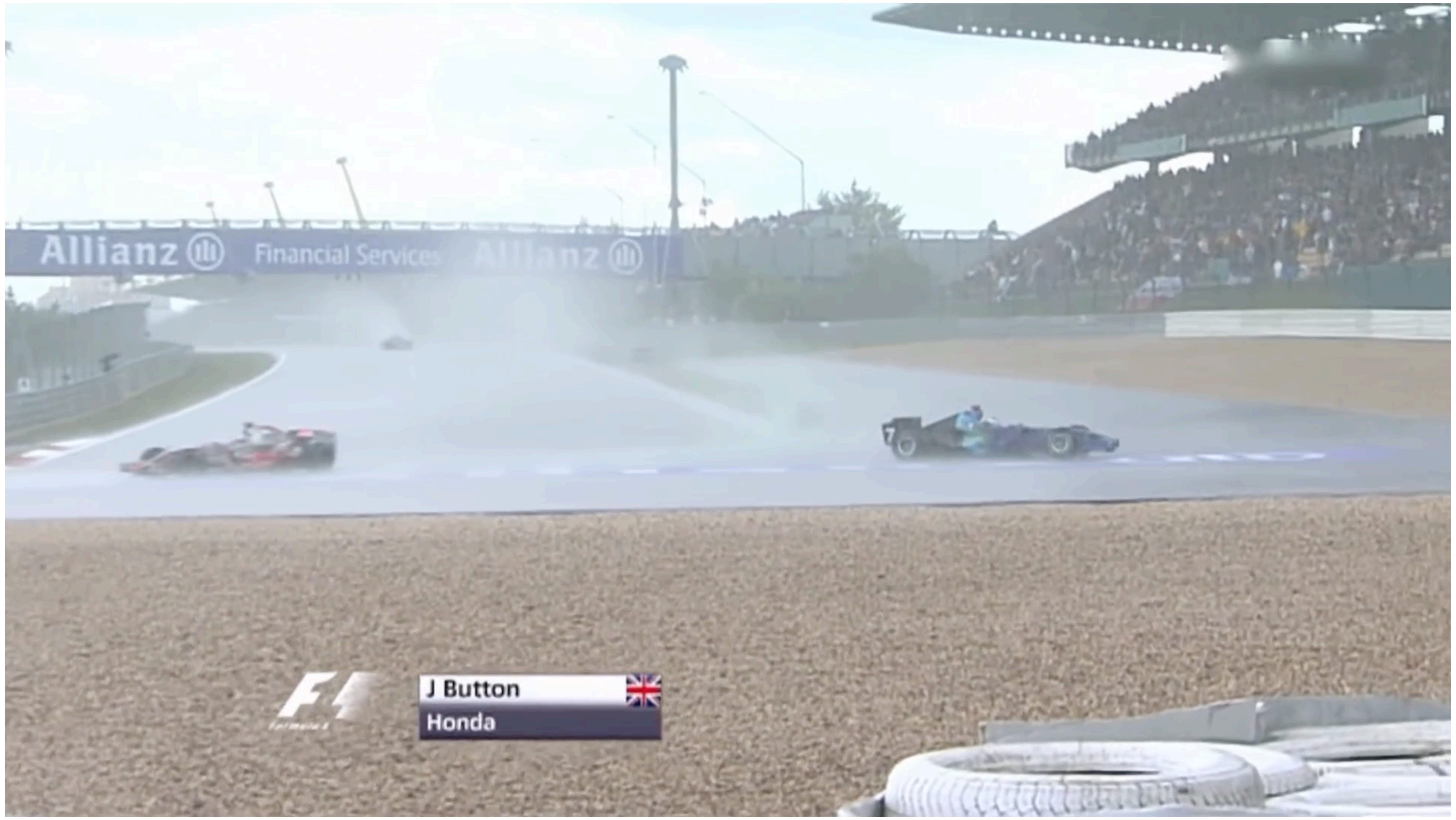
General Physics: Mechanics

PHYS-101(en)

