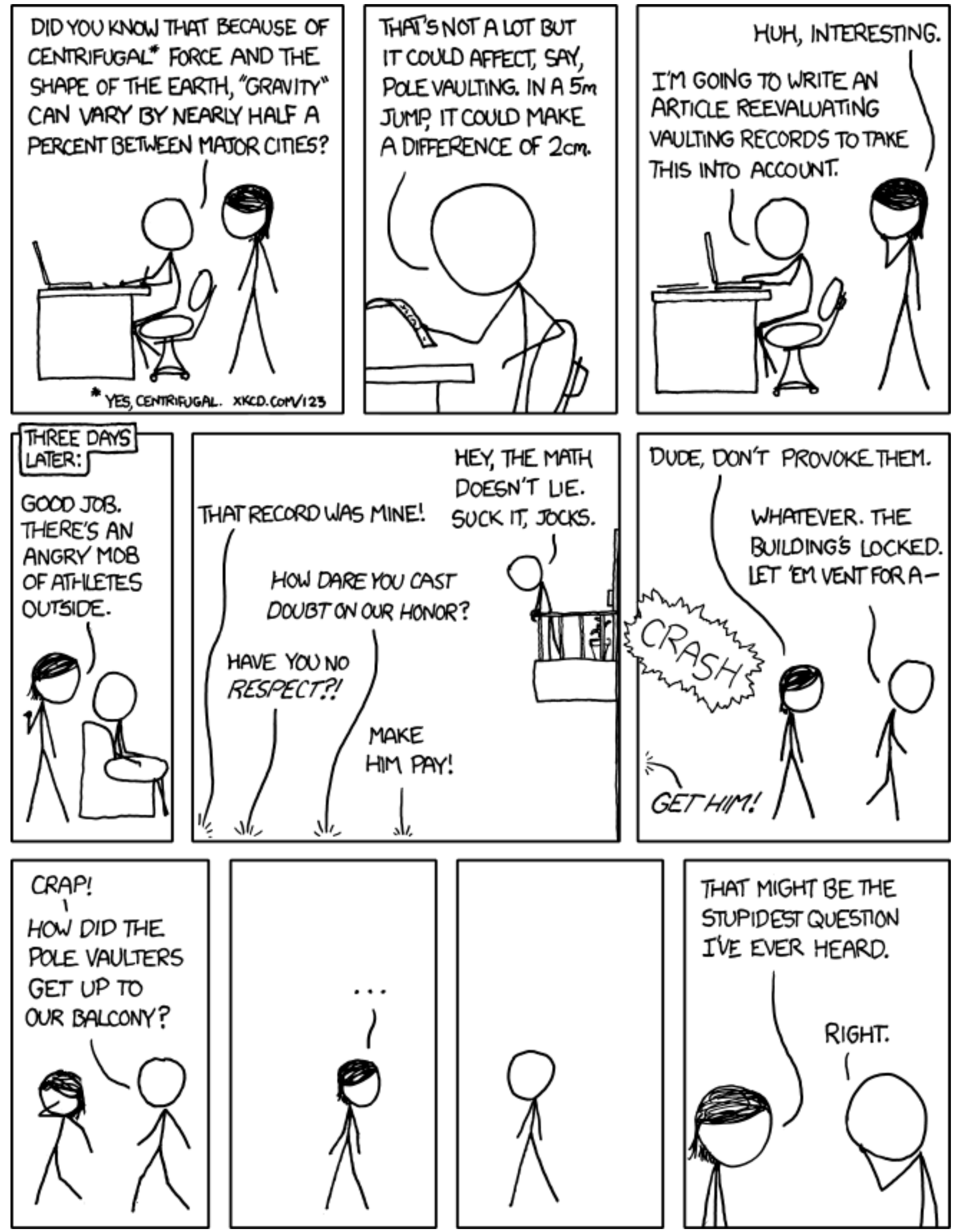
Swiss Plasma Center

General Physics: Mechanics



xkcd.com/852

PHYS-101(en)

Lecture 1b: Motion in two and three dimensions

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Today's agenda (MIT 3 and 4)

