General Physics: Mechanics

PHYS-101(en)
Lecture 1b: Motion in two and three dimensions

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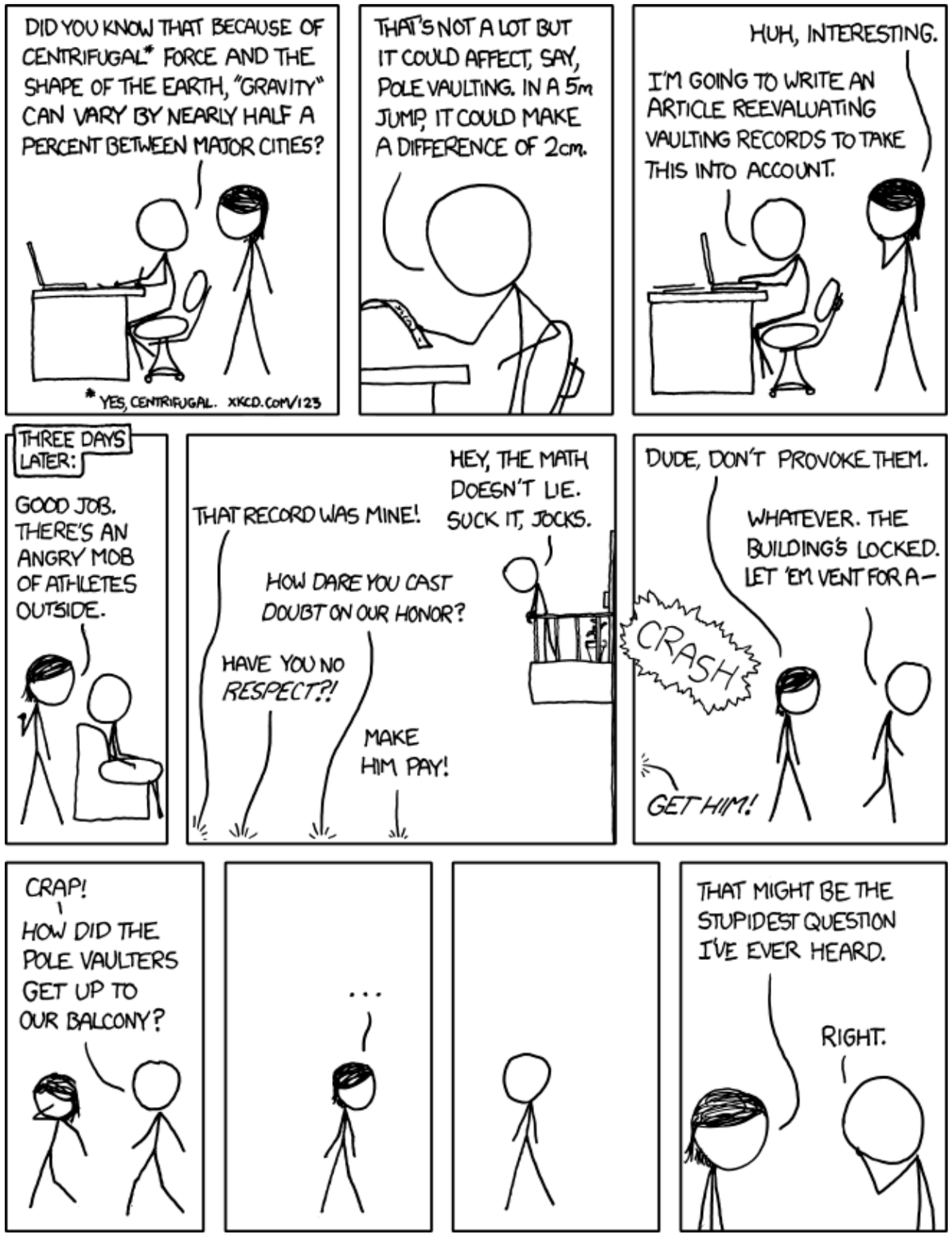
EPFL
Swiss Plasma Center



xkcd.com/852

Today's agenda (MIT 3 and 4)

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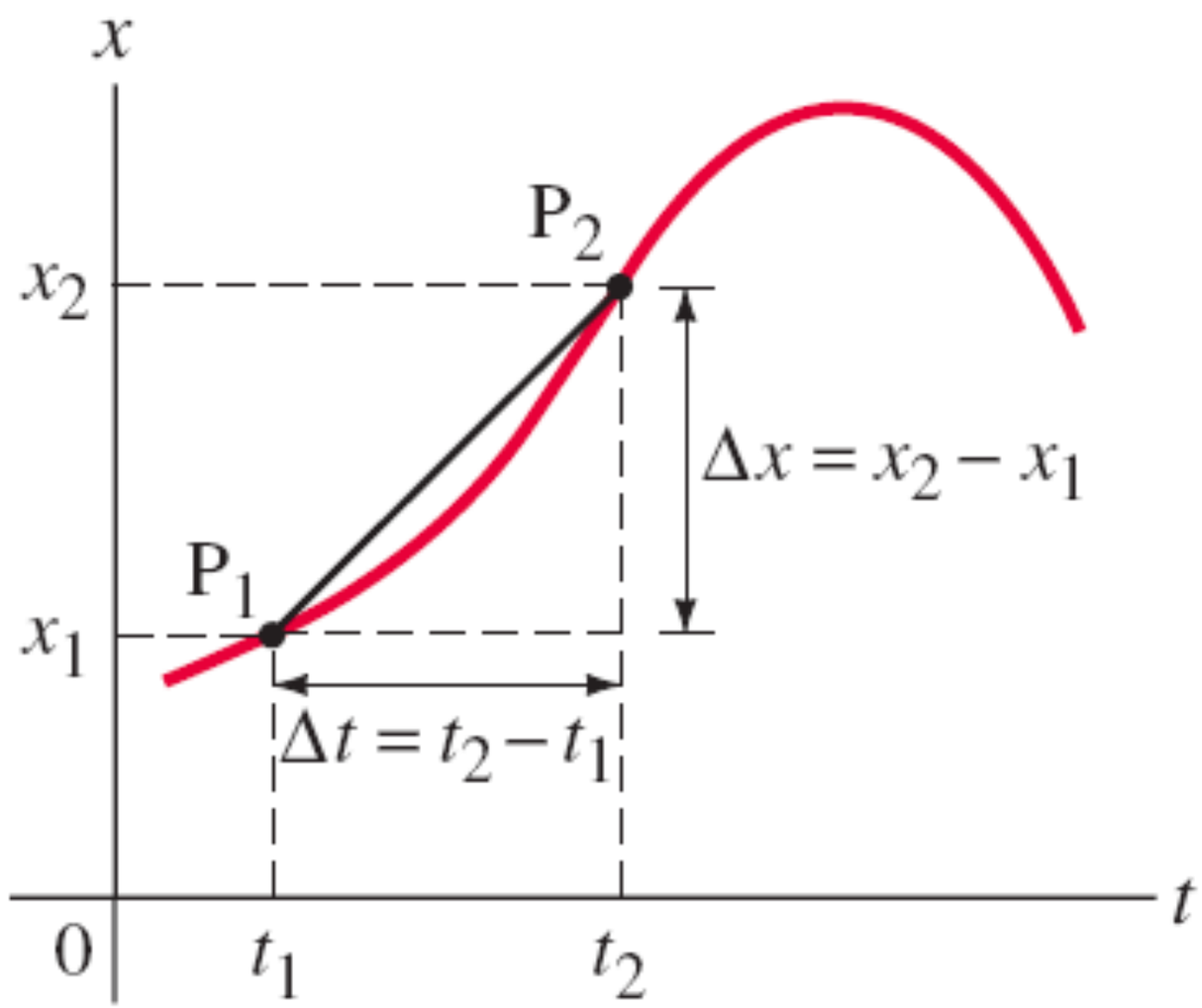
1. Revisit motion in one dimension

2. Motion in two and three dimensions in Cartesian coordinates

   - Acceleration due to gravity
   - Using vectors in equations
   - Projectile motion

Summary of motion in one dimension

- Position of an object as a function of time denoted by $x(t)$
- Average velocity: $\bar{v} = \dfrac{\text{change of position}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$



Summary of motion in one dimension

- Position of an object as a function of time denoted by $x(t)$
- Average velocity: $\bar{v} = \dfrac{\text{change of position}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$
- Instantaneous velocity: $v(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
- Average acceleration: $\bar{a} = \dfrac{\text{change of velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$
- Instantaneous acceleration:

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Conceptual question

A ball is thrown straight up. At its maximum height, its…

A. velocity is zero and acceleration is zero.

B. velocity is non-zero and acceleration is non-zero.

C. velocity is zero and acceleration is non-zero.

D. velocity is non-zero and acceleration is zero.

Conceptual question

A ball is launched straight up with initial velocity v_0 (neglect air resistance). If the initial velocity v_0 is doubled, the time to reach the apex of the trajectory…

A. doubles.

B. increases by a factor of 4.

C. halves.

D. Neither of these.

E. Not enough information given.

Projectile motion

1D → **2D**

A projectile is an object moving in 2D under the sole influence of the Earth's gravity

Review: scalars and vectors

- A **scalar** quantity consists of a single number
  - Examples: distance traveled, speed, mass, time
- A **vector** quantity is a set of numbers, which we will use to give direction
  - A vector quantity is often indicated by putting an arrow over the top (e.g. $\vec{v}$)
- You can visualize a vector as an arrow, which has a length (i.e. $|\vec{v}| = v$) together with an direction (e.g. $\hat{x}$)
- Becomes very important for 2D or 3D motion
- Examples: displacement, velocity, acceleration, force, momentum

