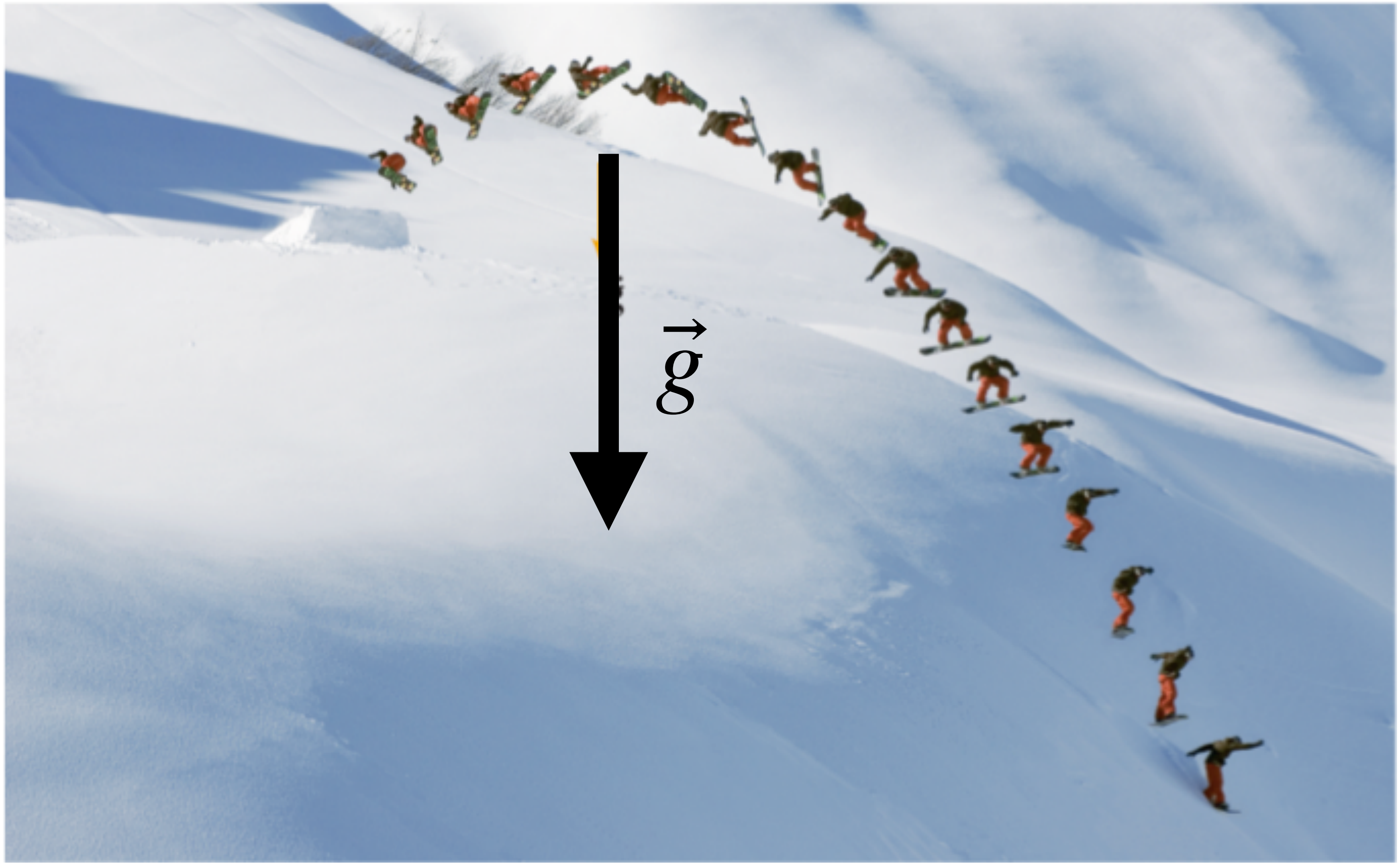
# General Physics: Mechanics

Swiss Plasma Center

## PHYS-101(en)

### Lecture 2b: Motion in two and three dimensions





Dr. Marcelo Baquero-Ruiz
marcelo.baquero@epfl.ch

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# Today's agenda (MIT 3 and 4)

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1. Motion in two and three dimensions in Cartesian coordinates

    - Acceleration due to gravity
    - Using vectors in equations
    - Projectile motion

# Review: dot product between two vectors

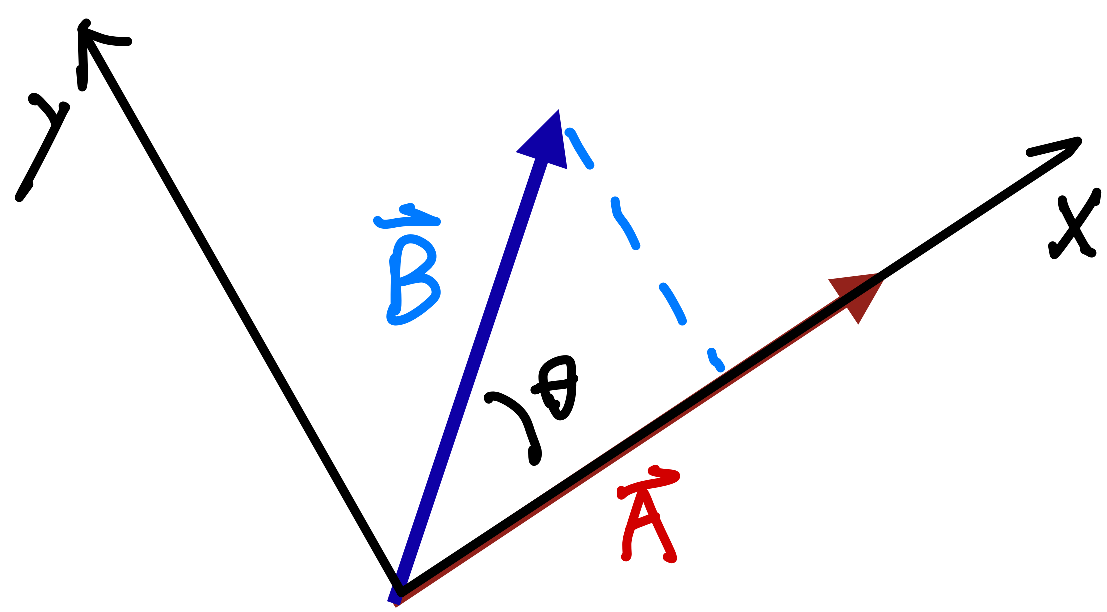
- Geometric interpretation of dot product