Lecture 4b: Circular motion



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Reminder: Supplementary Q&A

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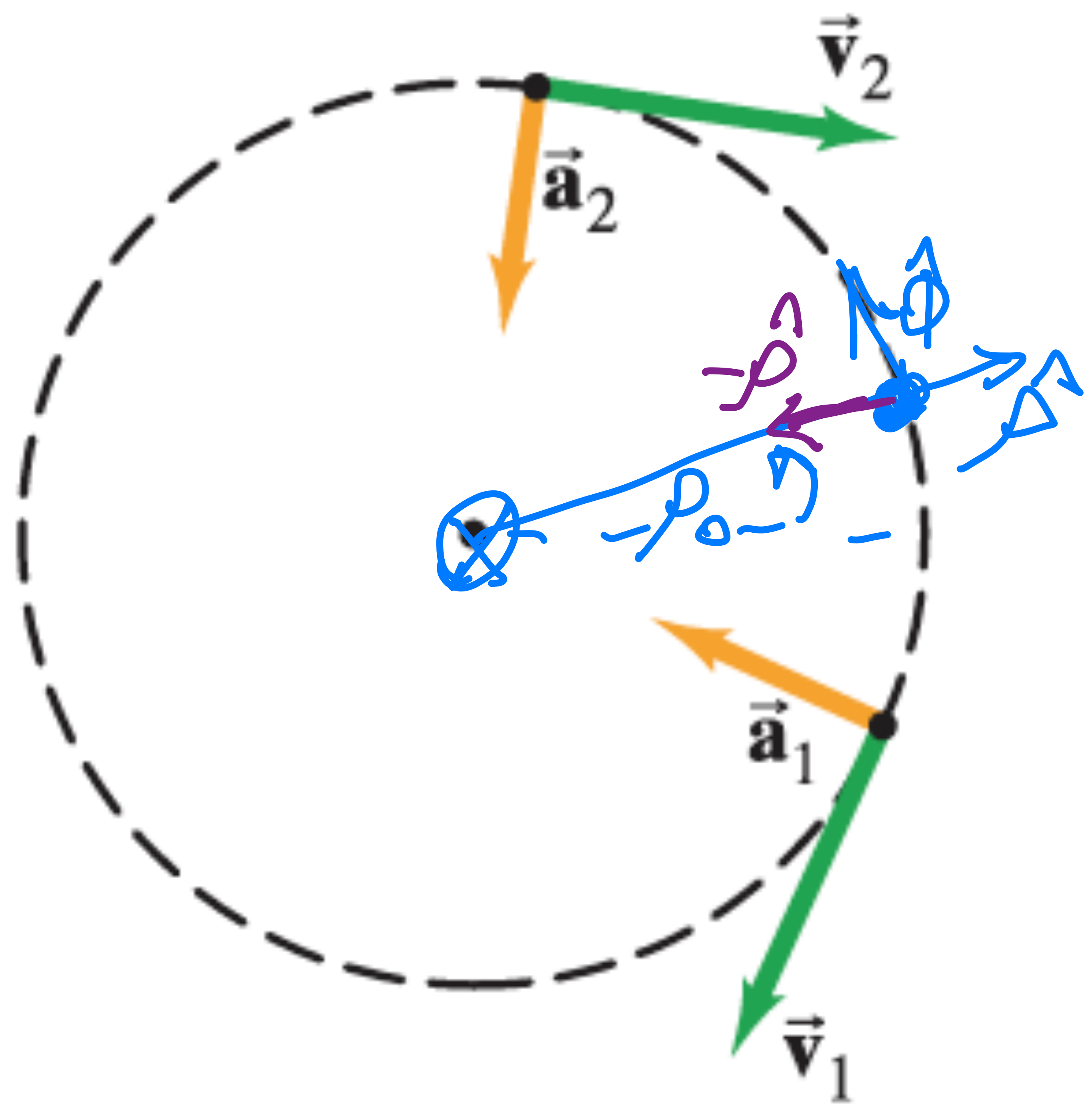
- Sessions start today and run until the end of the semester.
- They will take place Tuesdays 17:30-19:00 and Thursdays 18:00-19:30 in room **CM 1 105.**
- Additional resource for those of you who want to further discuss current and previous problem sets.

Summary: **Uniform** circular motion

- Motion in a circle of constant radius ρ_0 at constant angular velocity $\vec{\omega}$ (in rad/s) $\omega = \frac{d\phi}{dt} = \frac{2\pi}{T} = 2\pi f$
- Instantaneous velocity is always tangent to the circle and has magnitude $v = \rho_0 \omega$
- Acceleration always points to the axis of rotation and is called centripetal acceleration $\vec{a}_{cent} = \rho_0 \omega^2 (-\hat{\rho})$
- Cylindrical coordinates are extremely convenient to describe this motion!