Review: define coordinate system by vectors

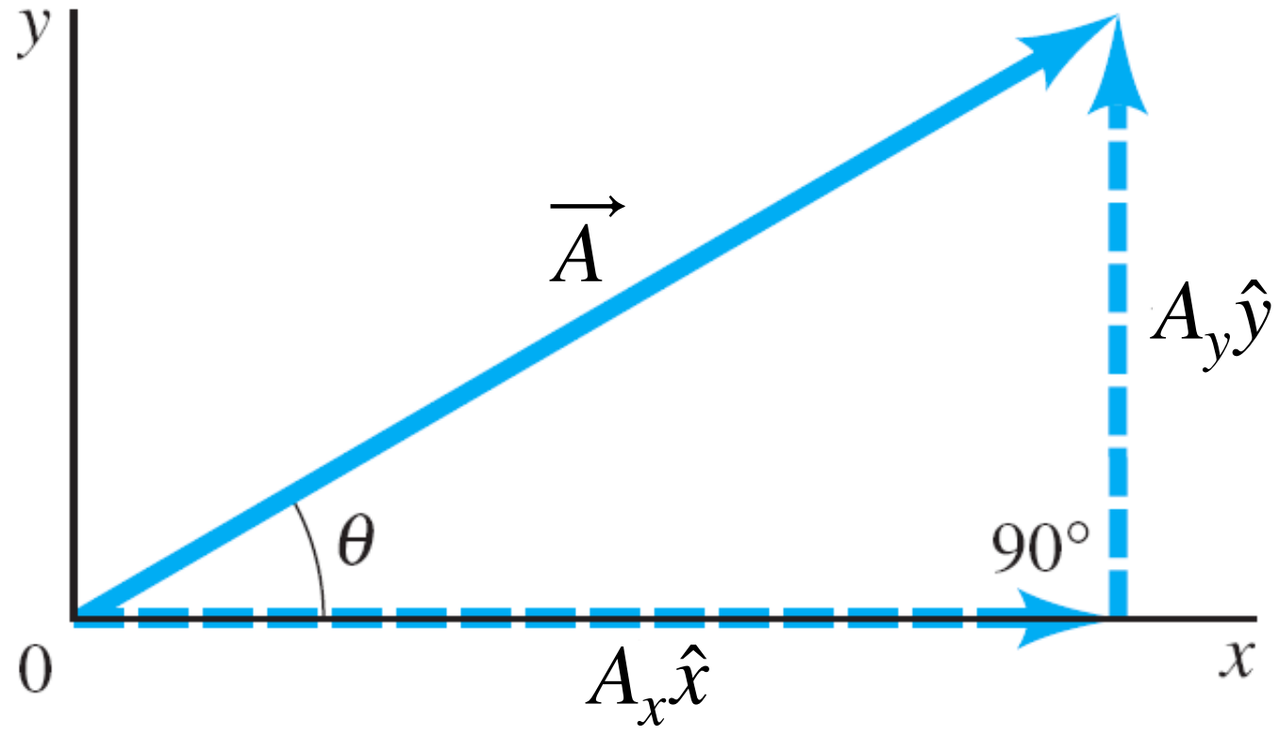
- A Cartesian coordinate system can be defined using a set of orthogonal *unit vectors* (i.e. vectors of length 1):

- Any vector can be expressed as a sum of its components parallel to the unit vectors

Review: getting components of vectors

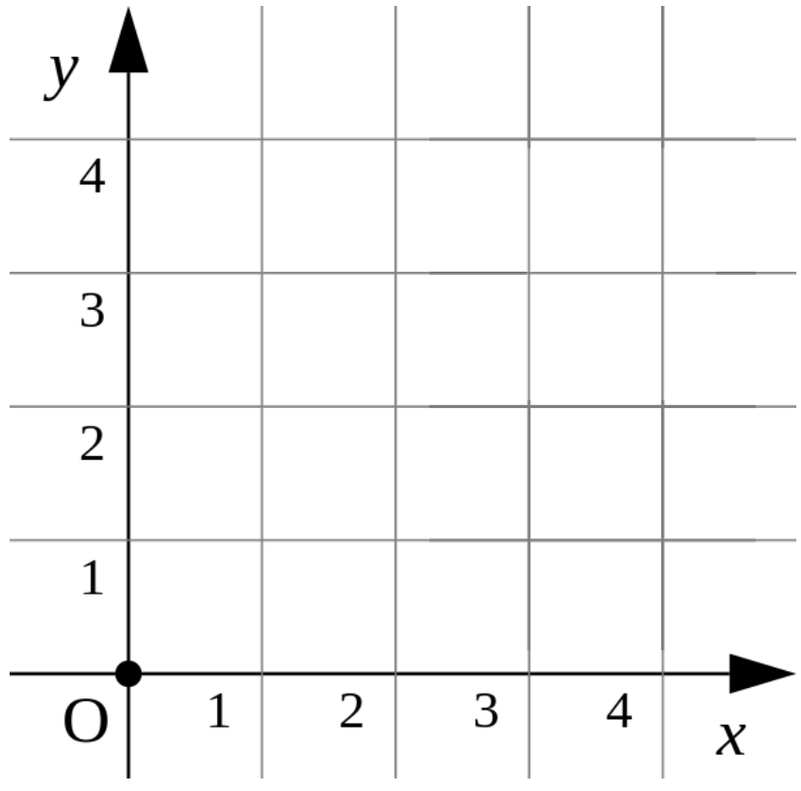
- Here $A = |\overrightarrow{A}|$ is the “norm” (i.e. length or magnitude) of a vector:

- Since the components are orthogonal, they are related by simple trigonometric functions:



Review: math with vectors

- Vector addition and subtraction are accomplished by adding or subtracting component-by-component



- Multiplying (or dividing) by a scalar

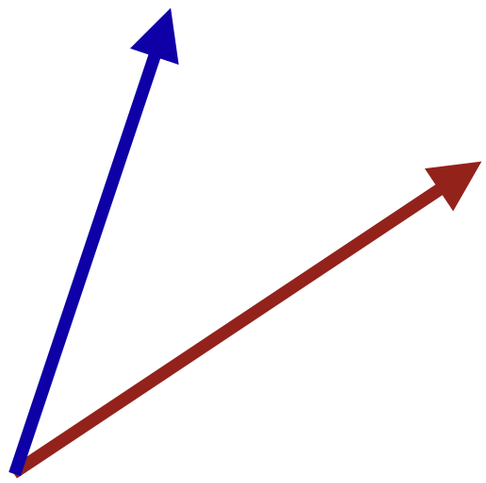
Review: dot product between two vectors

- Mathematical definition of a dot (scalar) product

- Dot product with a unit vector gives the component in that direction

Review: dot product between two vectors

- Geometric interpretation of dot product



Conceptual question

Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are shown below. The vector sum of these three vectors is $\vec{S} = \vec{A} + \vec{B} + \vec{C}$.

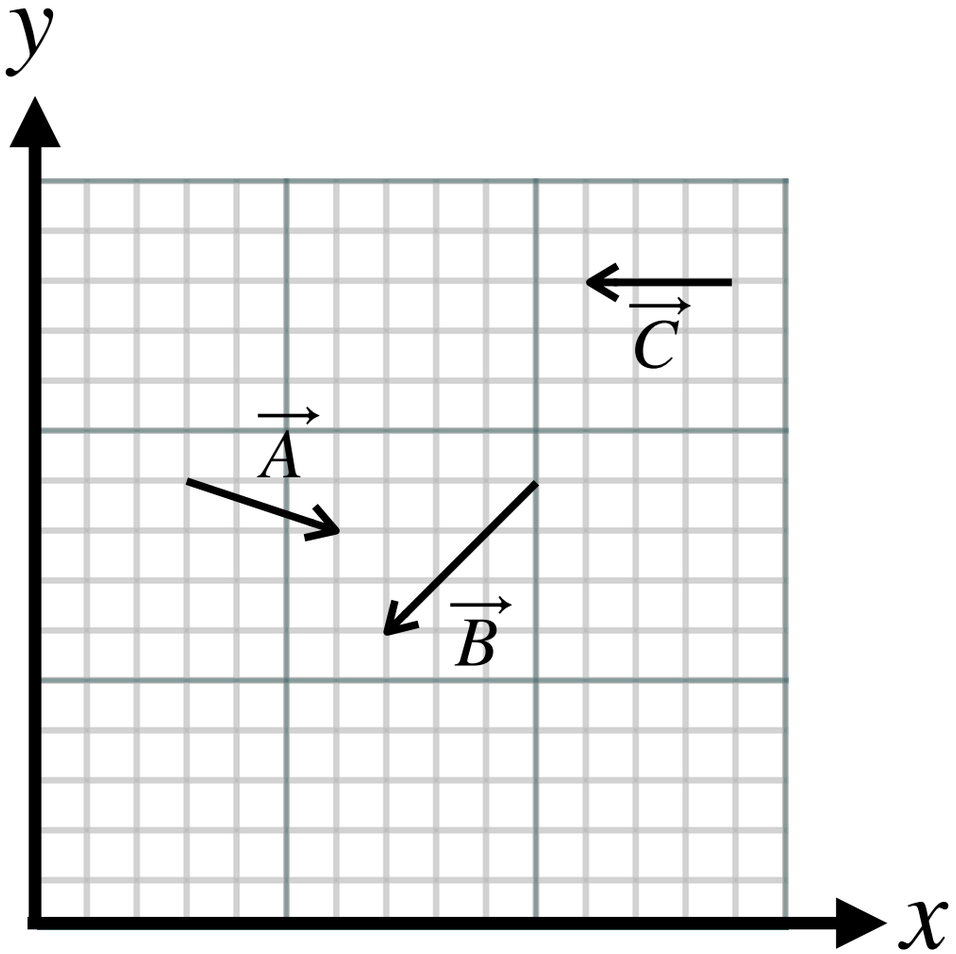
What is the value of S_x, the x component of $\vec{S}$?