$$\vec{A}\cdot\vec{B} = AB\cos(\theta)$$

$$\vec{A}\cdot\vec{B} = \underbrace{A_x}_{A_x = A} B_x + \underbrace{\cancel{A_y}}_{0} B_y$$

$$= A \underbrace{B_x}_{B\cos(\theta)}$$

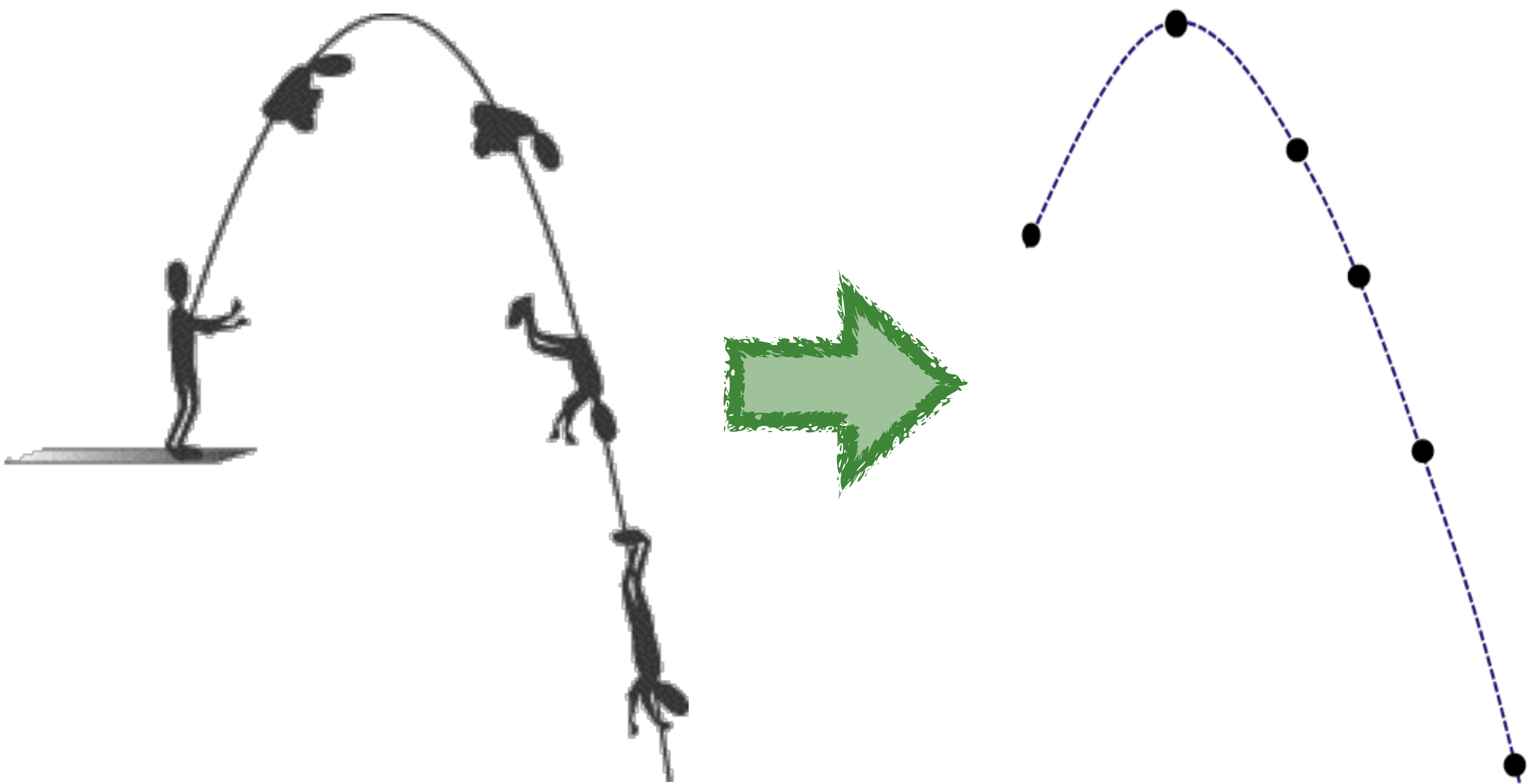
$$= AB\cos(\theta)$$



# Kinematics



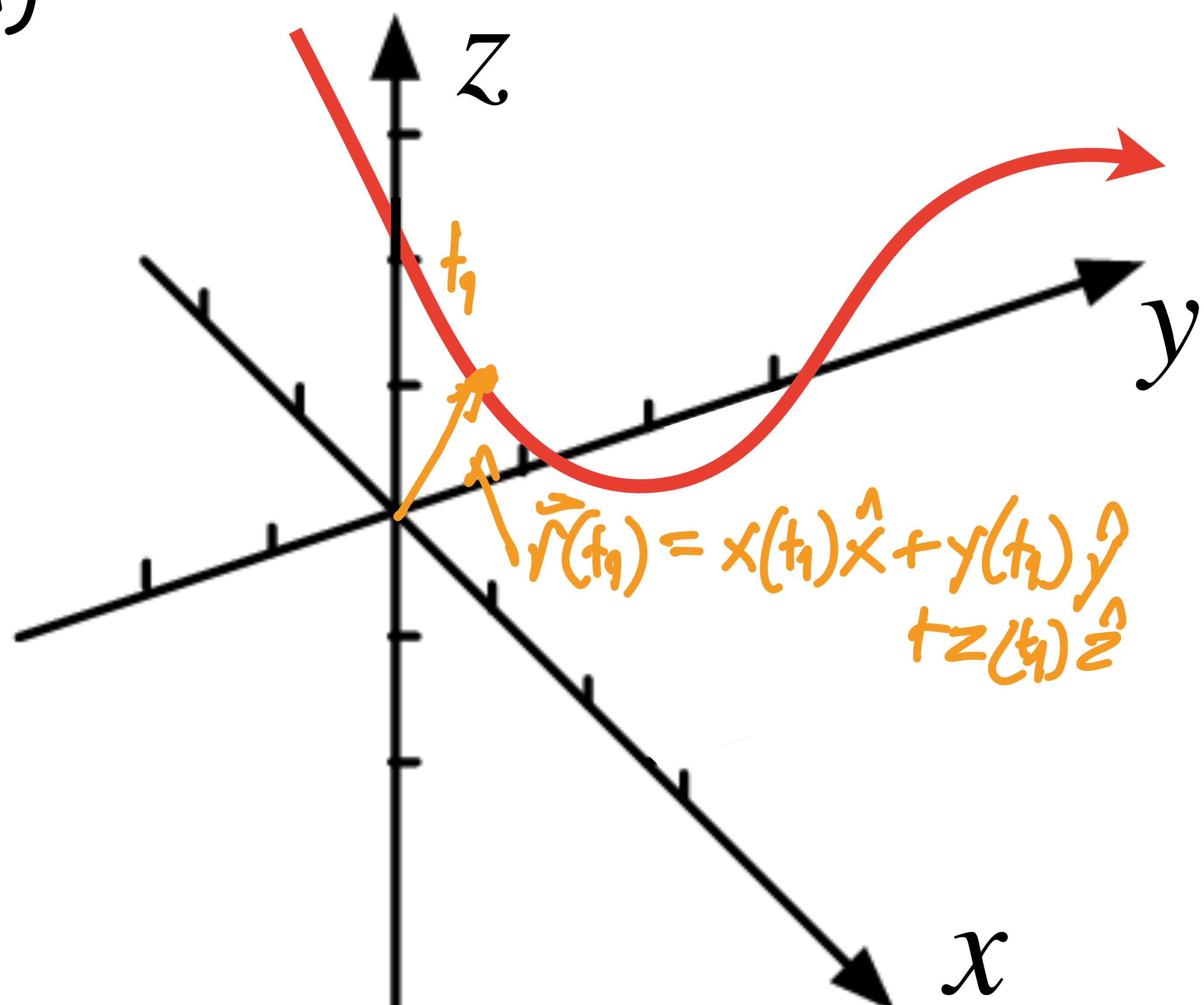
- A description of motion without considering forces
- We will approximate objects as point masses
- Need to go beyond one dimension