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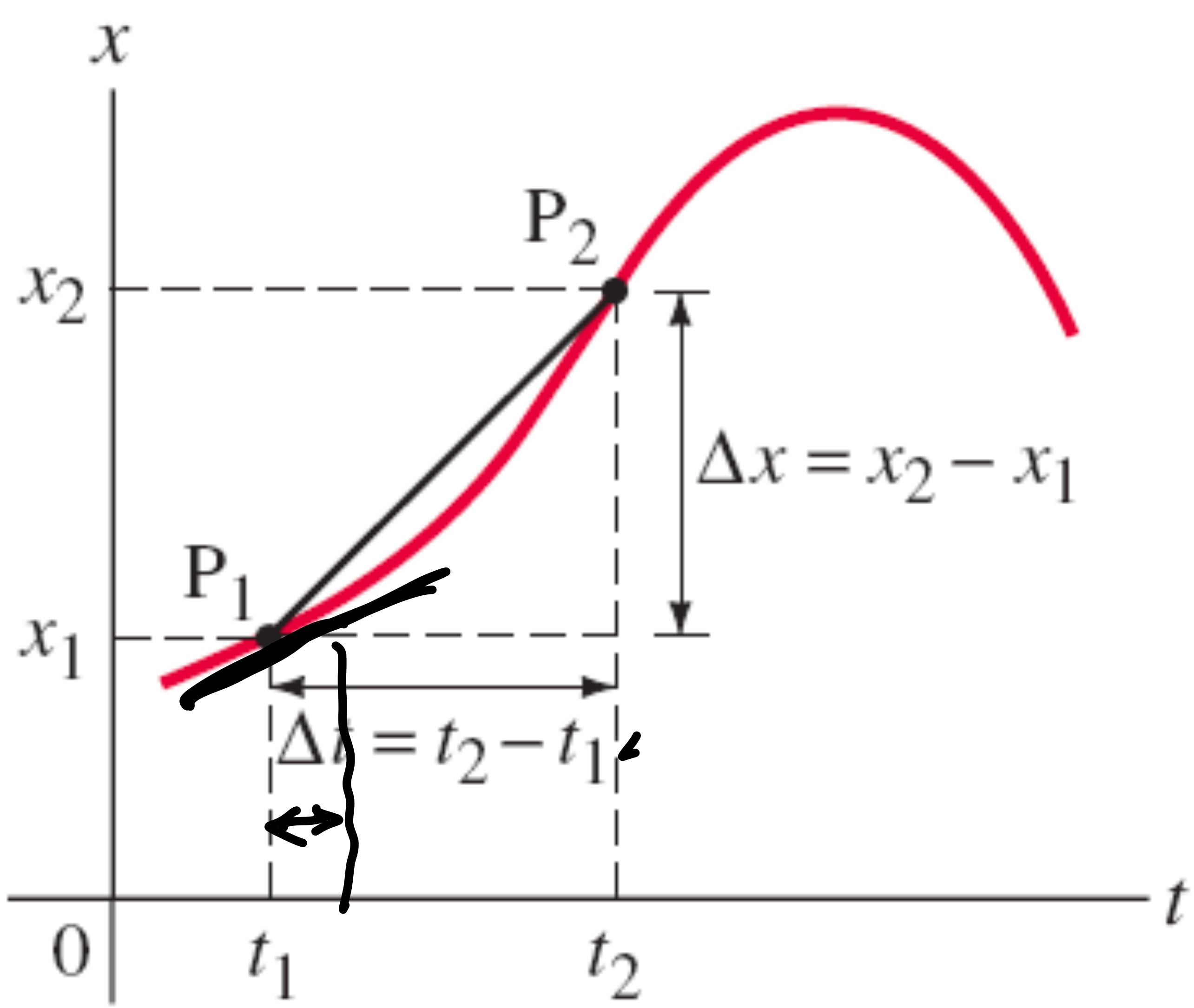
1. Revisit motion in one dimension
2. Motion in two and three dimensions in Cartesian coordinates
   - Acceleration due to gravity
   - Using vectors in equations
   - Projectile motion

Summary of motion in one dimension

- Position of an object as a function of time denoted by $x(t)$
- Average velocity: $\bar{v} = \dfrac{\text{change of position}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$



Summary of motion in one dimension

- Position of an object as a function of time denoted by $x(t)$

- Average velocity: $\bar{v} = \dfrac{\text{change of position}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$

- Instantaneous velocity: $v(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

- Average acceleration: $\bar{a} = \dfrac{\text{change of velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$

- Instantaneous acceleration:

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Conceptual question

responseware.eu
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A ball is thrown straight up. At its maximum height, its… $g \approx 9.81 \frac{m}{s^2}$