A. -6

B. -4

C. 3

D. -3



Conceptual question

A particle is moving while experiencing a constant acceleration. The velocity vector is shown below at two different times, an earlier time t_1 and a later time is t_2.

What is the direction of the average acceleration vector?

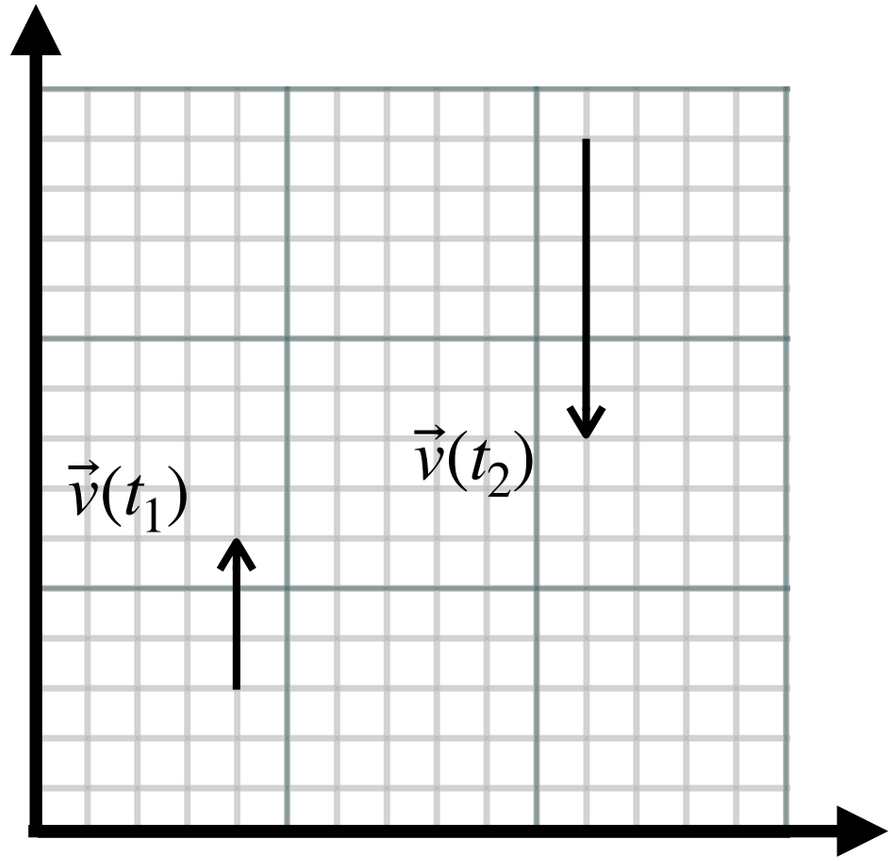
A. ↑

B. →

C. ↓

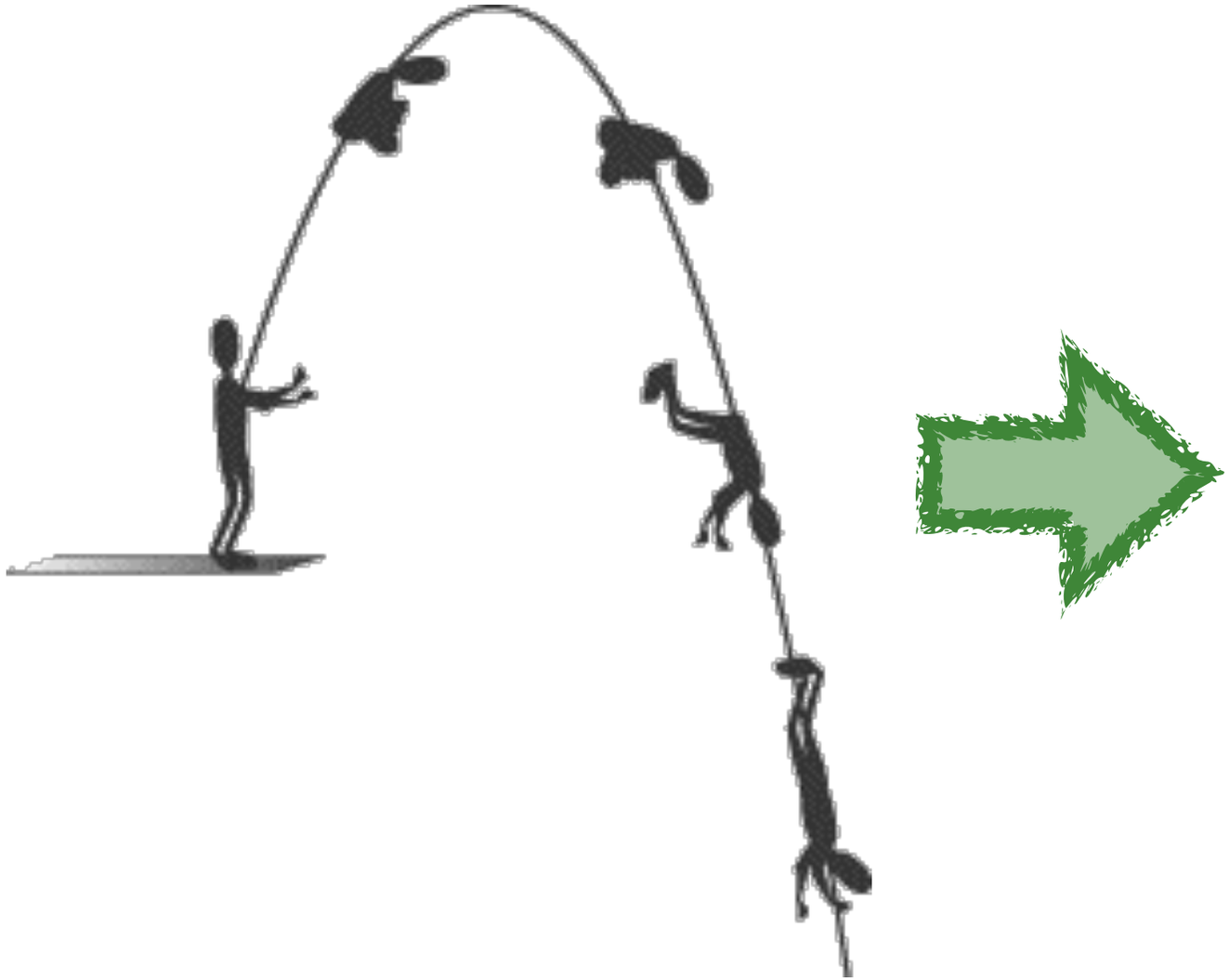
D. ←

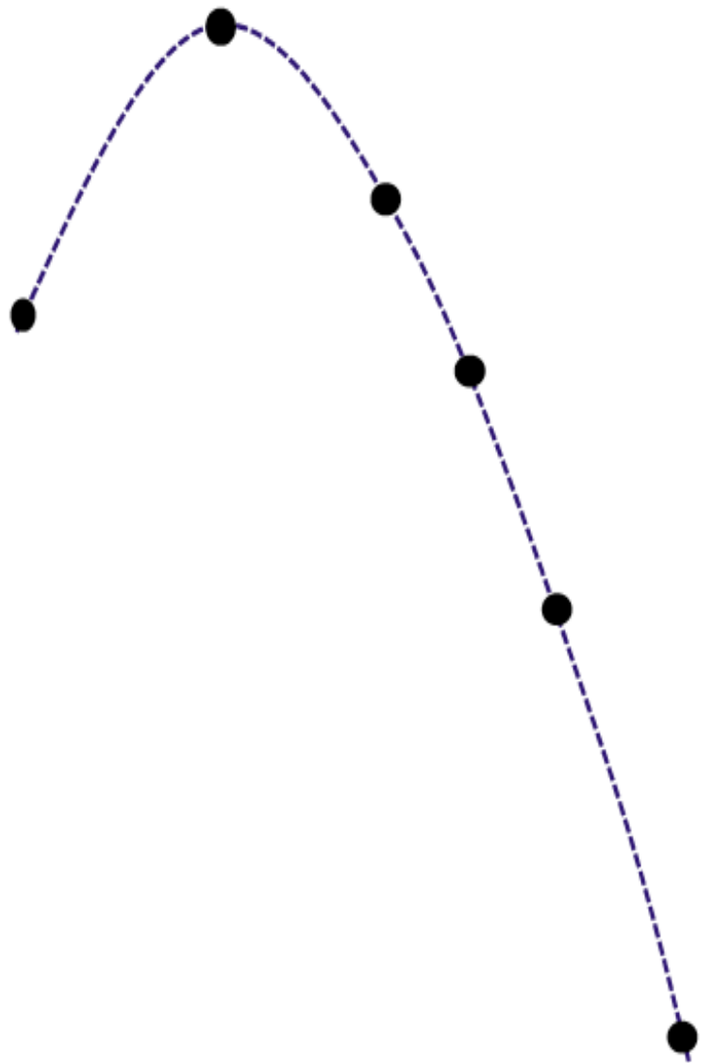
E. None of these.