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Example: Whirling stone

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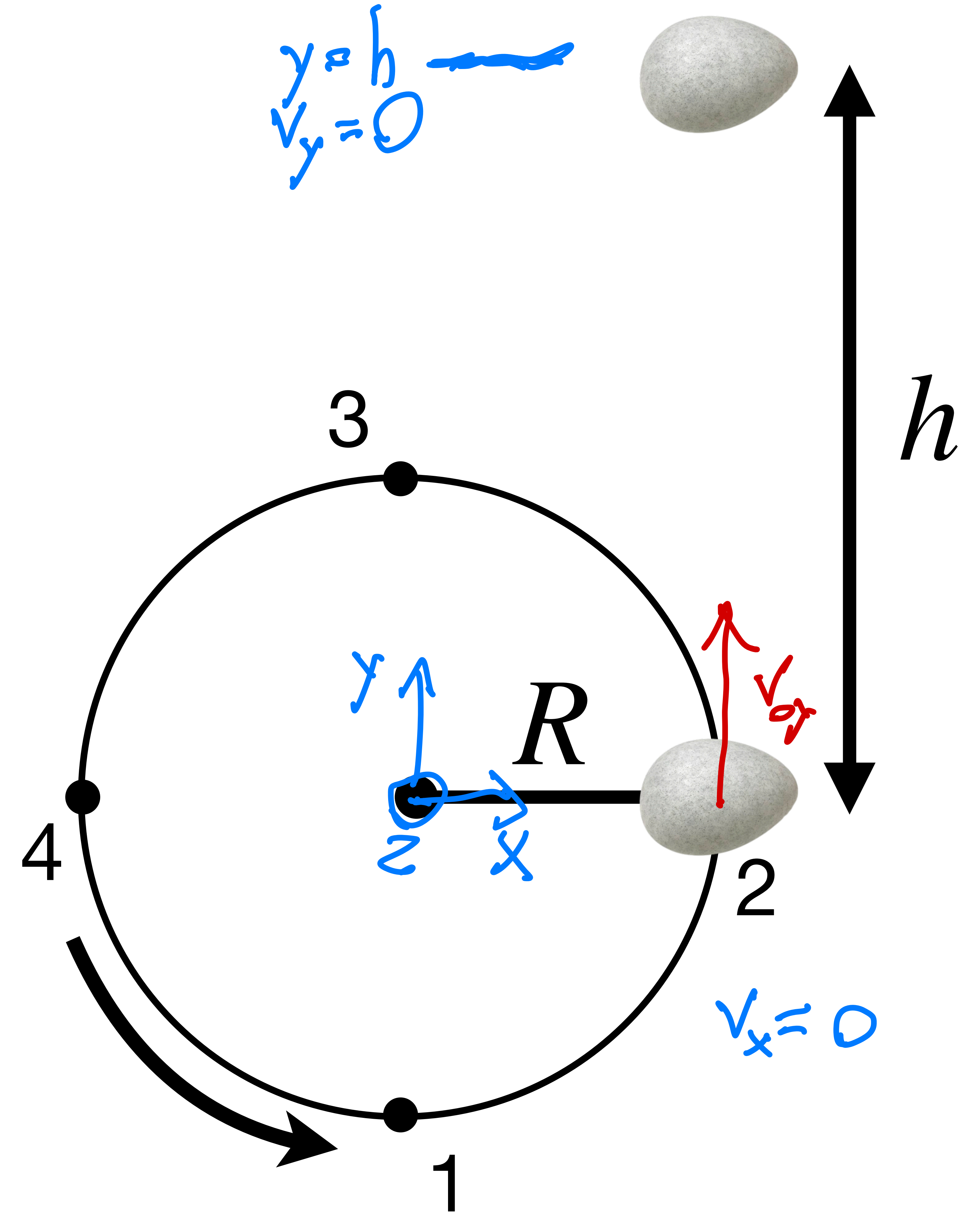
A stone is attached to a wheel and held in place by a string. It is whirled in circular trajectory of radius R in a vertical plane. Suppose the string is cut when the stone is at position 2, and the stone then rises to a maximum height h above the point at position 2. What was the angular velocity of the stone when the string was cut? Give your answer in terms of R, h, and g.