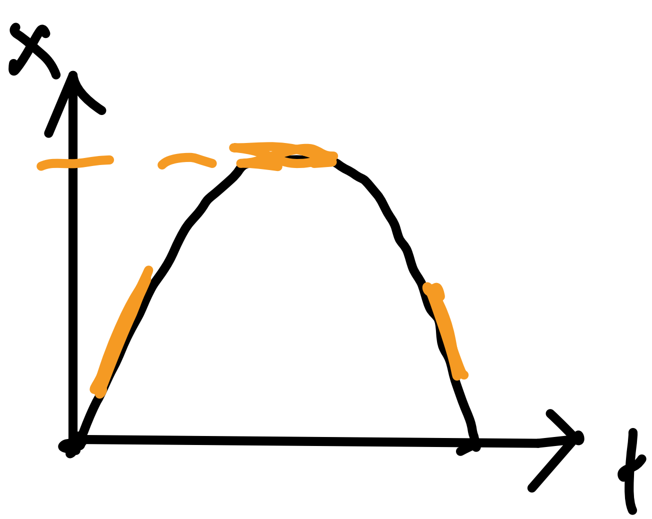
$a = -g$

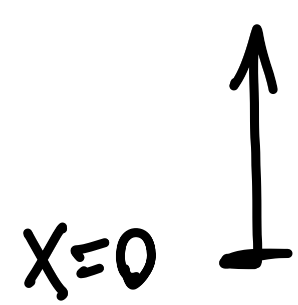
~~A.~~ velocity is zero and acceleration is zero.

B. velocity is non-zero and acceleration is non-zero.

C. velocity is zero and acceleration is non-zero.

~~D.~~ velocity is non-zero and acceleration is zero.





Conceptual question

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A ball is launched straight up with initial velocity v_0 (neglect air resistance). If the initial velocity v_0 is doubled, the time to reach the apex of the trajectory…

A. doubles.

B. increases by a factor of 4.

C. halves.

D. Neither of these.

E. Not enough information given.

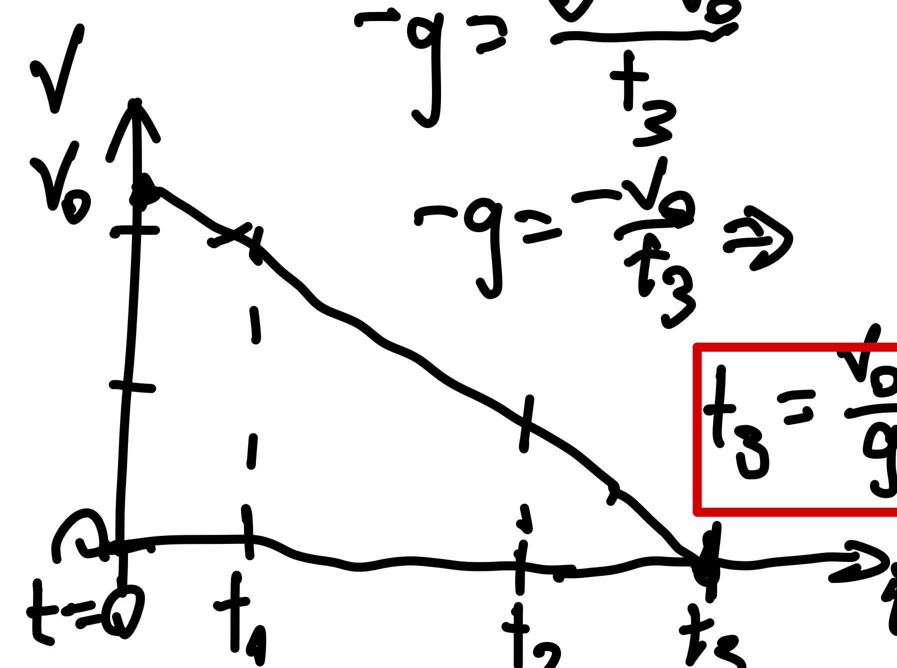
$$a = -g = \frac{dv}{dt} \approx \frac{\Delta v}{\Delta t}$$

$$\frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = -g$$

$$-g = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - \overbrace{v(t=0)}^{v_0}}{t_3 - 0}$$