# Vector position in Cartesian coordinates

- Position in 1D: $x(t)$

$$\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z}$$



# Vector displacement (Cartesian)

- Displacement in 1D: $\Delta x = x(t_2) - x(t_1)$ $t_2 > t_1$

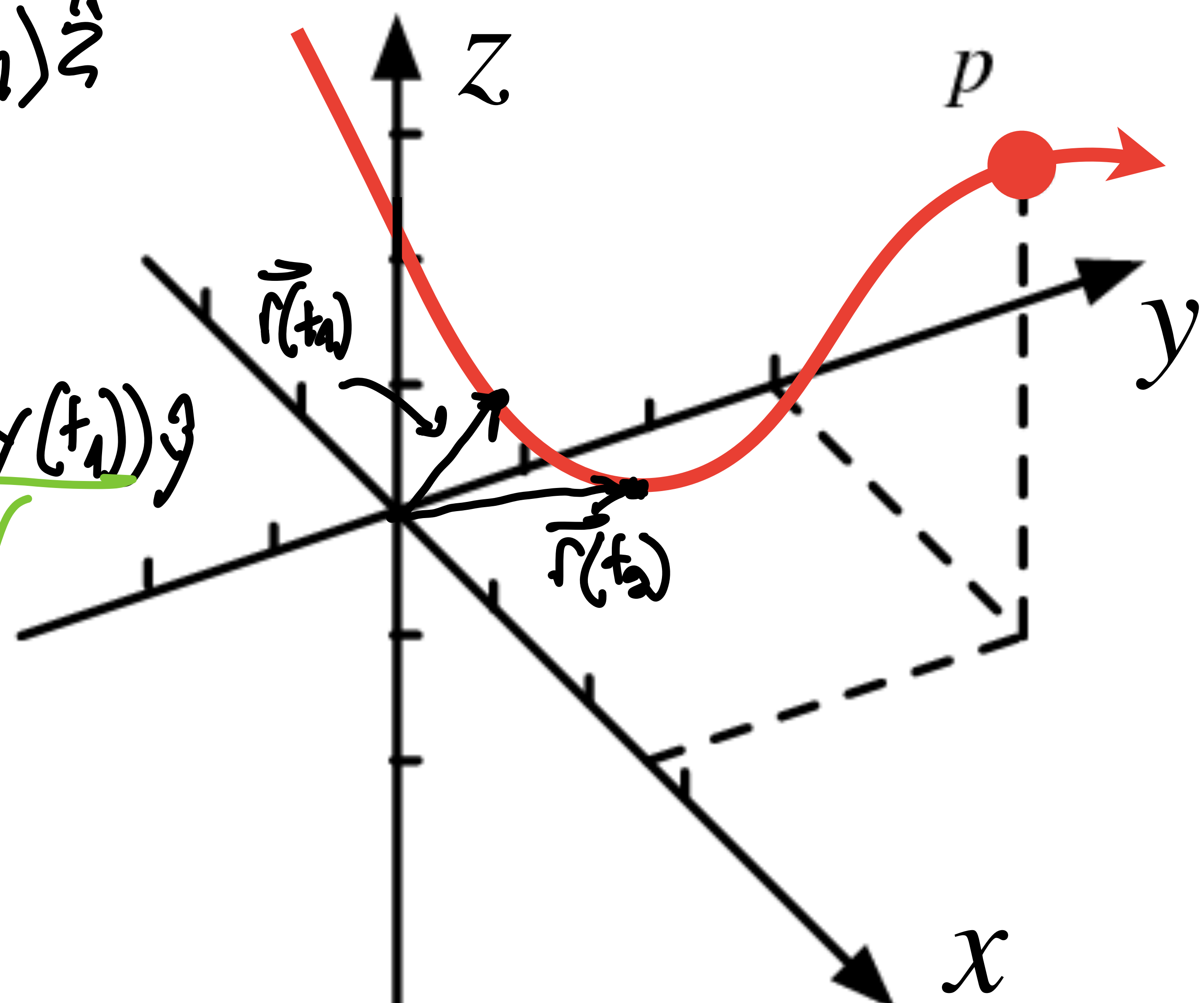
$$\vec{r}(t_2) = x(t_2)\,\hat{x} + y(t_2)\,\hat{y} + z(t_2)\,\hat{z}$$

$$\vec{r}(t_1) = x(t_1)\,\hat{x} + y(t_1)\,\hat{y} + z(t_1)\,\hat{z}$$

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

