General Physics: Mechanics

PHYS-101(en)

Lecture 14a: Damped oscillations, rolling and review

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December 16th, 2024

EPFL
Swiss Plasma Center

Announcements

- The average grade of the 2nd Mock exam final was 6.5 out of 16
  - Most of you only had time to work on one of the two problems
  - The average thus suggests that you did well, just maybe slowly
  - Be sure to check the solutions on the Moodle and the comments written by the TAs on your graded exams to find weaknesses that you can focus on
- Pick up your exam later today during one of the breaks

Announcements

- Several past final exams now on the Moodle
- There will be two Review sessions in January
  - First one on Thursday, Jan. 9th, from 10h to 12h
  - Second one on Tuesday, Jan. 14th, 10h - 12h
  - Room numbers to be announced on the Moodle
  - You can come and ask any questions that you might have regarding the lectures, the exercises or the exam

Announcements

- The final exam will be on Friday January 17th 2025, starting at 9h15 in the SwissTech Convention Center
  - It will be 3.5 hours long in English
  - No calculator or electronic devices allowed
  - Write in blue or black pen
  - You can bring a single cheat sheet (A4, front and back) that must be handwritten by you

Today’s agenda (Serway 10,15; MIT 20)

Swiss Plasma Center

1. **Course feedback**
2. Damped oscillations and resonance
3. Rolling, circular motion
4. Review

Today’s agenda (Serway 10,15; MIT 20)

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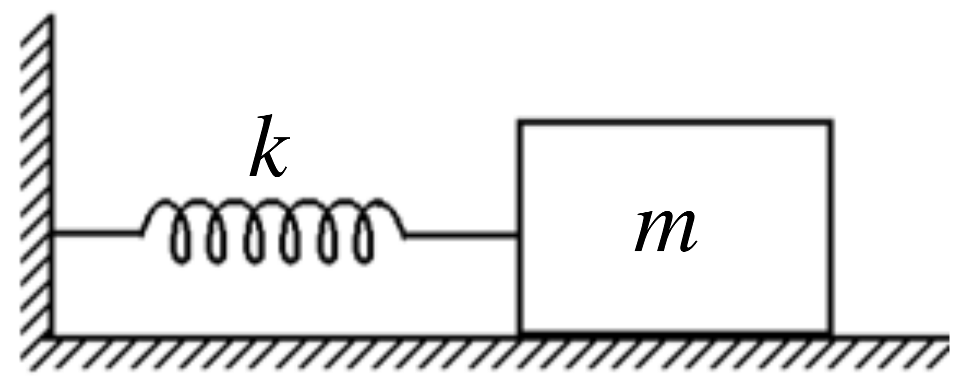
Conceptual question

A block of mass m is attached to a spring with spring constant k. It is free to slide along a horizontal frictionless surface. At $t = 0$ the block-spring system is released from rest at a displacement $x_0 > 0$ from the equilibrium position. What is the velocity of the block when it first passes through the equilibrium position?



A. $v = -2x_0\sqrt{k/m}$

B. $v = -x_0\sqrt{k/m}$

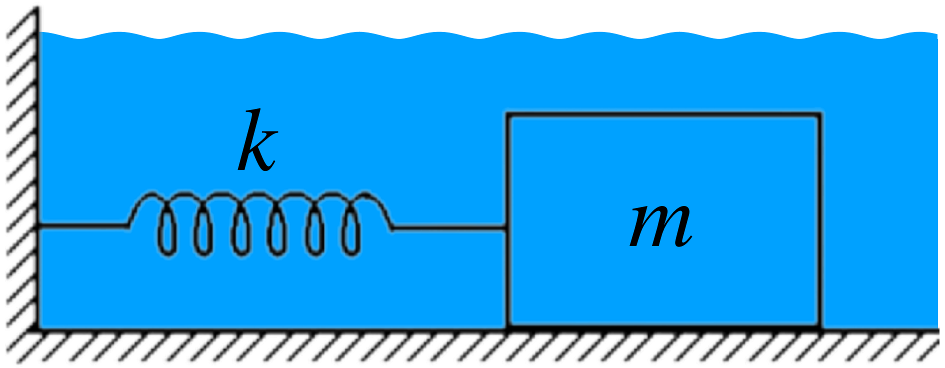
C. $v = x_0\sqrt{k/m}$

D. $v = 2x_0\sqrt{k/m}$

Damped oscillation

- Remember viscous drag (lecture 6)?

$$\vec{F}_{drag} = -\beta v \hat{v}$$



Damped oscillation

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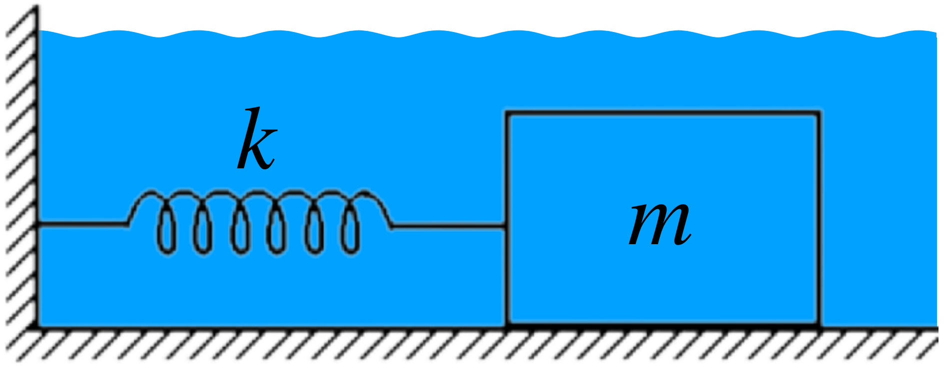