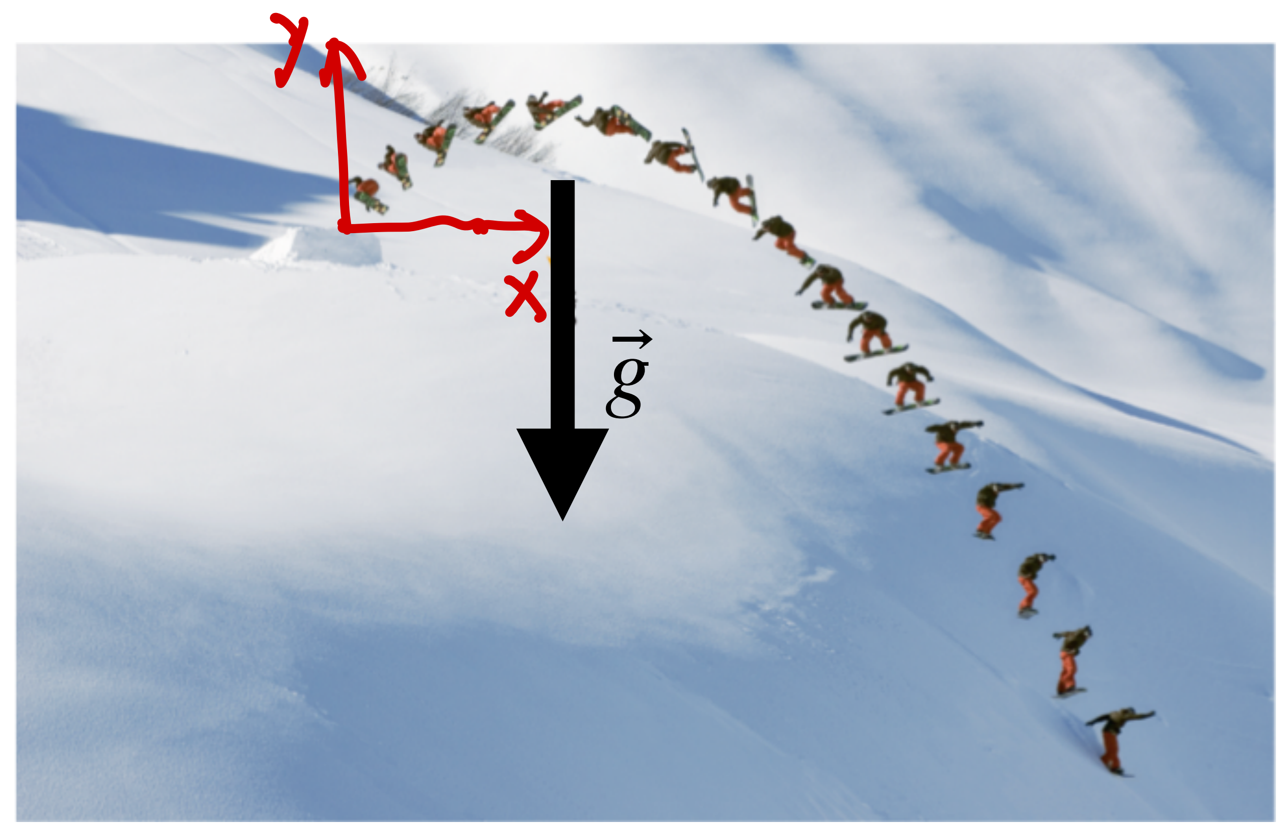
$$-g = \frac{0 - v_0}{t_3}$$

$$-g = \frac{-v_0}{t_3} \Rightarrow \boxed{t_3 = \frac{v_0}{g}}$$



Projectile motion

1D → 2D



A projectile is an object moving in 2D under the sole influence of the Earth's gravity

Review: scalars and vectors

- A **scalar** quantity consists of a single number
  - Examples: distance traveled, speed, mass, time
- A **vector** quantity is a set of numbers, which we will use to give direction
  - A vector quantity is often indicated by putting an arrow over the top (e.g. $\vec{v}$)
  - You can visualize a vector as an arrow, which has a length (i.e. $|\vec{v}| = v$) together with an direction (e.g. $\hat{x}$)
  - Becomes very important for 2D or 3D motion
  - Examples: displacement, velocity, acceleration, force, momentum