$$= \underbrace{\left(x(t_2) - x(t_1)\right)}_{\Delta x}\hat{x} + \underbrace{\left(y(t_2) - y(t_1)\right)}_{\Delta y}\hat{y} + \underbrace{\left(z(t_2) - z(t_1)\right)}_{\Delta z}\hat{z}$$

$$= \Delta x\,\hat{x} + \Delta y\,\hat{y} + \Delta z\,\hat{z}$$



# Vector velocity (Cartesian)

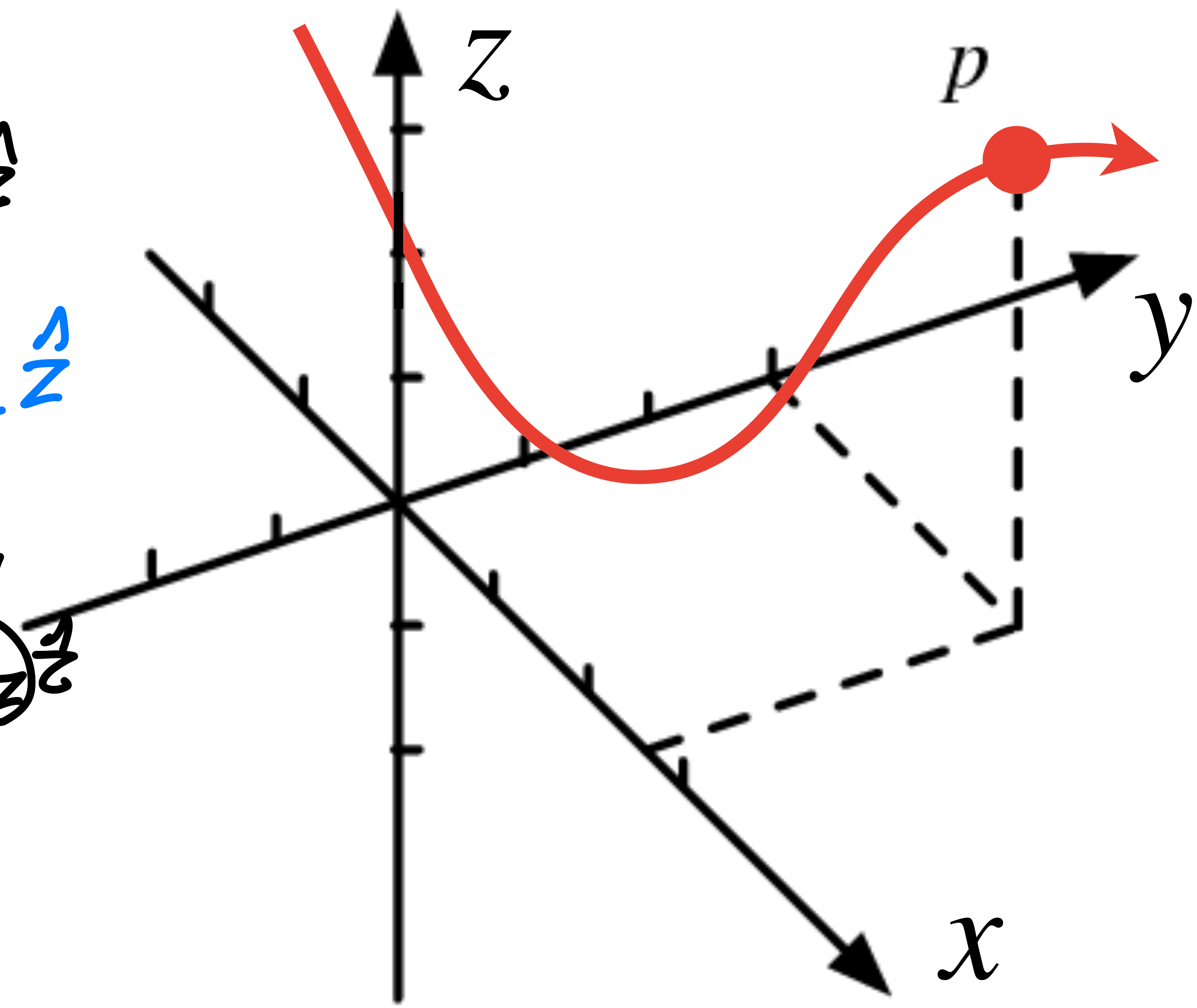
- Average velocity in 1D: $\bar{v} = \dfrac{\text{displacement}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$