- This represents damping, which is realistic for many physical systems

Damped oscillation

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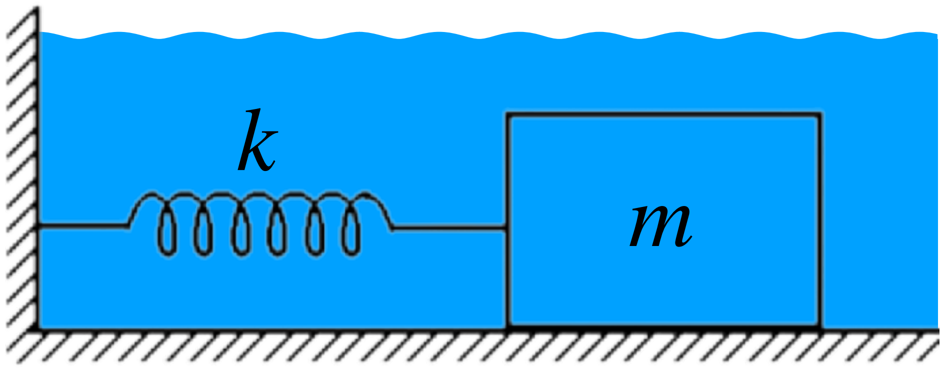

- This represents damping, which is realistic for many physical systems

- Equation of motion becomes

$$-\beta \frac{dx}{dt} - kx = m \frac{d^2 x}{dt^2}$$

Damped oscillation

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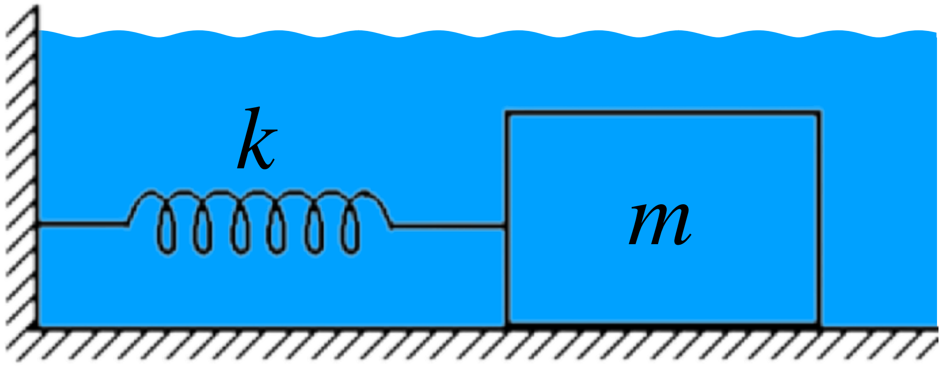

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- Equation of motion becomes

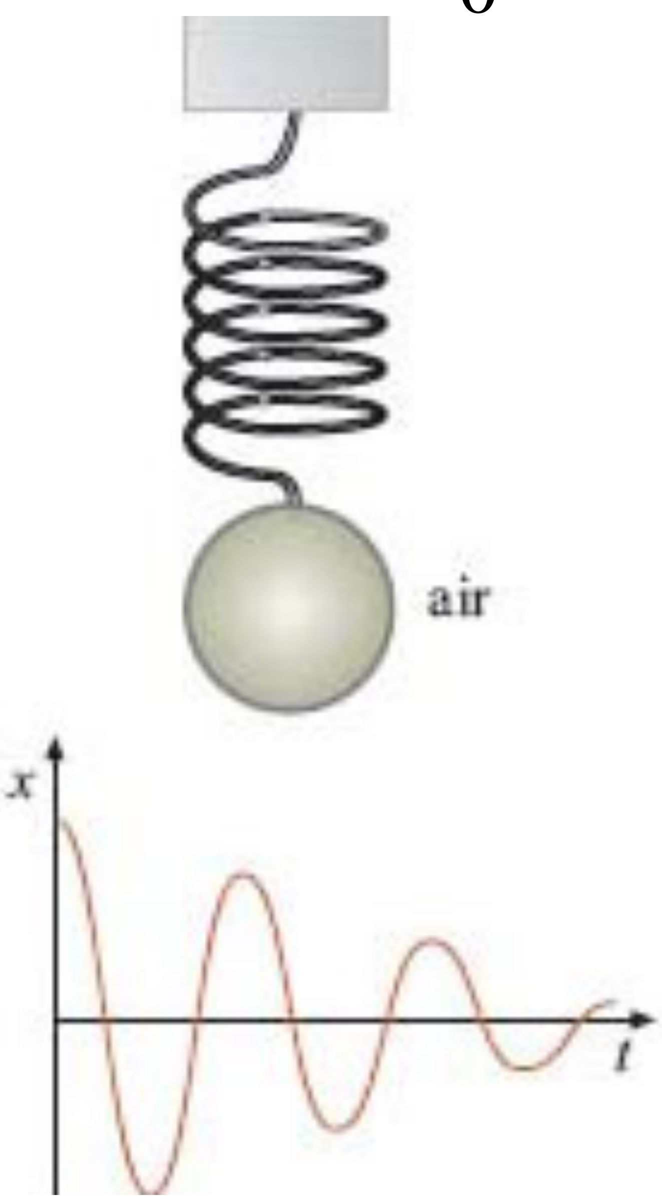
$$-\beta \frac{dx}{dt} - kx = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega_0^2 x = 0$$