Review: define coordinate system by vectors

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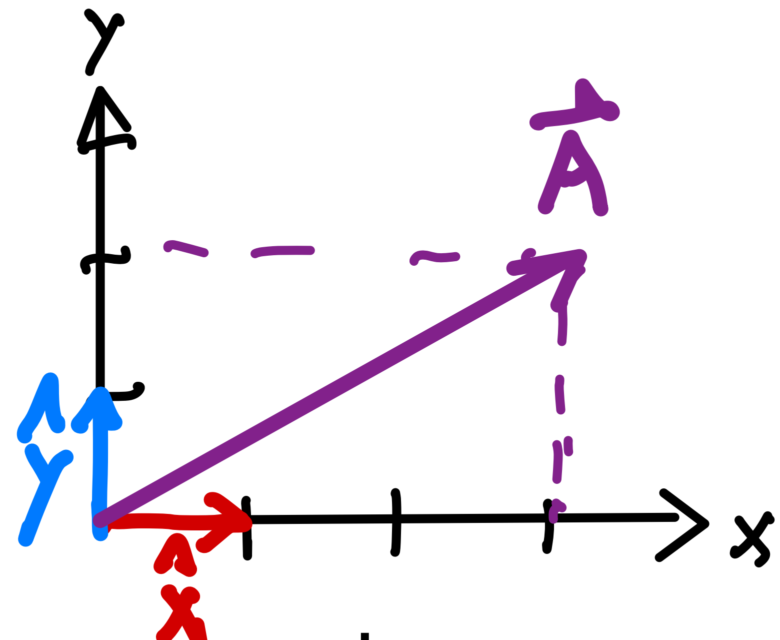
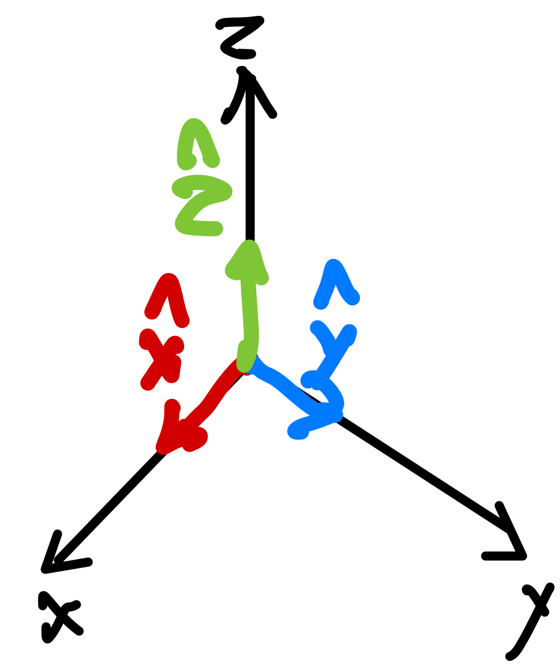
- A Cartesian coordinate system can be defined using a set of orthogonal *unit vectors* (i.e. vectors of length 1):

$$\vec{A} = 3\hat{x} + 2\hat{y}$$





- Any vector can be expressed as a sum of its components parallel to the unit vectors

$$\vec{A} = A_x\hat{x} + A_y\hat{y} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Review: getting components of vectors

- Here $A = |\vec{A}|$ is the "norm" (i.e. length or magnitude) of a vector:

