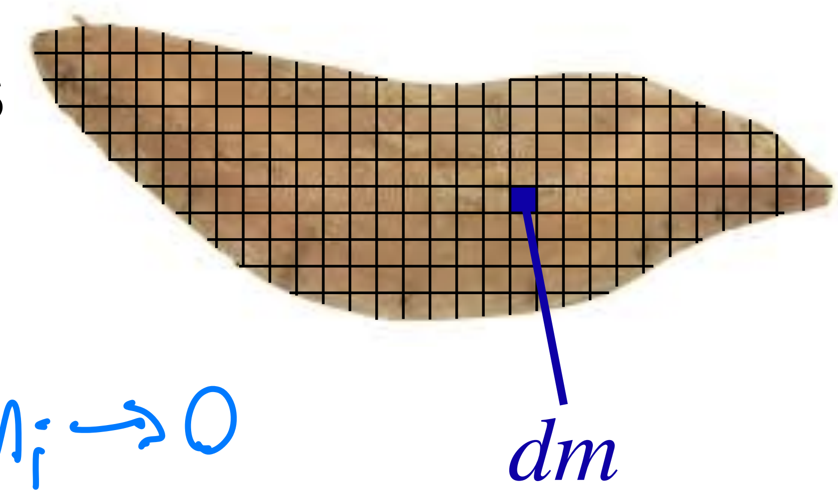

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i} \approx \frac{\sum_i \vec{r}_i \, dm_i}{\sum_i dm_i}$$

$dm_i \to 0$

- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\vec{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r} dm}{M}$$

1D: $\lambda = \frac{dm}{dx}$

$dm = \lambda \, dx$

2D: $\sigma = \frac{dm}{dA}$

3D: $\rho_V = \frac{dm}{dV}$

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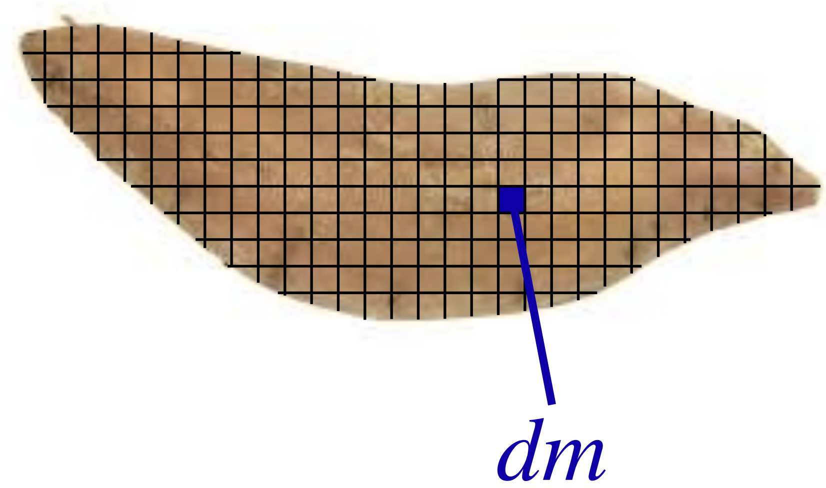
- The previous formula actually still holds



$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i}$$

- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\vec{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r} dm}{M}$$

- Use density to convert mass to spatial integral

DEMO (148)

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Finding the center of mass:

Hanging

Kinematics of the center of mass

- Given a system with constant masses, take the derivative of

$$\vec{R}_{CM} = \frac{d}{dt}\left[\frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}\right] \quad \text{or} \quad \vec{R}_{CM} = \frac{\int_M \vec{r}\,dm}{M}$$

to find

$$\frac{d}{dt}\left[\sum m_i \vec{r}_i\right] = \sum m_i \frac{d\vec{r}_i}{dt} = \sum m_i \vec{v}_i$$

$$\vec{V}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M} \quad \text{or} \quad \vec{V}_{CM} = \frac{\int_M \vec{v}\,dm}{M}$$