Three cases of damped oscillation

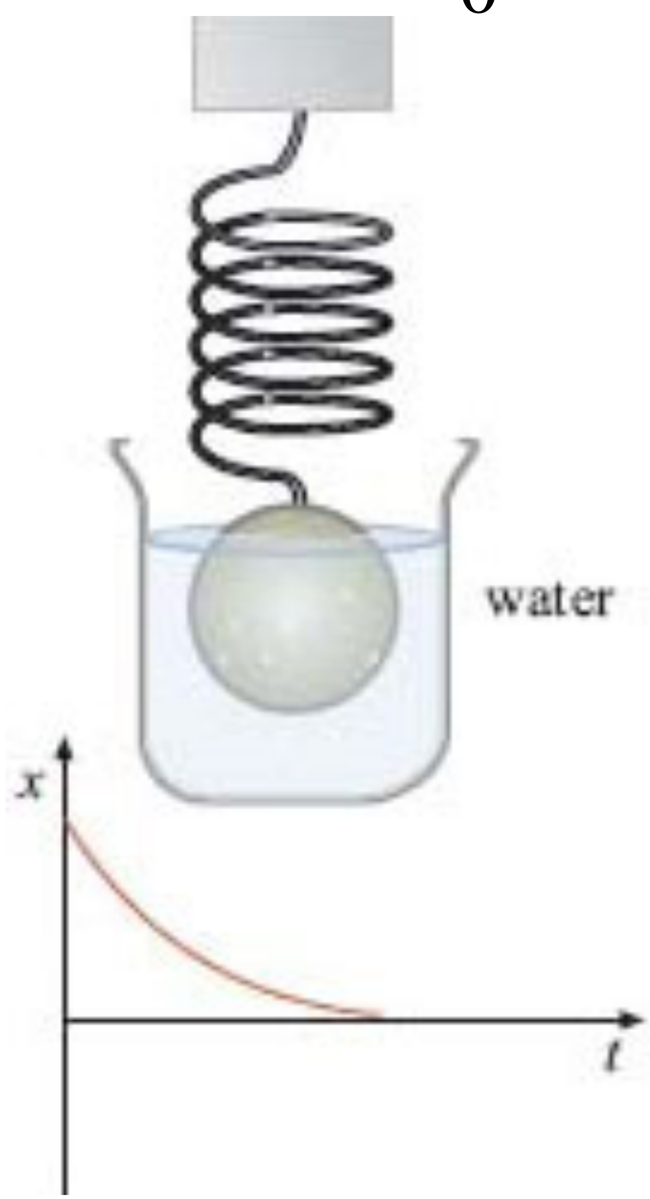
Weak damping

$$\lambda^2 < \omega_0^2$$



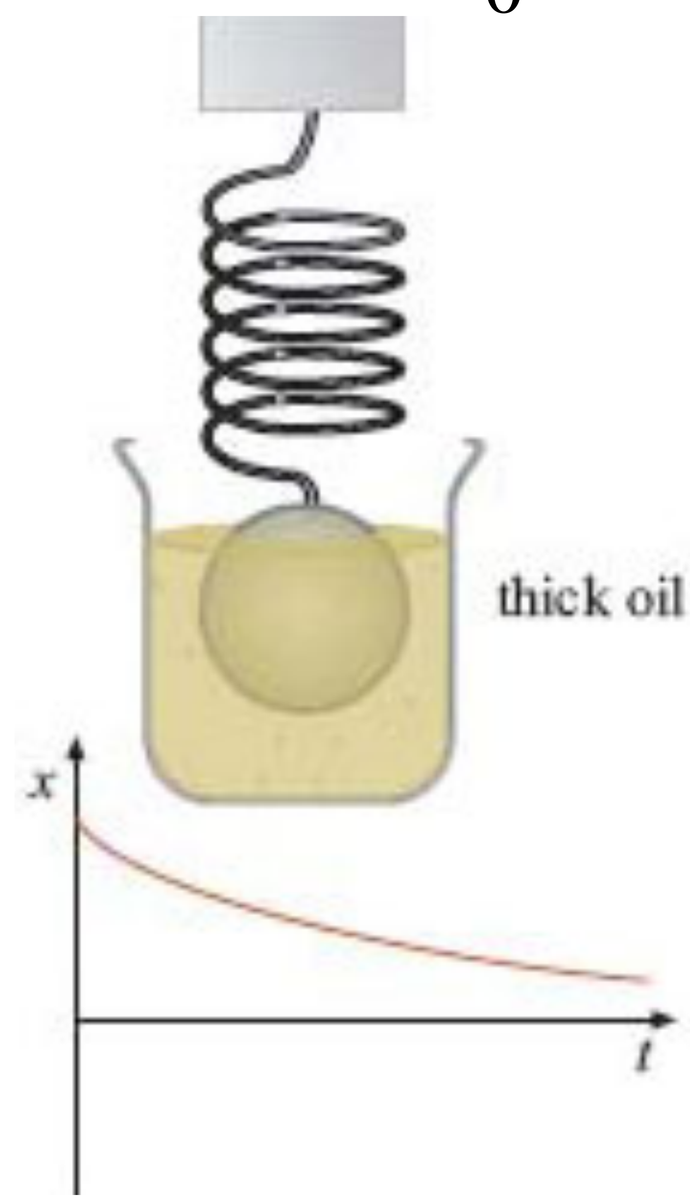
Critical damping

$$\lambda^2 = \omega_0^2$$



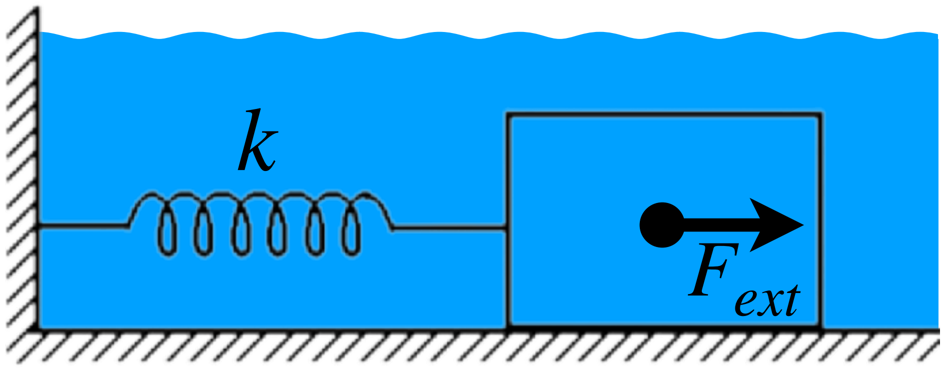
Strong damping

$$\lambda^2 > \omega_0^2$$



Forced oscillation

- Now let's add yet another term, an external driving force



$$F_{ext}(t) = F_d \cos(\omega_d t)$$

- This represents the influence of an externally applied force with amplitude F_d and angular frequency ω_d

Forced oscillation

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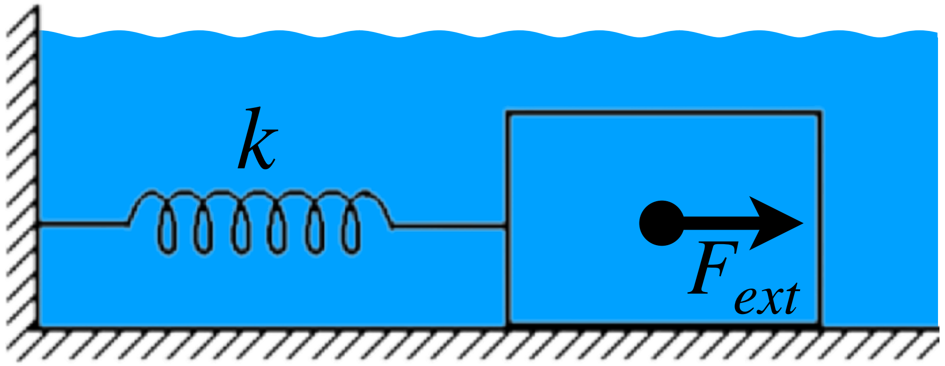