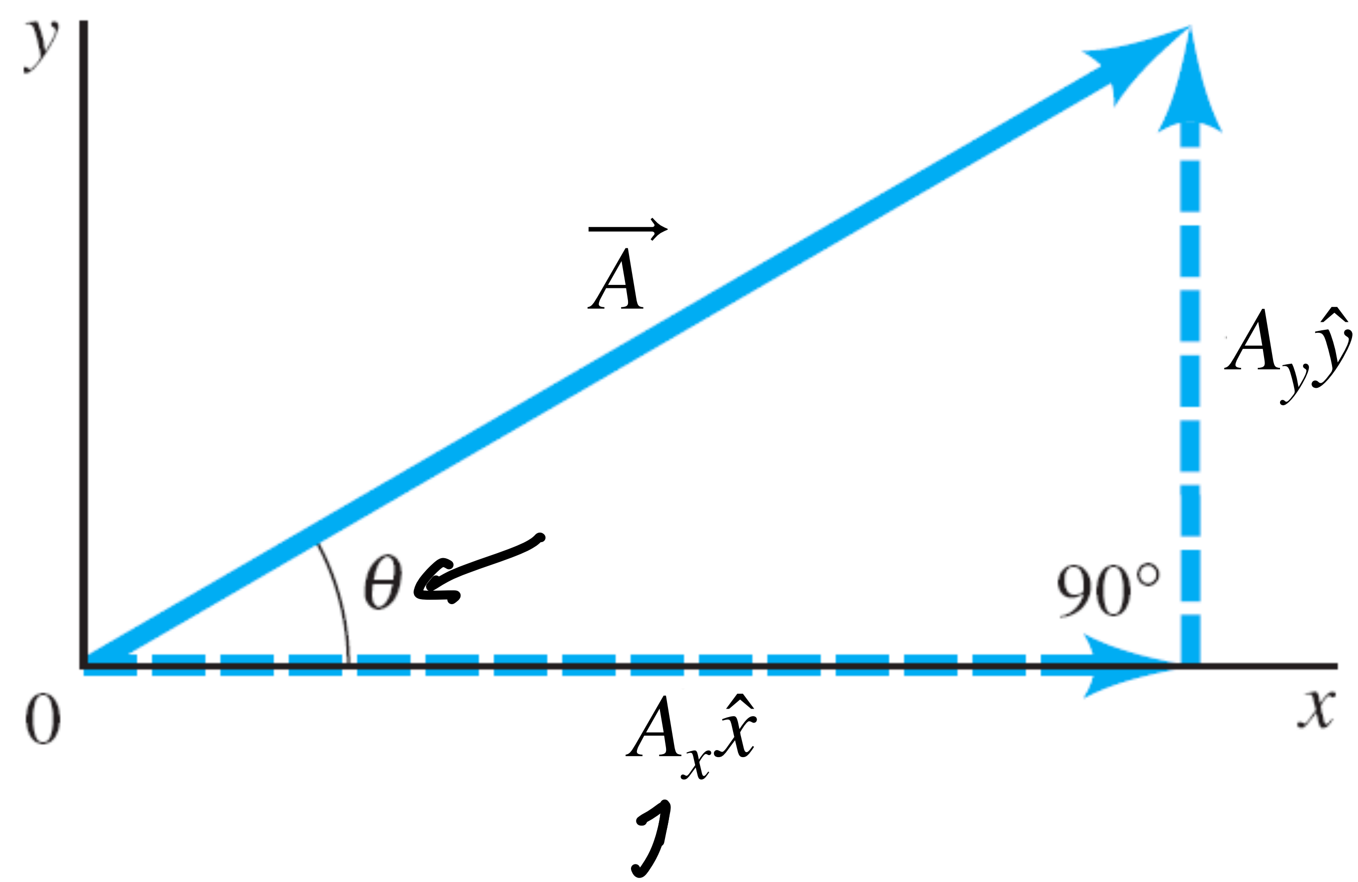
$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- Since the components are orthogonal, they are related by simple trigonometric functions:

$$\sin(\theta) = \frac{A_y}{A}$$

$$\cos(\theta) = \frac{A_x}{A}$$

$$\tan(\theta) = \frac{A_y}{A_x}$$



Review: math with vectors

- Vector addition and subtraction are accomplished by adding or subtracting component-by-component