Kinematics

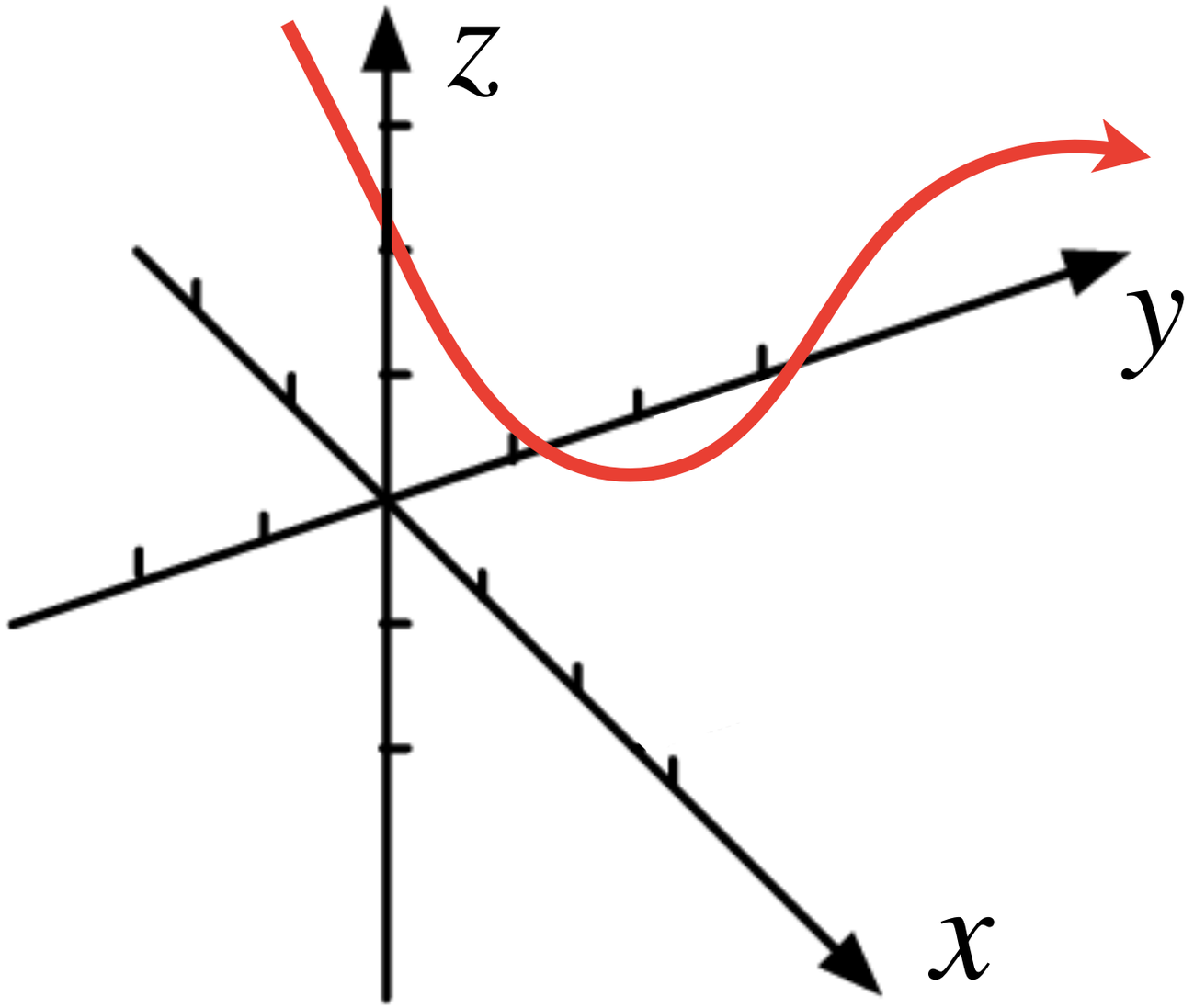
- A description of motion without considering forces
- We will approximate objects as point masses
- Need to go beyond one dimension





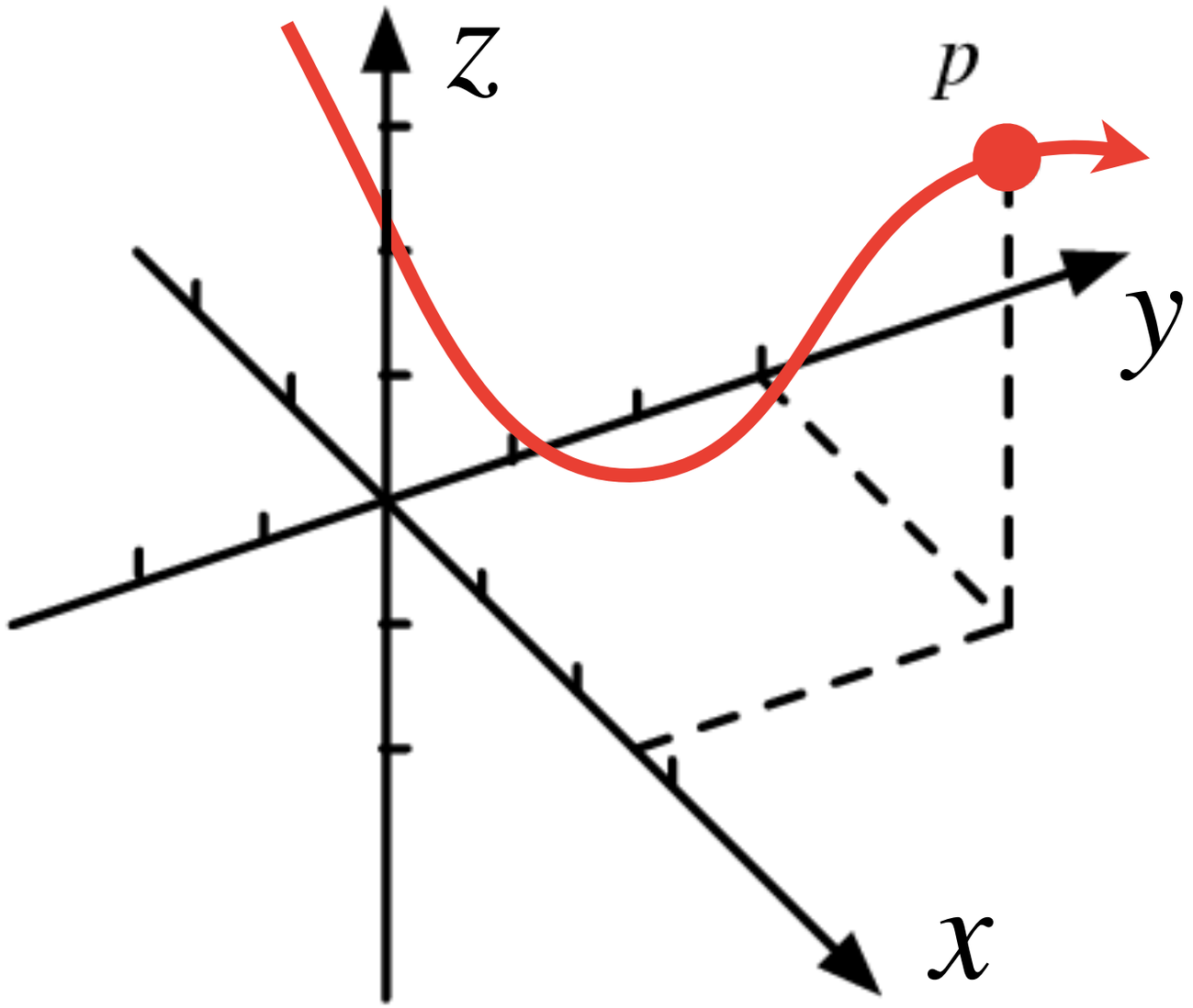
Vector position in Cartesian coordinates