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- Equation of motion becomes

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = \frac{F_d}{m}\cos(\omega_d t)$$

Forced oscillation

- The solution is

$$x(t) = \begin{pmatrix} \text{homogeneous} \\ \text{solution} \end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

Forced oscillation

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- The solution is

$$x(t) = \begin{pmatrix}\text{homogeneous}\\ \text{solution}\end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

$$A_d = A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

Forced oscillation

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and the *forced* phase is

$$\varphi_d = \varphi_d(\omega_d) = \tan^{-1}\left(\frac{2\lambda\omega_d}{\omega_d^2 - \omega_0^2}\right)$$

Resonance

- When the driving frequency and natural frequency are very close, $\omega_d \approx \omega_0$, weird stuff can happen

$$A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

Today's agenda (Serway 10,15; MIT 20)

Swiss Plasma Center

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3. **Rolling, circular motion**
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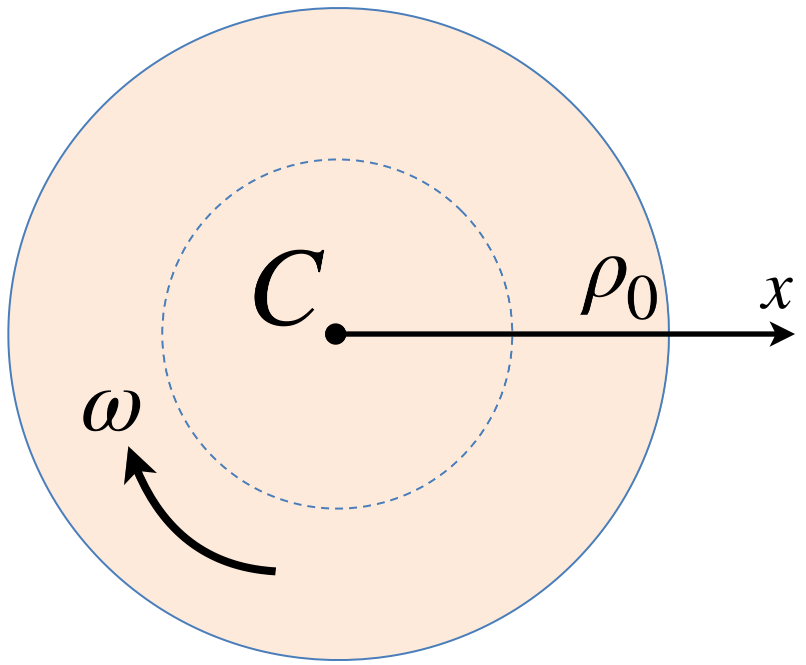
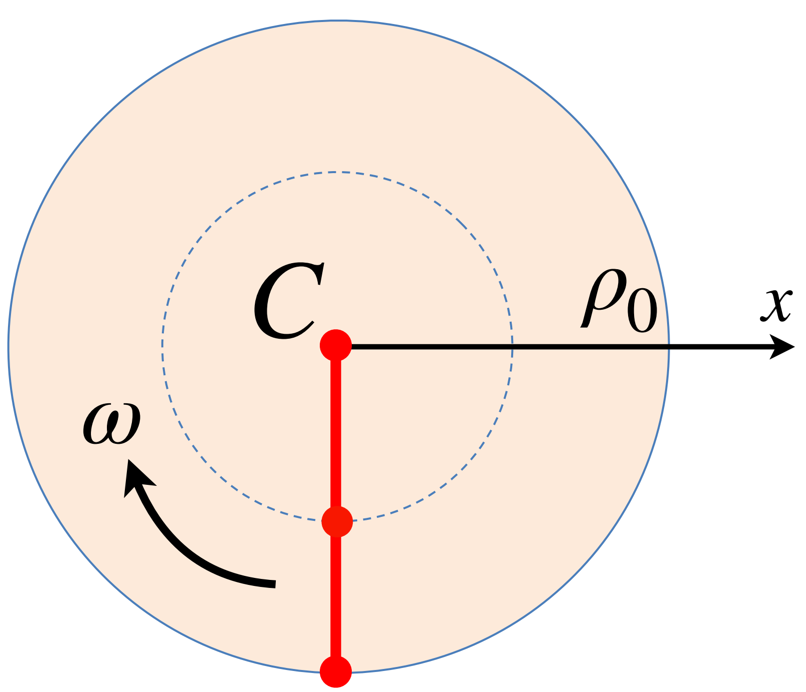
Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C



Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt



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- Therefore, every point has the same value of ω and α