$\Delta t = t_2 - t_1$

$\dfrac{1}{\Delta t}\Delta\vec{r} = \dfrac{\Delta x}{\Delta t}\hat{x} + \dfrac{\Delta y}{\Delta t}\hat{y} + \dfrac{\Delta z}{\Delta t}\hat{z}$

$\lim_{\Delta t \to 0} \dfrac{1}{\Delta t}\Delta\vec{r} = \dfrac{dx}{dt}\hat{x} + \dfrac{dy}{dt}\hat{y} + \dfrac{dz}{dt}\hat{z}$

$\dfrac{d\vec{r}}{dt} = \vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$



# Vector velocity (Cartesian)

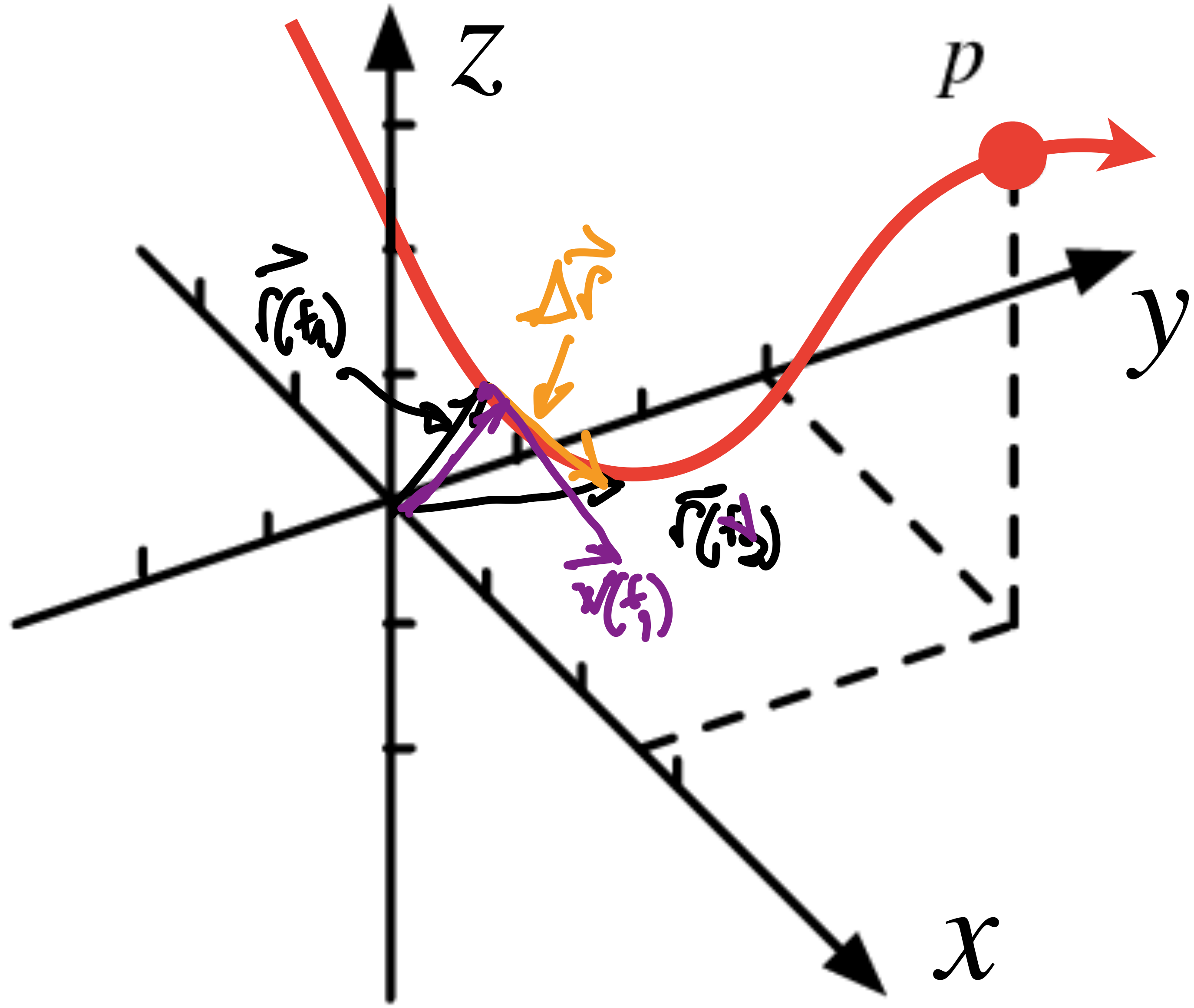
- Speed (i.e. magnitude of velocity):