Example: Whirling stone

$y(t) = \cancel{y_0}^{\,0} + v_{oy} t - \frac{g}{2} t^2$ $\quad$ $v_y(t) = v_{oy} - gt$

At max: $v_y = 0 = v_{oy} - g t_h \Rightarrow t_h = \frac{v_{oy}}{g}$

$y(t_h) = h = v_{oy} t_h - \frac{g}{2} t_h^2 = v_{oy}\left(\frac{v_{oy}}{g}\right) - \frac{g}{2}\left(\frac{v_{oy}}{g}\right)^2 = \frac{v_{oy}^2}{g} - \frac{v_{oy}^2}{2g} = \frac{v_{oy}^2}{2g}$

$\Rightarrow h = \frac{v_{oy}^2}{2g}$

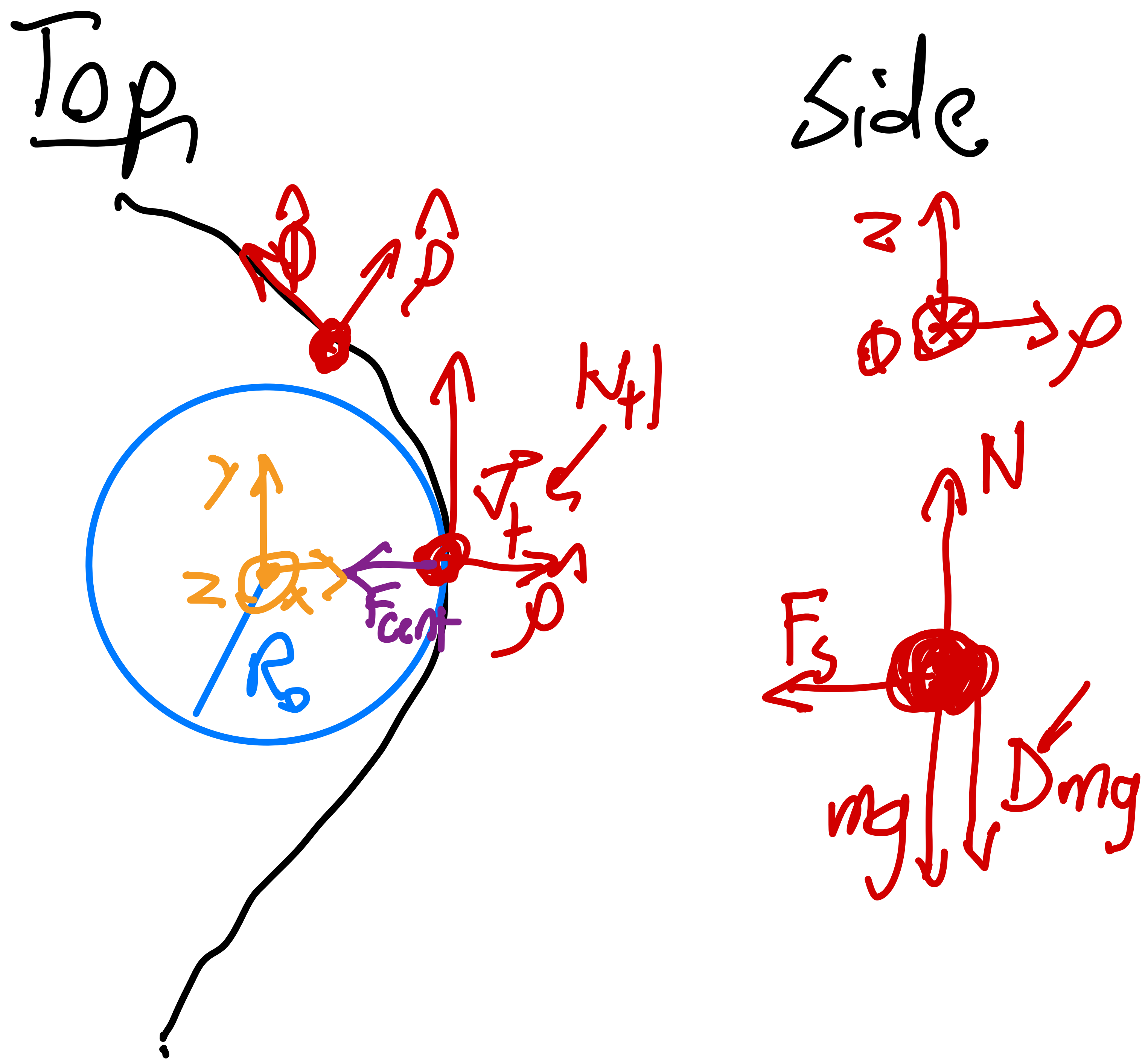
Circ. motion: $v_{oy} = R\omega$

$\Rightarrow 2gh = (R\omega)^2 \Rightarrow \omega = \frac{\sqrt{2gh}}{R}$

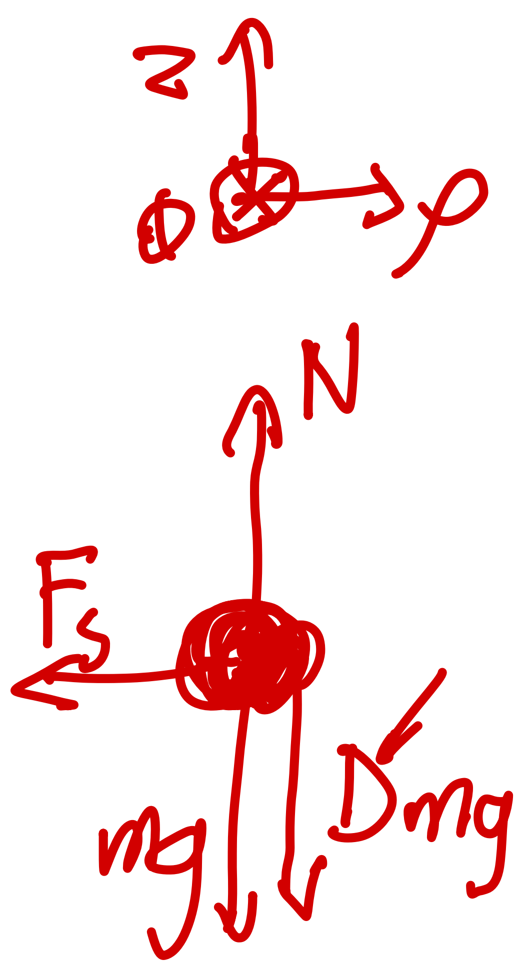
$$\boxed{\bar{\omega} = \frac{\sqrt{2gh}}{R}\frac{1}{2}}$$

Example: Formula 1 downforce

At top speed, an F1 car can generate as much as 5 g's of *downforce* (i.e. a downwards force 5 times its own weight). This comes from *airfoils* designed into the car, which are effectively upside down airplane wings. Imagine navigating a flat turn of radius R_0. How much faster can an F1 car *with* airfoils drive, compared to one *without*?