Pure rotation of a rigid body

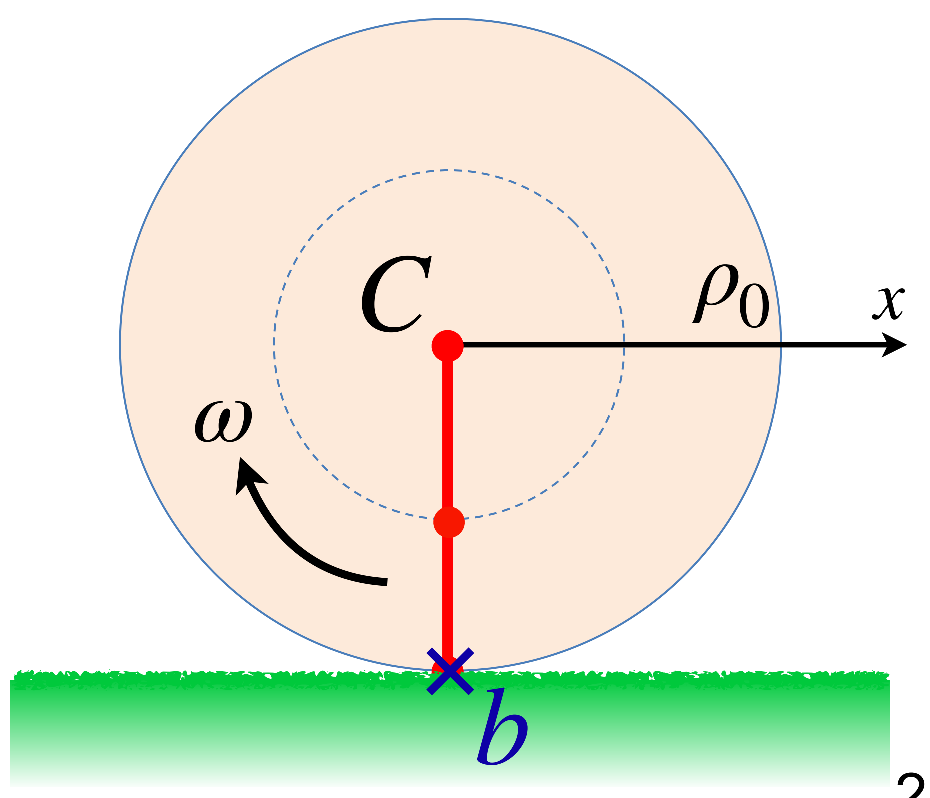
- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α
- The distance they move is the arc length $\Delta\ell(\rho) = \rho\Delta\phi$, so any point p has $\vec{v}_{Cp} = \dfrac{\Delta\ell}{\Delta t}\hat{\phi} = \dfrac{\rho\Delta\phi}{\Delta t}\hat{\phi} = \rho\omega\,\hat{\phi}$ <u>in the CM frame</u>

Rolling without slipping

- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

$$\vec{v}_{gb} = 0$$

<u>in the ground frame of reference</u>



Rolling without slipping

- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

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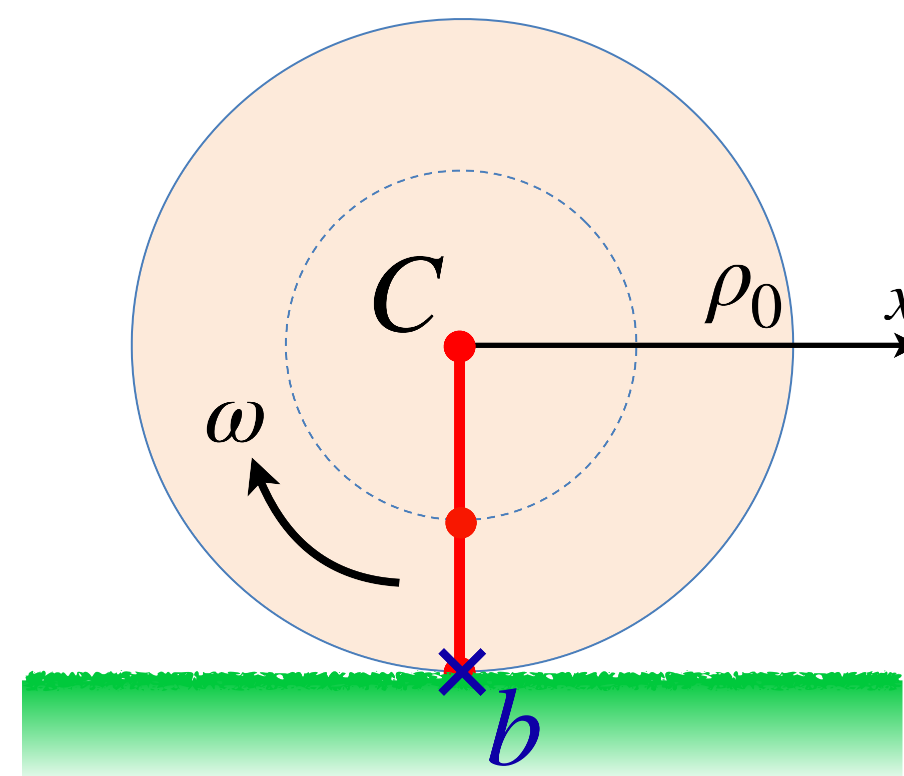
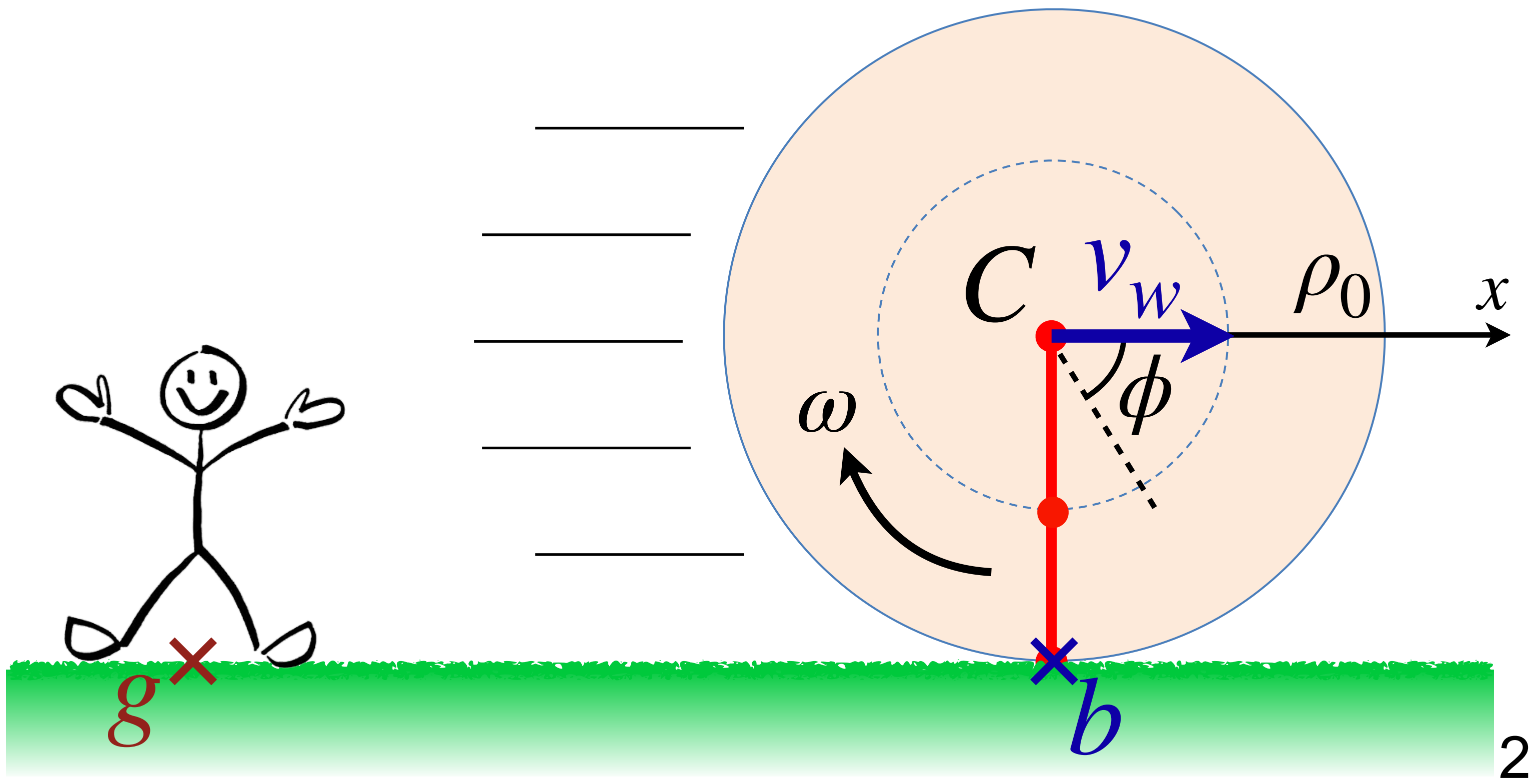
<u>in the ground frame of reference</u>