- Position in 1D: $x(t)$



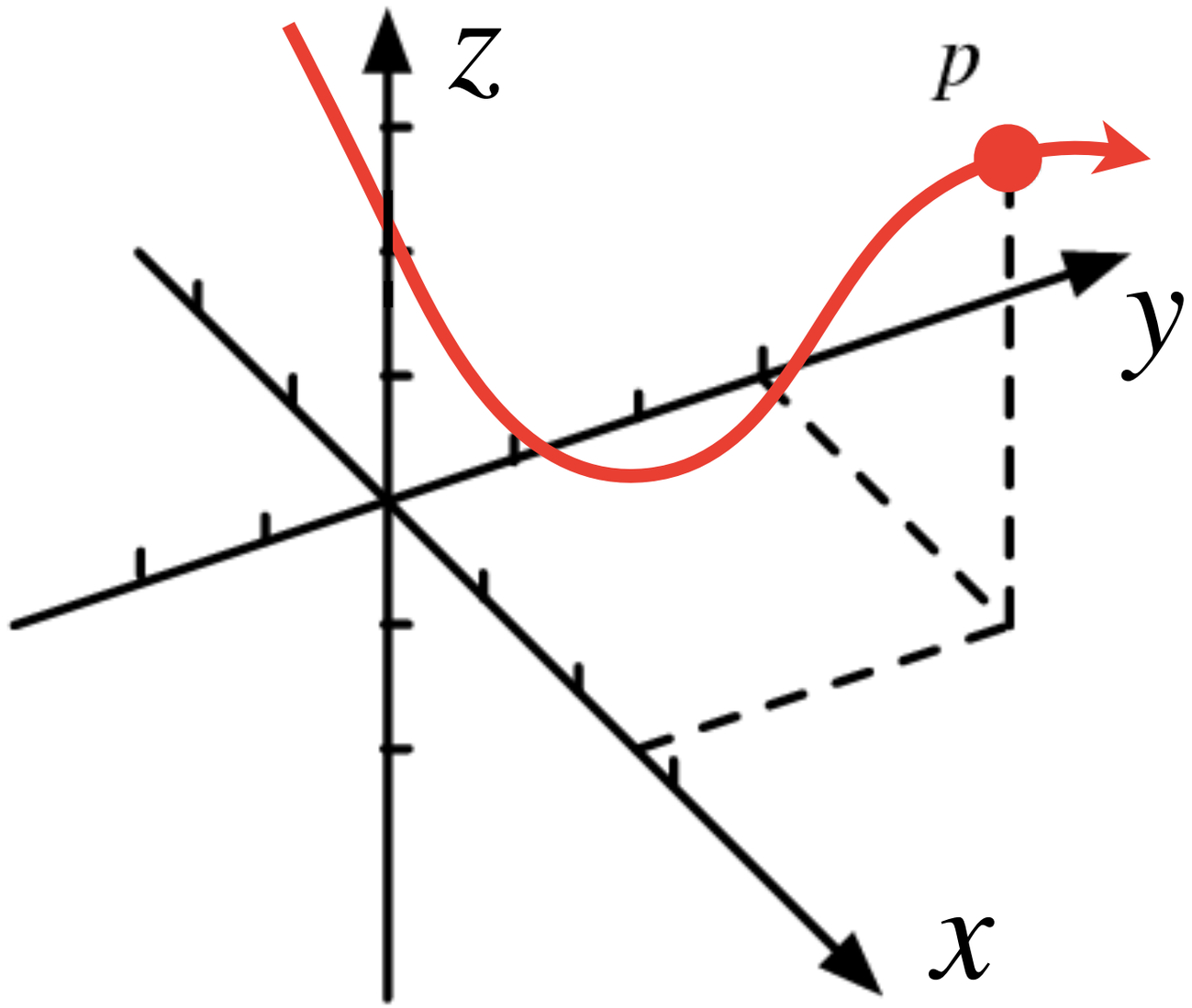
Vector displacement (Cartesian)

- Displacement in 1D: $\Delta x = x(t_2) - x(t_1)$



Vector velocity (Cartesian)

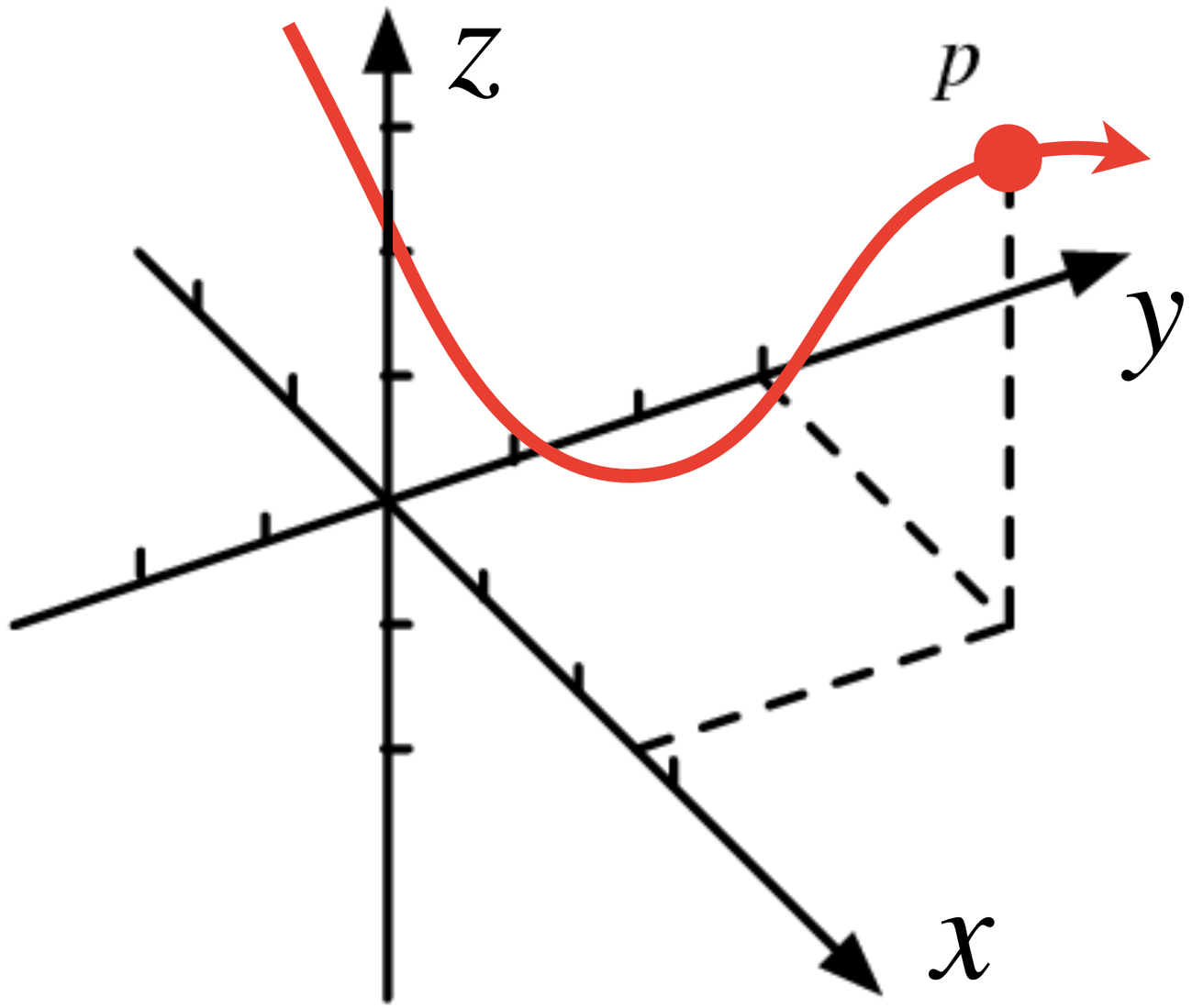
- Average velocity in 1D: $\bar{v} = \dfrac{\text{displacement}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$



Vector velocity (Cartesian)

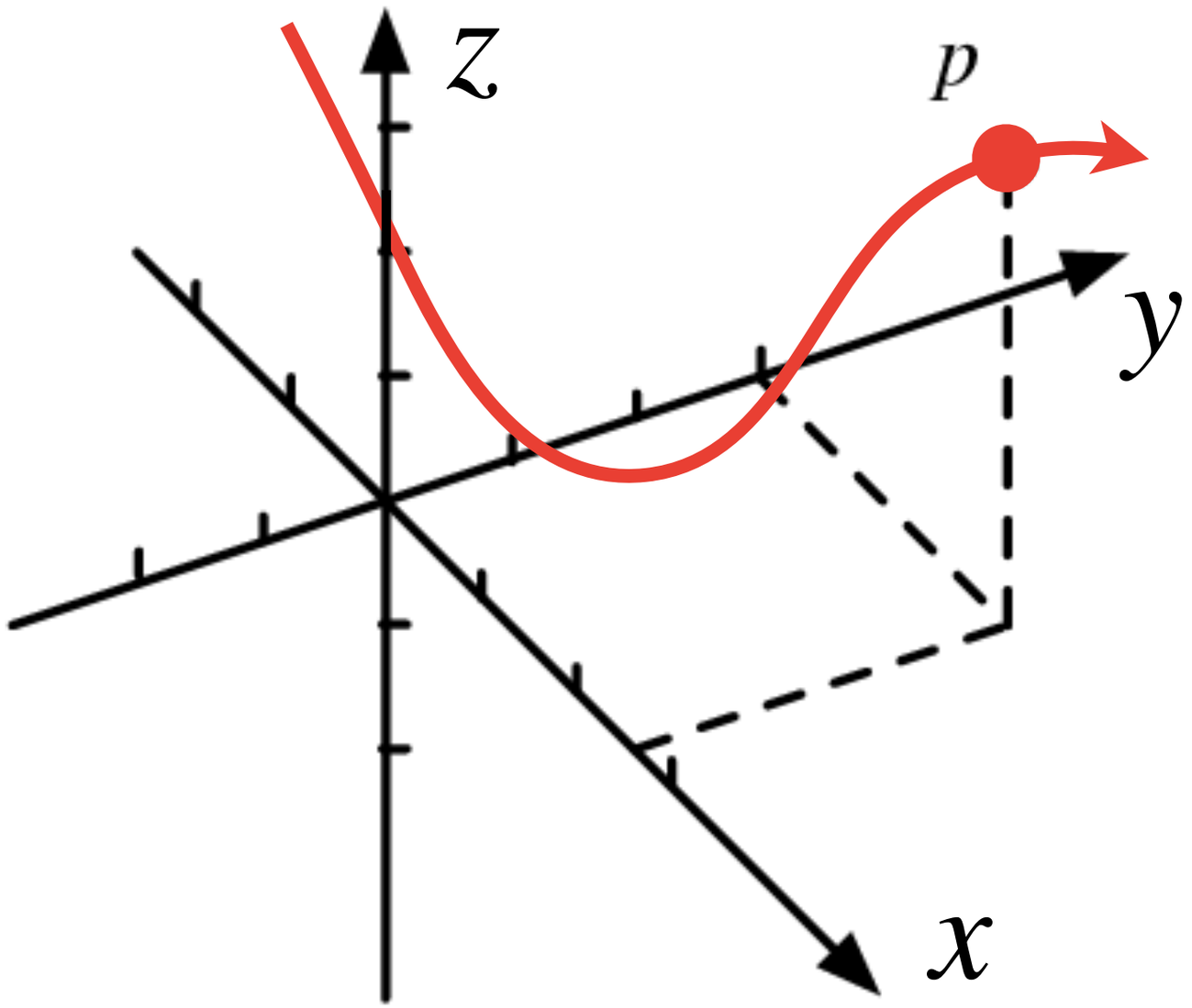
- Speed (i.e. magnitude of velocity):