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

- Direction:

Tangent to trajectory

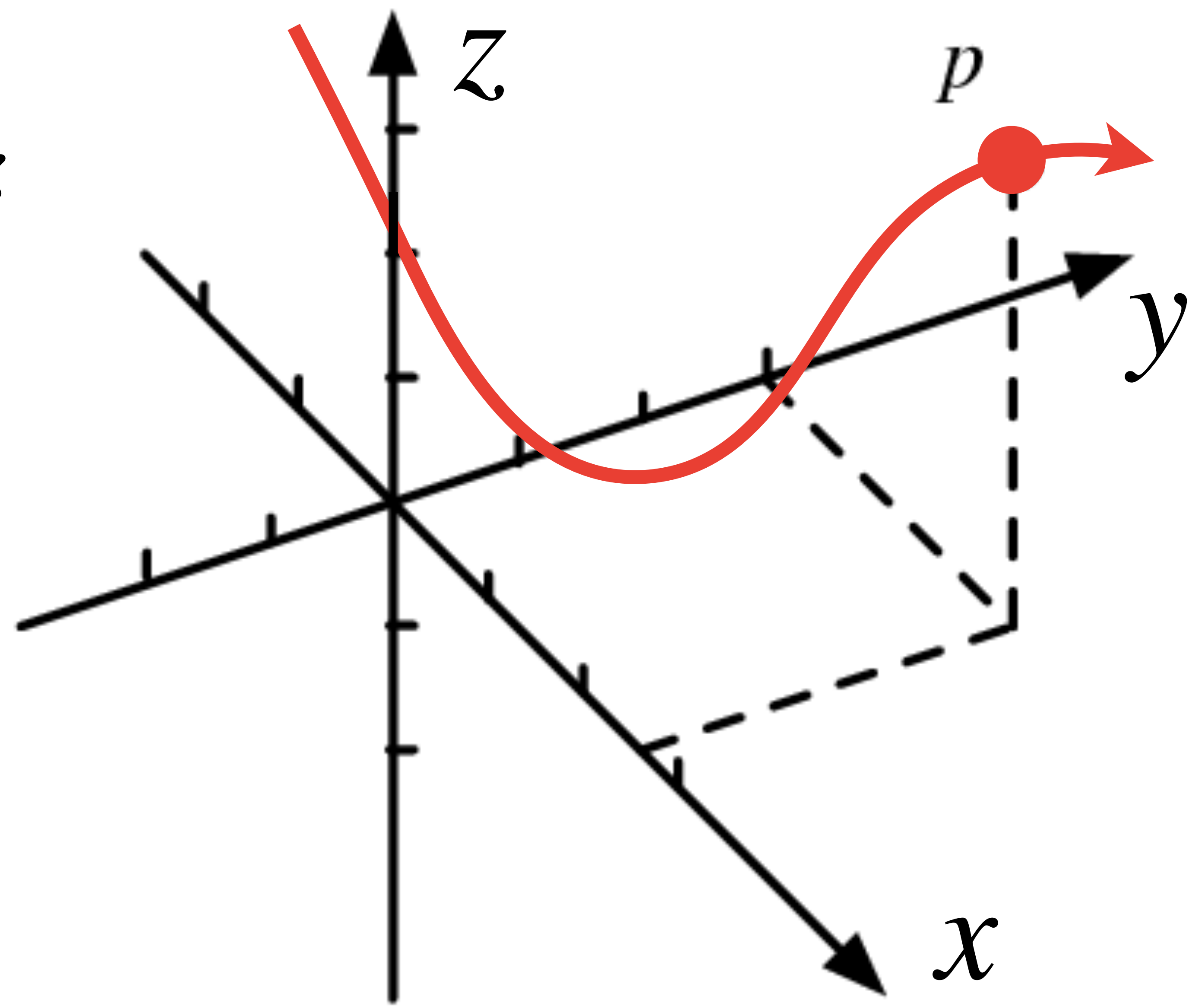


# Vector acceleration (Cartesian)

- Average acceleration in 1D: $\bar{a} = \dfrac{\text{change in velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} + \frac{dv_z}{dt}\hat{z}$$