- This means the friction is static, points in the opposite direction to translational acceleration, and does no work



Rolling velocity and switching reference frames

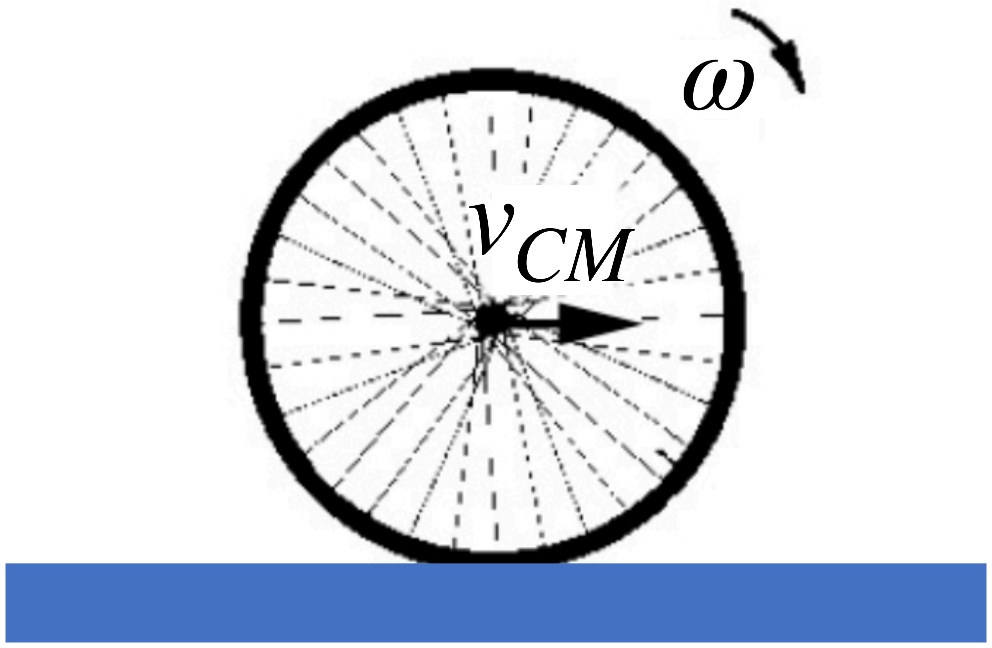
- In the CM frame, all points on the rim have $\vec{v}_{Cp} = \rho_0 \omega \, \hat{\phi}$
- In the frame of reference of the ground, the point touching the ground b has $\vec{v}_{gb} = 0$



Conceptual question

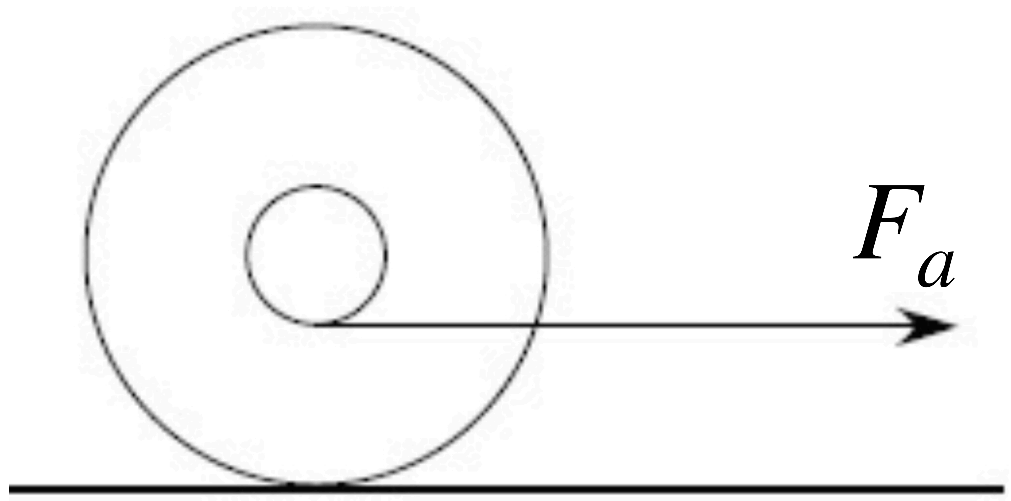
A bicycle wheel is initially spinning in the air and then is put into contact with a rough surface (see figure). It slips against the surface. What is the direction of the kinetic friction force acting on the wheel?

A. Points to the right
B. Points to the left
C. Points up
D. Points down



Example: Yo-yo

A yo-yo is placed on a rough surface and rolls without slipping. It is composed of two disks separated by a spindle with a smaller diameter. A string is wound around the spindle and pulled with a force F_a. In which direction does it move? To the right, winding up the string, or to the left, unwinding the string?