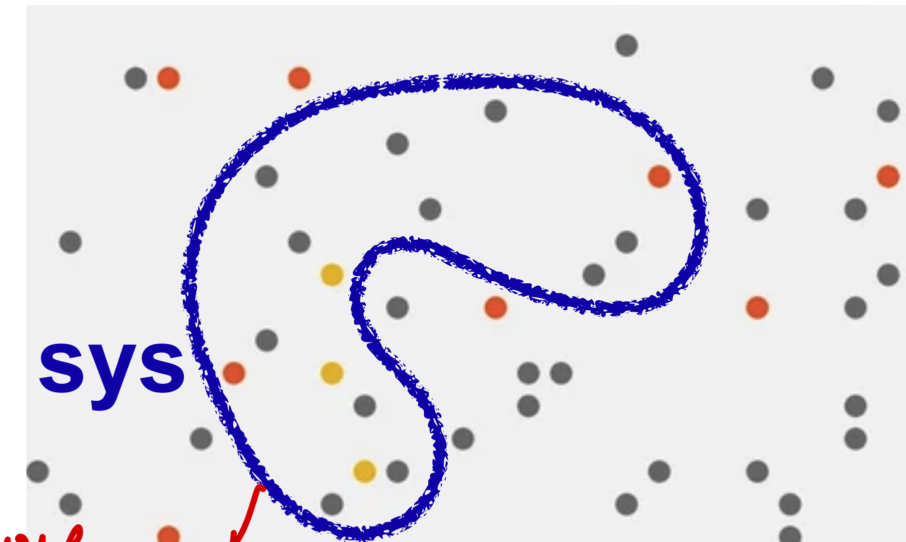
and again to find

$$\vec{A}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M} \quad \text{or} \quad \vec{A}_{CM} = \frac{\int_M \vec{a}\,dm}{M}$$

Forces are applied at the center of mass

- The center of mass can prove it's own usefulness

$$\vec{A}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M} = \frac{\sum_{i\in sys} m_i \vec{a}_i}{M}$$



$$M\vec{A}_{CM} = \sum_{i\in sys} m_i \vec{a}_i$$

By Newtons 2nd law

$$m_i\vec{a}_i = \sum_j \vec{F}_{ji} = \sum_{j\in sys} \vec{F}_{ji} + \sum_{j\notin sys} \vec{F}_{ji}$$

$$M\vec{A}_{CM} = \sum_{i\in sys}\left[\sum_{j\in sys} \vec{F}_{ji} + \sum_{j\notin sys} \vec{F}_{ji}\right]$$

$$= \underbrace{\sum_{i\in sys}\sum_{j\in sys} \vec{F}_{ji}}_{=0} + \sum_{i\in sys}\sum_{j\notin sys} \vec{F}_{ji} = \vec{F}^{ext}_{tot}$$

By Newton's 3rd law

Forces are applied at the center of mass

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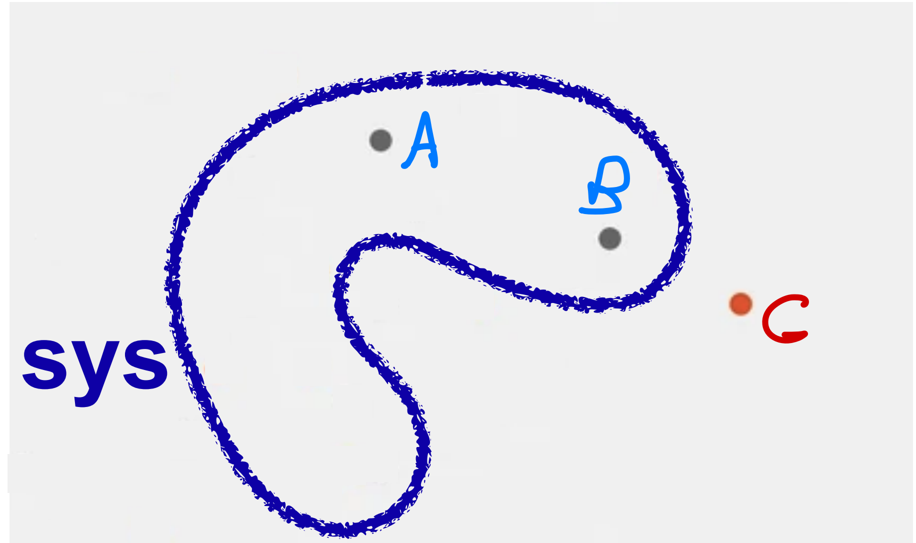
$$M\overrightarrow{A}_{CM} = \overrightarrow{F}^{ext}_{net}$$

- Thus, we can pretend the entire system is located at the center of mass

Forces are applied at the center of mass

- Separate **internal** and **external** forces

$$M\vec{A}_{CM} = \sum_{i\in sys}\left(\sum_{j\in sys}\vec{F}_{ji} + \sum_{j\notin sys}\vec{F}_{ji}\right)$$



$$= \underbrace{\vec{F}_{AB} + \vec{F}_{BA}}_{=0} + \vec{F}_{CA} + \vec{F}_{CB}$$

$$= \vec{F}_{CA} + \vec{F}_{CB} = \vec{F}^{ext}_{net}$$

See you tomorrow!