- Direction:



Vector acceleration (Cartesian)

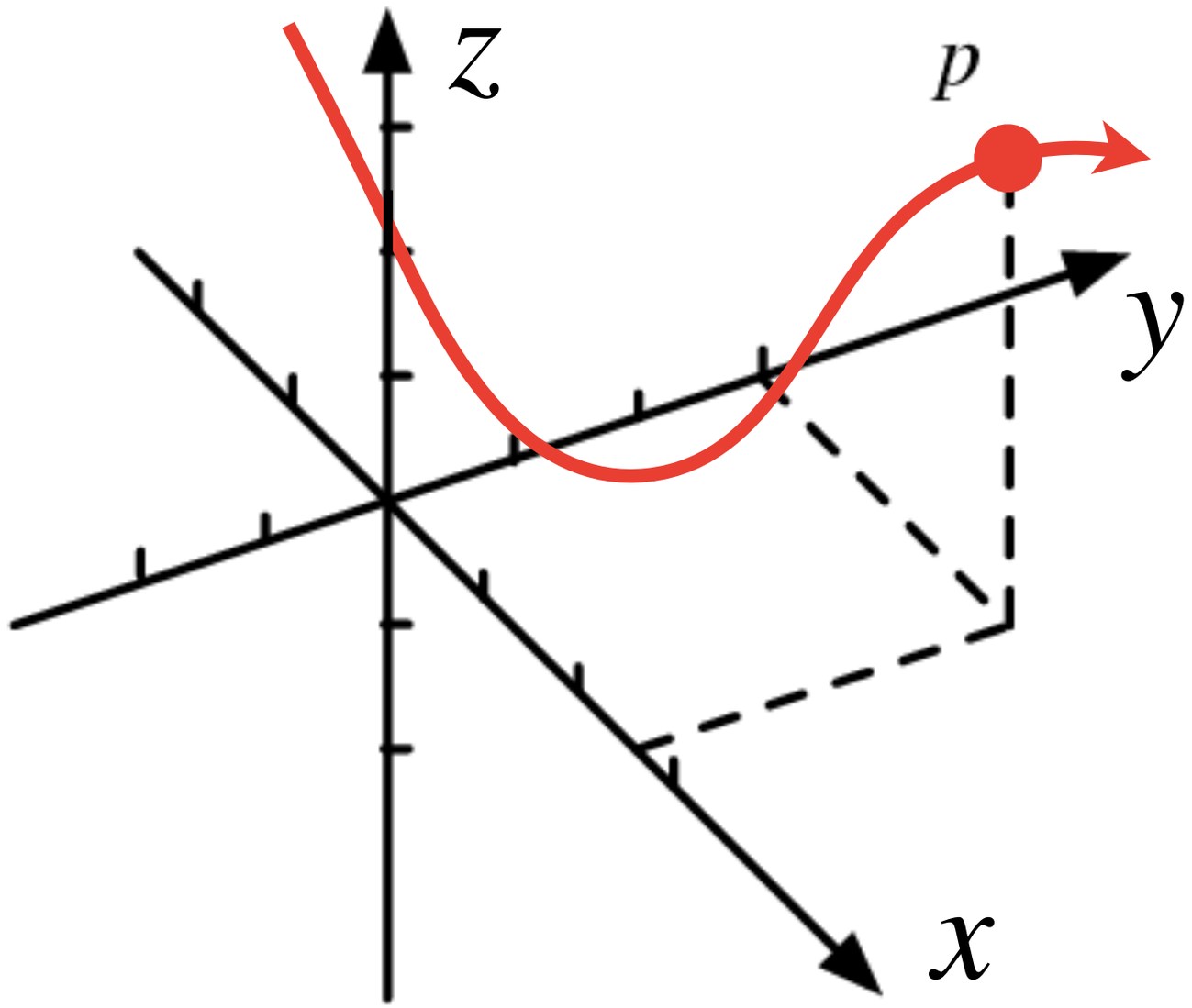
- Average acceleration in 1D: $\bar{a} = \dfrac{\text{change in velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$



Vector acceleration (Cartesian)

- Magnitude of the acceleration:

- Direction:



DEMO (55)

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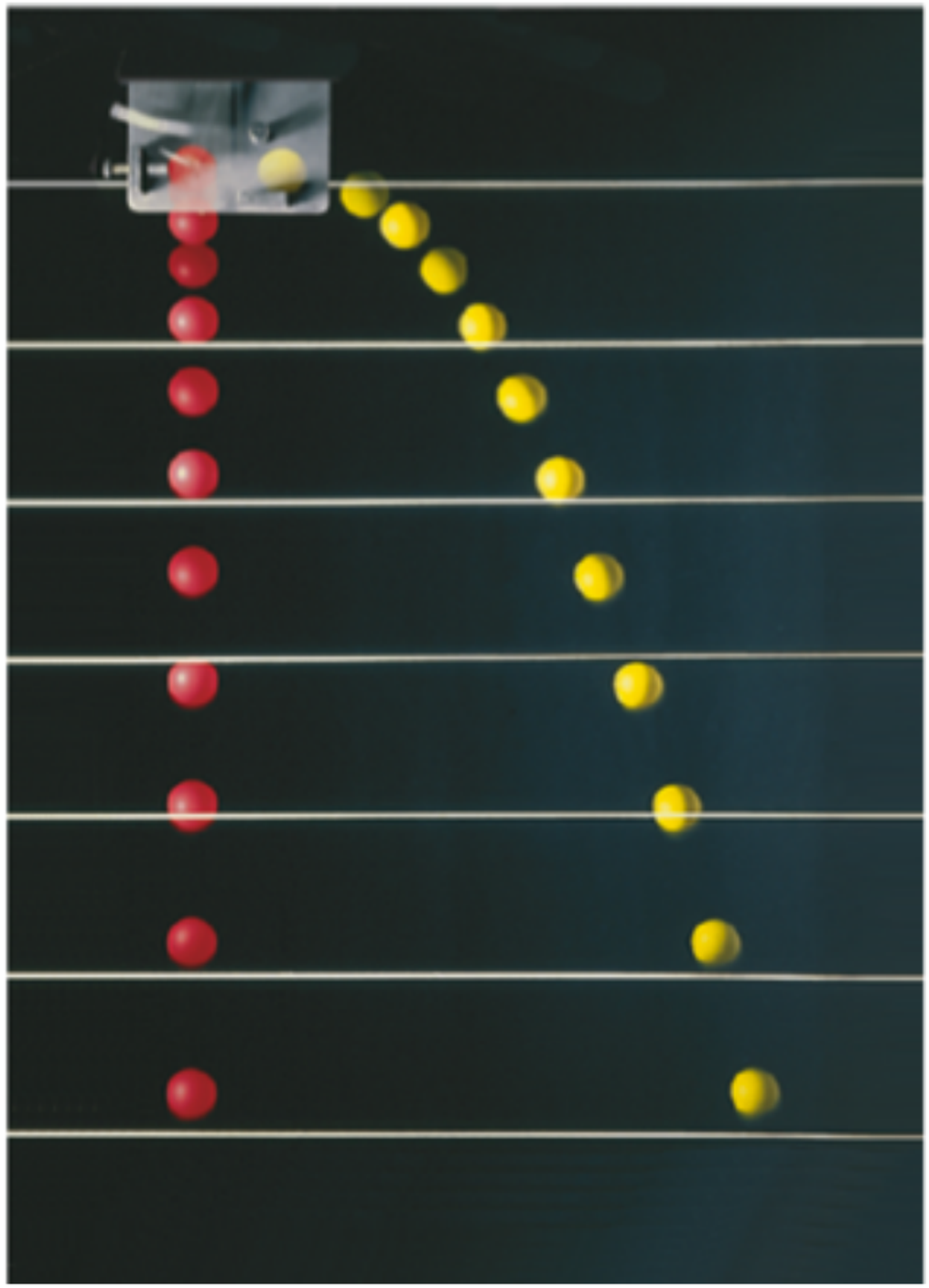
Projectile motion