Example: Formula 1 downforce



$$\sum F_z: \quad N - mg - Dmg = 0 \quad \Rightarrow \quad N = mg + Dmg = (D+1)mg$$

$$\sum F_\rho: \quad -F_s = ma_{cent} = -mR_0\omega^2 \quad \Rightarrow \quad F_s = mR_0\omega^2$$

$$F_s \leq \mu_s N$$

$$\cancel{m}R_0\omega^2 \leq \mu_s(D+1)\cancel{m}g \quad \Rightarrow \quad \omega^2 \leq \frac{\mu_s g(D+1)}{R_0}$$

But $v_t = R_0\omega \Rightarrow \omega^2 = \frac{v_t^2}{R_0^2} \Rightarrow v_t^2 \leq \mu_s g R_0(D+1)$

$$\Rightarrow \boxed{v_t \leq \sqrt{\mu_s g R_0(D+1)}}$$

Example: Formula 1 downforce

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If $\mu_s = 0.9$, $g = 10\,\frac{m}{s^2}$, $R_0 = 100\,m$ $\Rightarrow$ $V_N^{max} = 110\,\frac{km}{h}$ $\quad V_D^{max} = 260\,\frac{km}{h}$

$$V_f \leq \sqrt{\mu_s g R_0 (D+1)}$$

$$V_D^{max} = \sqrt{\mu_s g R_0 (D+1)}$$

$$V_N^{max} = \sqrt{\mu_s g R_0}$$

$$\Rightarrow \frac{V_D^{max}}{V_N^{max}} = \frac{\sqrt{\mu_s g R_0 (D+1)}}{\sqrt{\mu_s g R_0}} = \sqrt{\frac{\cancel{\mu_s g R_0}(D+1)}{\cancel{\mu_s g R_0}}} = \sqrt{D+1}$$

If $D=5$, $\sqrt{D+1} \approx 2.4$