$$\vec{A} = 3\hat{x} + 1\hat{y} = 3\hat{x} + \hat{y}$$

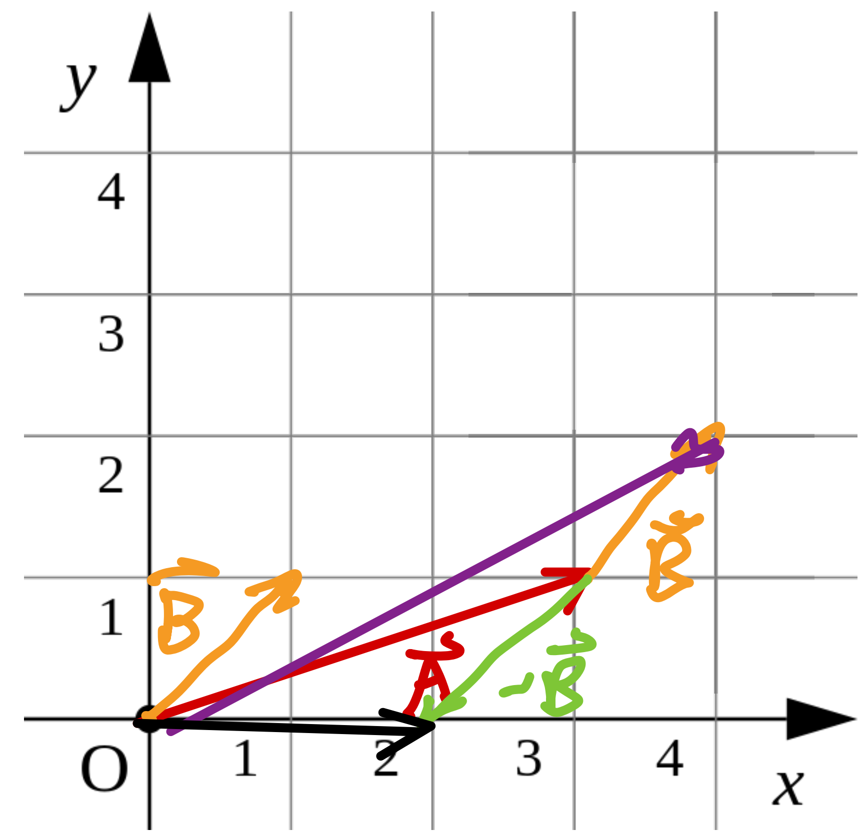
$$\vec{B} = \hat{x} + \hat{y}$$

$$\vec{A} + \vec{B} = (3\hat{x} + \hat{y}) + (\hat{x} + \hat{y})$$

$$= 3\hat{x} + \hat{x} + \hat{y} + \hat{y} = \underline{4\hat{x}} + \underline{2\hat{y}}$$

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$= (3\hat{x} + \hat{y}) - (\hat{x} + \hat{y}) = 2\hat{x} + 0\hat{y} = \underline{2\hat{x}}$$



- Multiplying (or dividing) by a scalar

$$c \cdot \vec{A} = cA_y\hat{x} + cA_y\hat{y}$$

Review: dot product between two vectors

- Mathematical definition of a dot (scalar) product

$\hat{x}\cdot\hat{x} = 1 = \hat{y}\cdot\hat{y}$

$\hat{x}\cdot\hat{y} = 0 = \hat{y}\cdot\hat{x}$

$$\vec{A}\cdot\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$$

$$= A_xB_x\underbrace{\hat{x}\cdot\hat{x}}_{=1} + \cancel{A_yB_x\hat{y}\cdot\hat{x}} + A_yB_y\underbrace{\hat{y}\cdot\hat{y}}_{=1} + \cancel{A_xB_y\hat{x}\cdot\hat{y}}$$

$$= A_xB_x + A_yB_y + A_zB_z$$

- Dot product with a unit vector gives the component in that direction

$\vec{B} = \hat{x} \Rightarrow B_x = 1 \quad B_y = 0$

$\vec{A}\cdot\hat{x} = A_x + 0 = A_x$

$\vec{A}\cdot\hat{y} = A_y$

$\vec{A} = 3\hat{x} + \hat{y}$

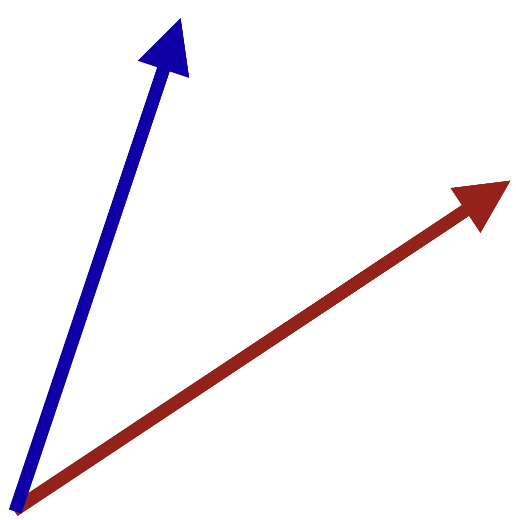
$\vec{A}\cdot\hat{x} = 3\underbrace{\hat{x}\cdot\hat{x}}_{=1} + \cancel{\hat{y}\cdot\hat{x}} = 0$

$= 3$

Review: dot product between two vectors

- Geometric interpretation of dot product

$$\vec{A} \cdot \vec{B} = AB\cos(\theta)$$



See you at the exercises tomorrow!

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- Wednesdays from 17:15 to 19:00
  - Don’t forget to sign up for a tutoring group on Moodle
  - Exercises will be found on the Moodle (bring your own paper copy or way to access them digitally)