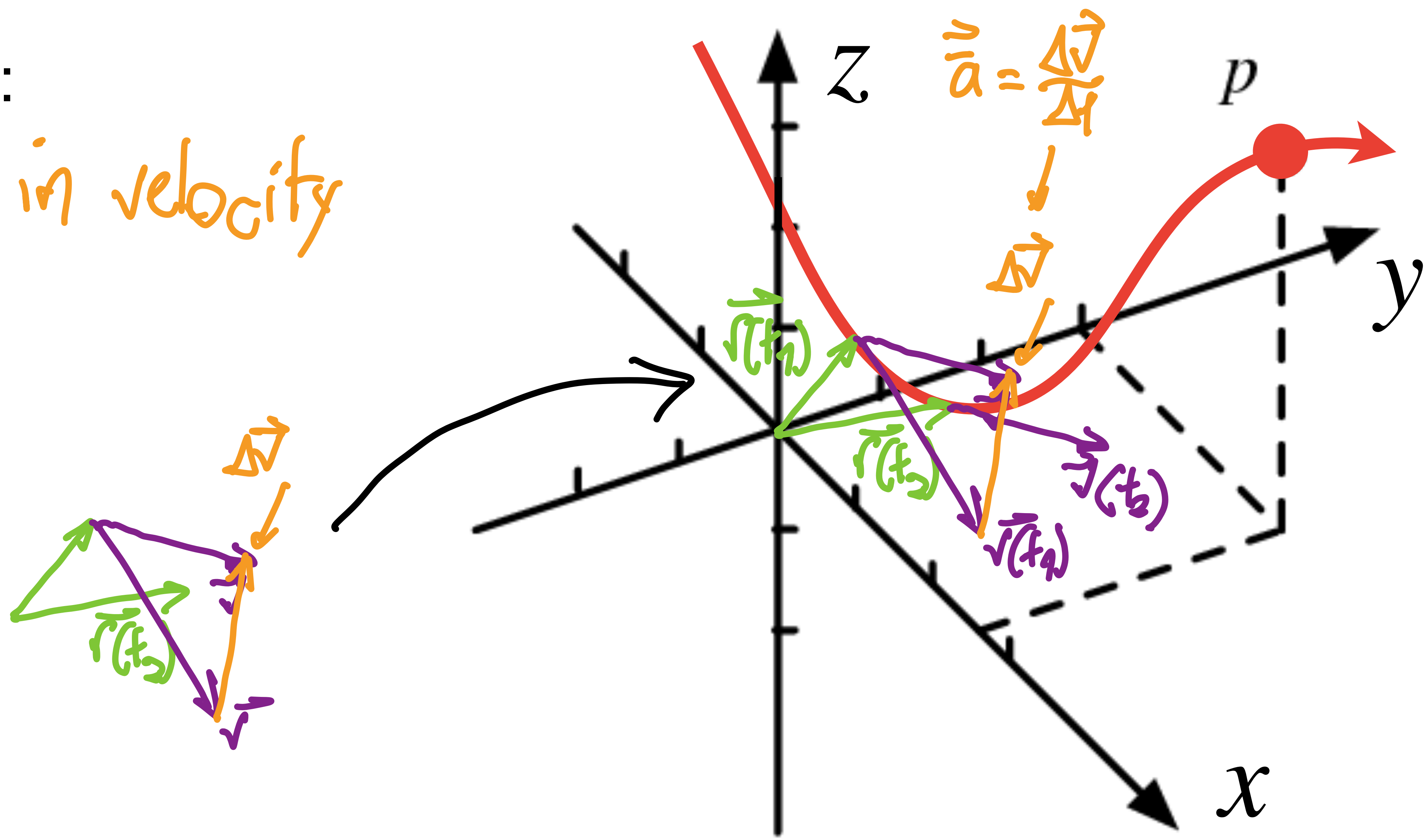
# Vector acceleration (Cartesian)

- Magnitude of the acceleration:

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

- Direction:

change in velocity



# DEMO (55)



Projectile motion

# Projectile motion

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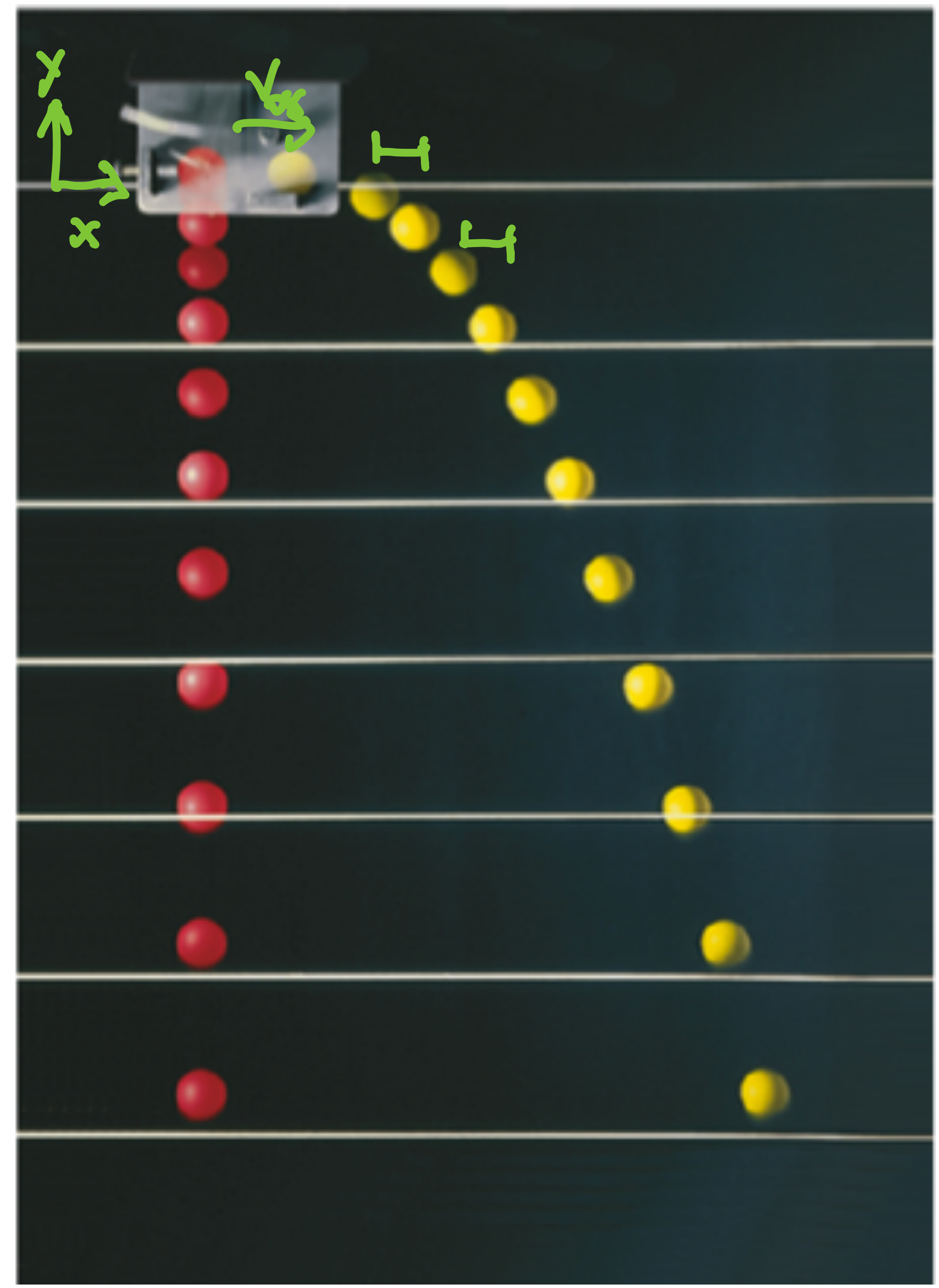
- Two balls are released simultaneously under gravity
- What causes the difference in their motions?
- What equations of motion need modified?