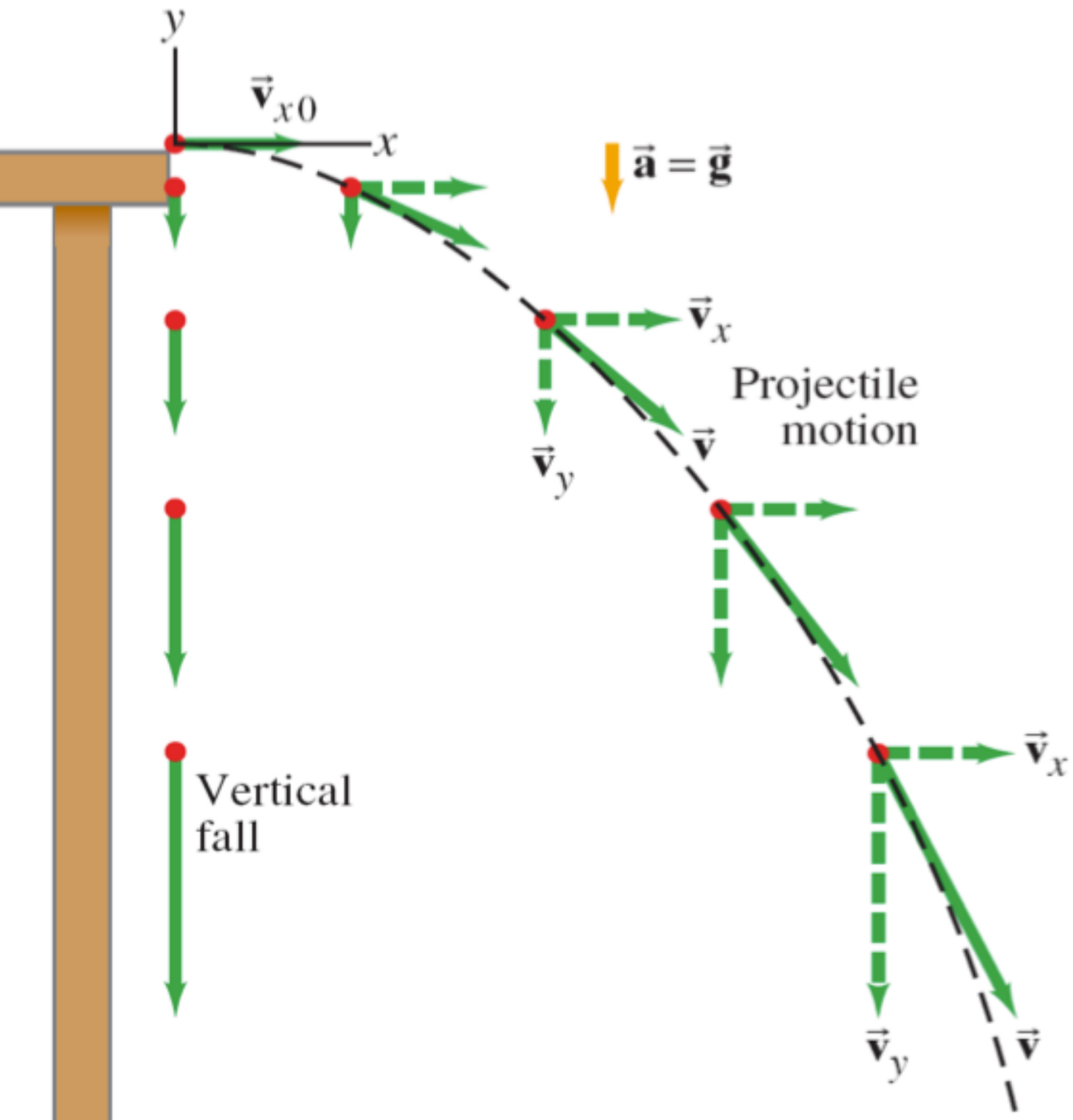
Projectile motion

- Two balls are released simultaneously under gravity
- What causes the difference in their motions?
- What equations of motion need modified?