DEMO (45)

Rolling with slipping

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Conceptual question

Which one of the following physical quantities is **not** a *vector*?

A. Position
B. Impulse
C. Torque
D. Work
E. Displacement

Conceptual question

Which one of the following *scalar* quantities **can** be negative?

A. Mass
B. Moment of inertia
C. Work
D. Kinetic energy
E. Spring constant

Conceptual question

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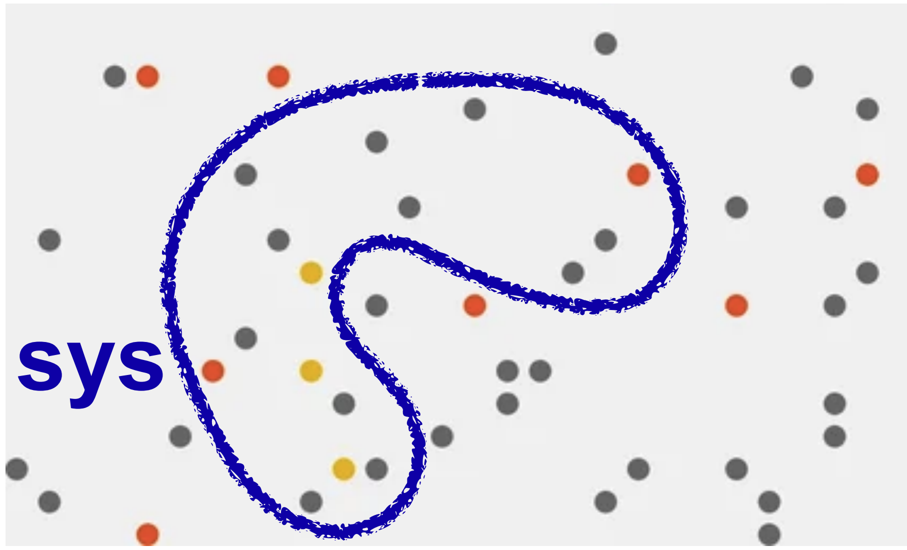
The net *force* acting on sys (the physical system on the right) is

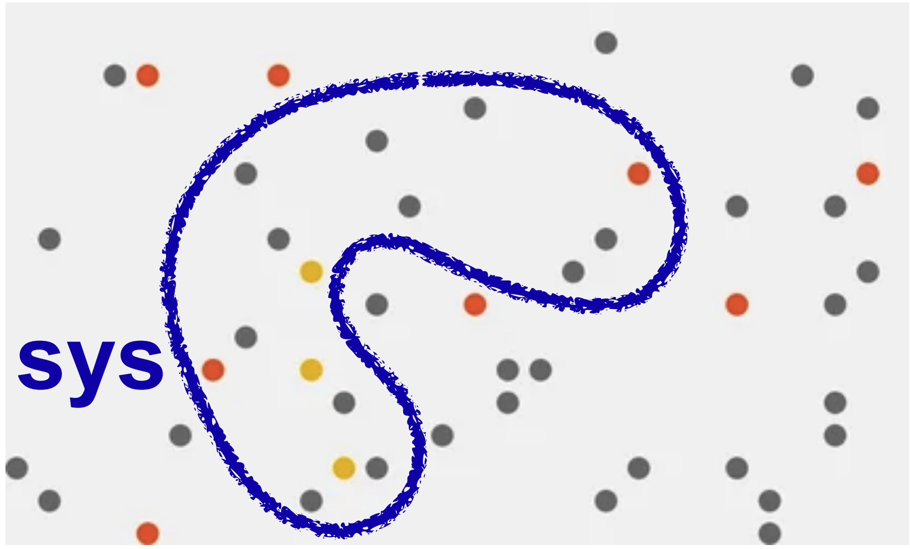
A. The largest force exerted on any point anywhere
B. The largest force exerted on any point in sys
C. Zero
D. The sum of all forces internal to sys
E. The sum of all forces exerted by points **not** in sys on points in sys



Conceptual question

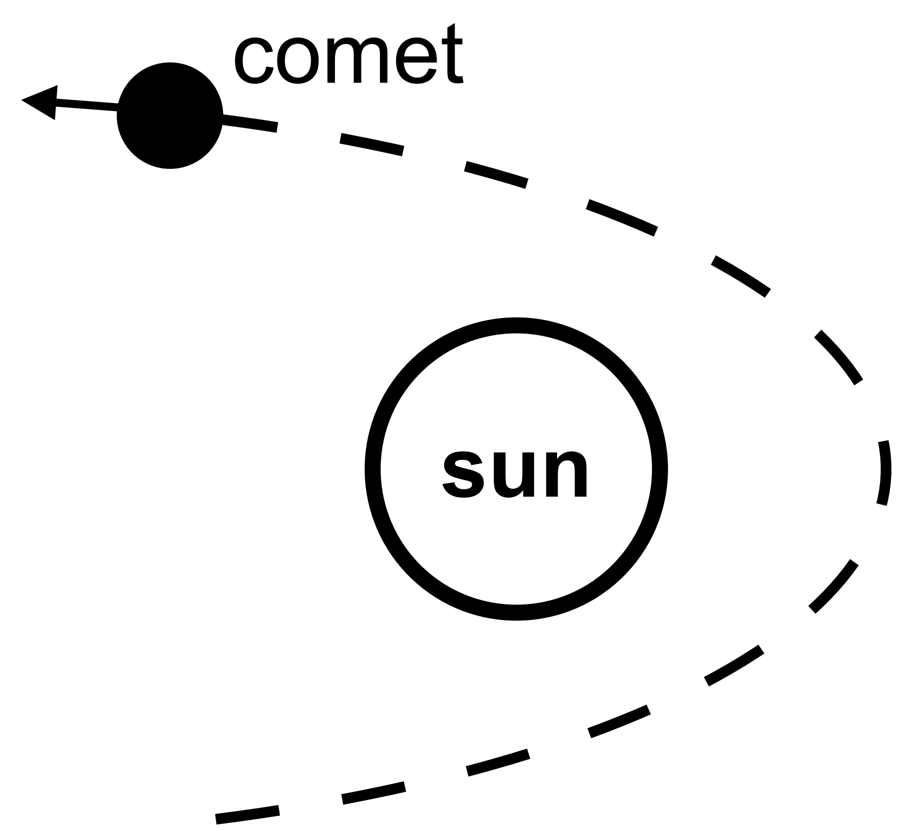
We look at sys from an inertial reference frame and see that $\vec{F}^{ext}_{net} = 0$. There is no matter exchange with the outside. Which one of the following statements is **not** true?