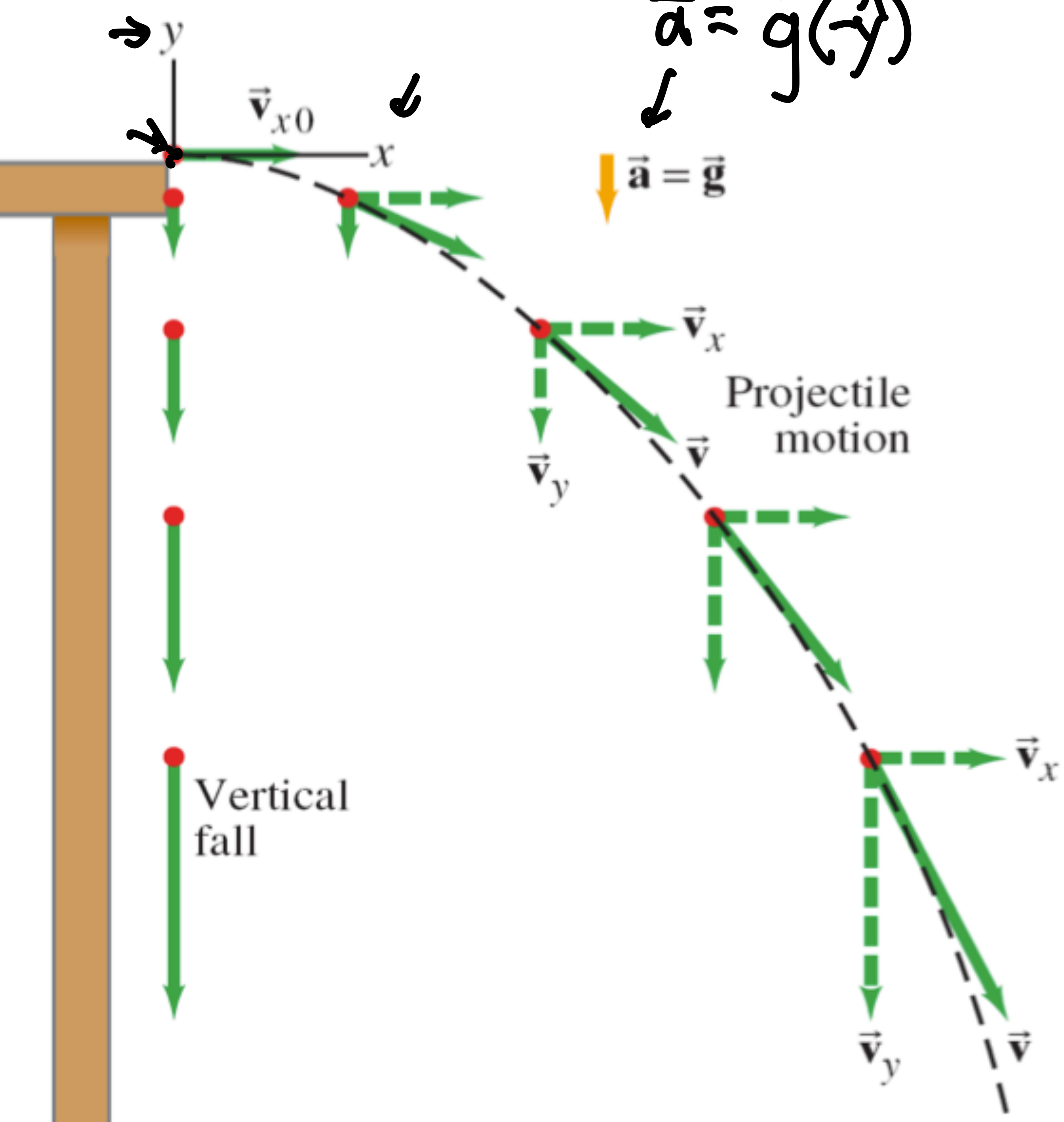


# Velocity throughout projectile motion

- Motion in horizontal and vertical components are decoupled and independent

Along y

$\vec{a} = g(-\hat{y})$, $g = 9.81\ \frac{m}{s^2}$



# See you at the exercises tomorrow!

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