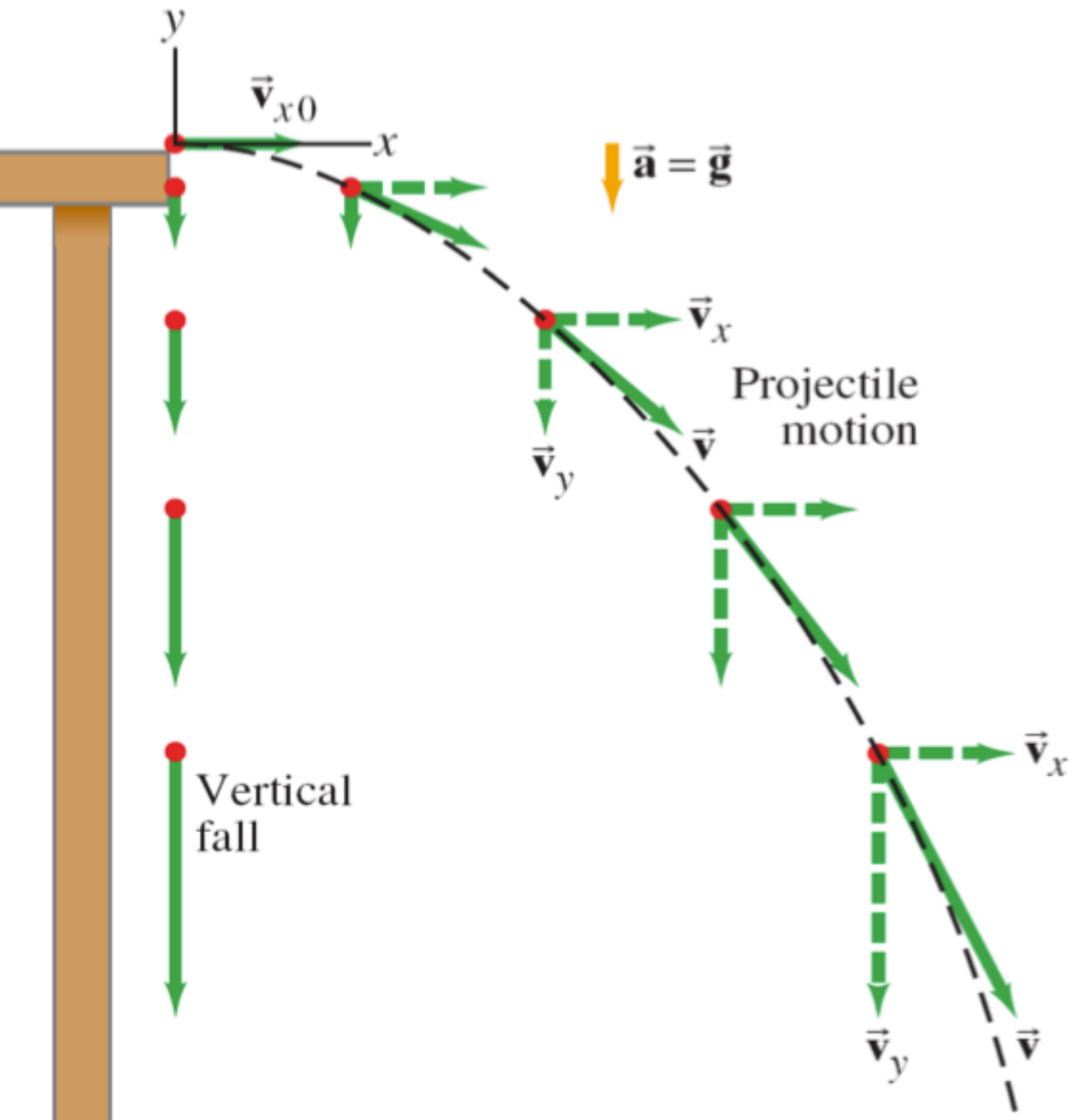
Velocity throughout projectile motion

- Motion in horizontal and vertical components are decoupled and independent



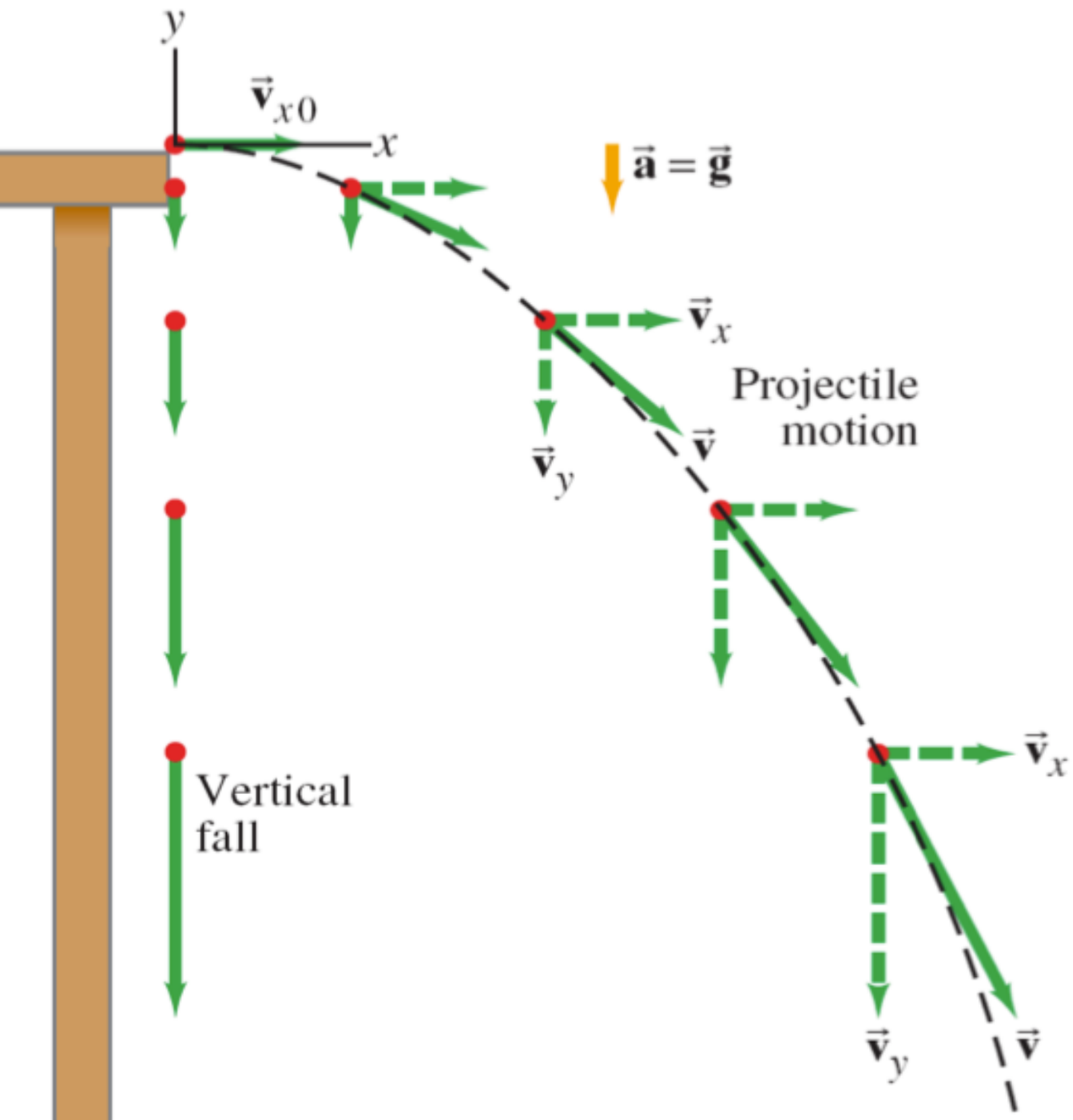
Position throughout projectile motion

- Motion in horizontal and vertical components are decoupled and independent



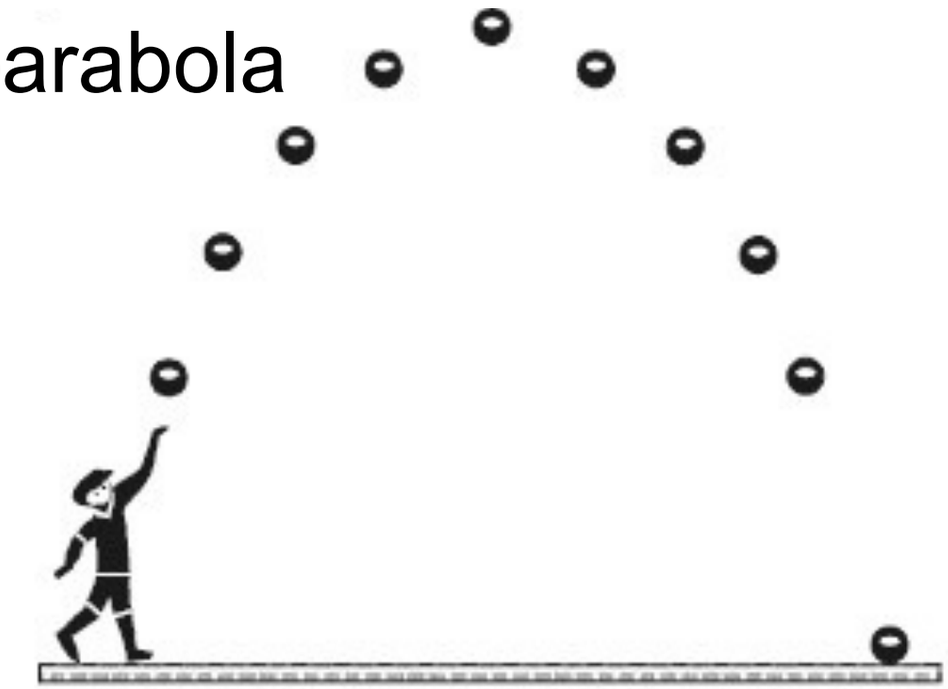
Calculating “g”

- Motion in horizontal and vertical components are decoupled and independent



Trajectory of projectile motion

- The path of a projectile is always a parabola



Conceptual question

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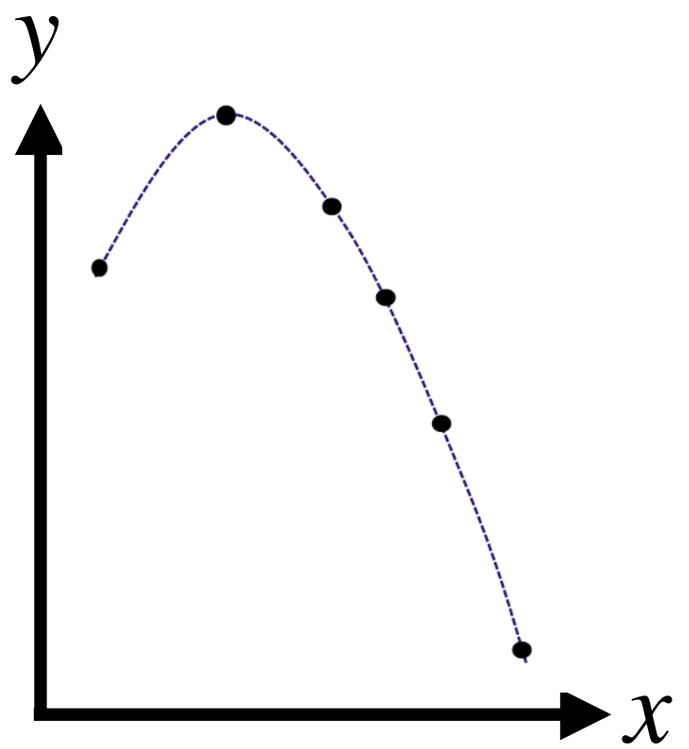
A projectile moves through the air under gravitational acceleration. At the highest point in its arc, which of the following is true? Neglect the effects of air resistance.

A. The speed and acceleration are zero.
B. The speed is equal to zero, and the acceleration is constant and not equal to zero.
C. The speed is at a minimum but not equal to zero and the acceleration is zero.
D. The speed is at a minimum but not equal to zero and the acceleration is constant and not equal to zero.
E. Neither the acceleration nor speed has yet attained its minimum value.



Summary of projectile motion

- The acceleration is

$$\vec{a}(t) = -g\hat{y}$$

- By integrating, we find the velocity is

$$\vec{v}(t) = v_{x0}\hat{x} + (-gt + v_{y0})\hat{y}$$

where v_{x0} and v_{y0} are the initial velocities in each direction

- Integrating again, the position is

$$\vec{r}(t) = (v_{x0}t + x_0)\hat{x} + (-\frac{g}{2}t^2 + v_{y0}t + y_0)\hat{y}$$

where x_0 and y_0 are the initial positions in each dimension

DEMO (762)

The monkey and the hunter

See you at the exercises tomorrow!

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- Wednesdays from 17:15 to 19:00
  - Don’t forget to sign up for a tutoring group on Moodle
  - Exercises will be found on the Moodle (bring your own paper copy or way to access them digitally)

Conceptual question

You are standing at the edge of a cliff and throw one ball straight up and another ball straight down at the same initial *speed*.

Neglecting air resistance, which ball hits the ground below the cliff with the *greater* speed?

A. The ball thrown upwards

B. The ball thrown downwards

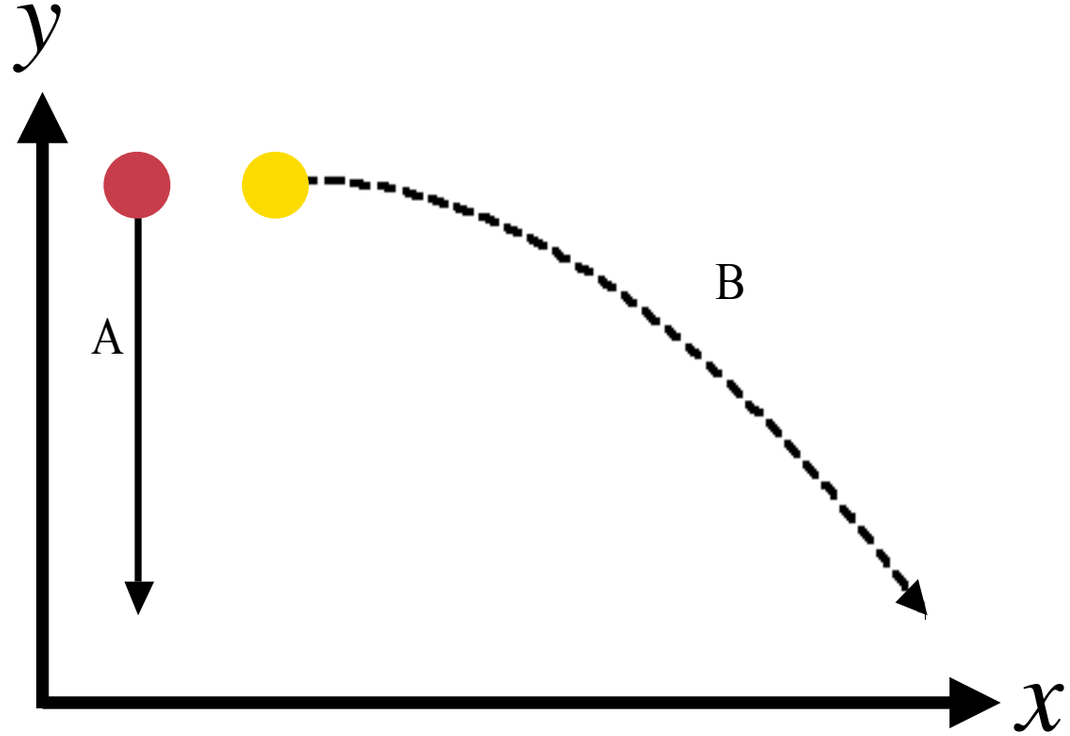
C. Neither — they both hit at the same speed

Conceptual question

Two balls begin at rest at the same height (but not at the same place).

At time $t = 0$, the red ball is simply released and the yellow ball is launched straight horizontally.

Which one hits the ground first?



A. Ball A hits the ground first.

B. Ball B hits the ground first.

C. They both hit simultaneously.

D. It depends on which one is heavier.