A. The internal forces add up to zero
B. The total momentum of sys is conserved
C. The momentum of each individual particle is guaranteed to stay constant
D. The position of the center-of-mass (CM) is well defined
E. The velocity of the CM is constant



Conceptual question

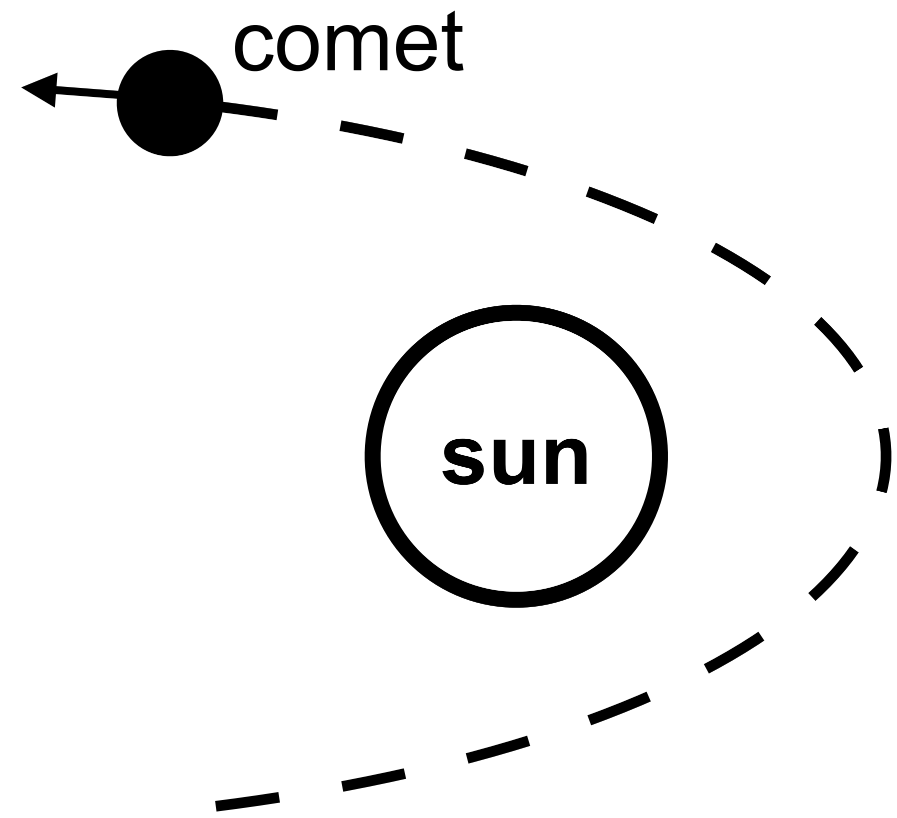
A comet is on a hyperbolic orbit around the Sun. While the comet is moving <u>away</u> from the Sun, the work done by the Sun on the comet is…



A. positive.
B. zero.
C. negative.

Conceptual question

A comet is on a hyperbolic orbit around the Sun. While the comet is moving away from the Sun, what happens with its speed?



A. It increases
B. It stays the same
C. It decreases
D. There is not enough info

Conceptual question

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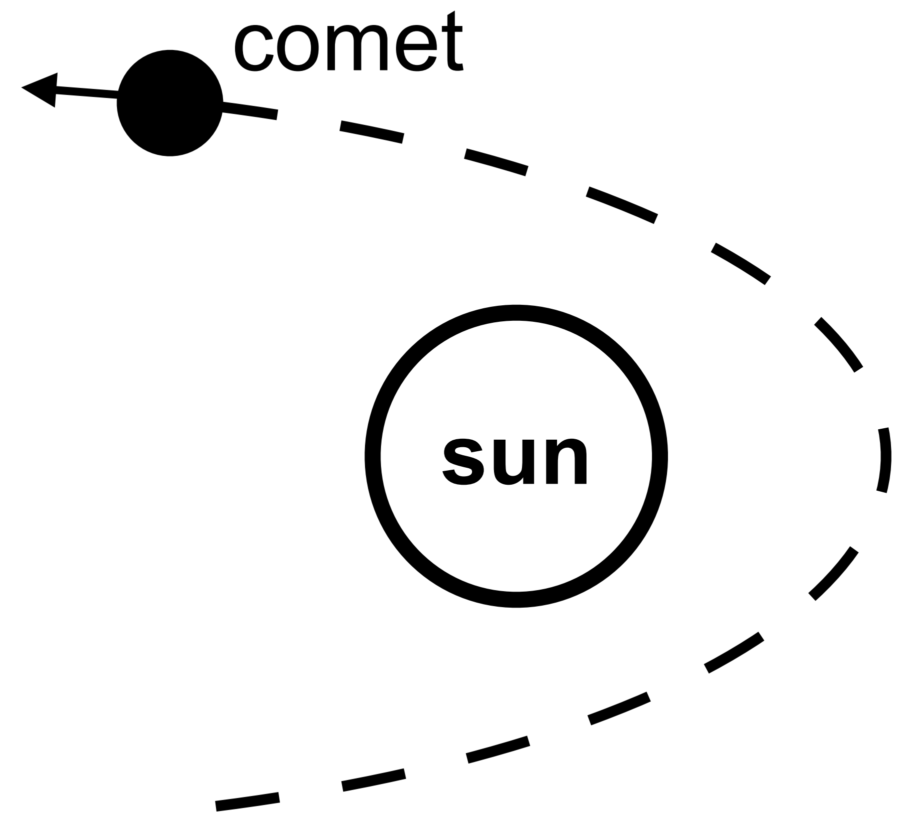
A comet is on a hyperbolic orbit around the Sun. While the comet is moving away from the Sun, what happens with its gravitational potential energy?



A. It increases
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Conceptual question

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A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls her arms in, she reduces her moment of inertia and her angular speed increases. Compared to her initial **angular momentum**, her **angular momentum** after she has pulled her arms in must be…

A. the same.
B. larger.
C. smaller.