General Physics: Mechanics

Swiss Plasma Center

PHYS-101(en)

Lecture 14a: Damped oscillations, rolling and review

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December 16th, 2024

Announcements

Swiss Plasma Center

- The average grade of the 2nd Mock exam final was 6.5 out of 16
    - Most of you only had time to work on one of the two problems
    - The average thus suggests that you did well, just maybe slowly
    - Be sure to check the solutions on the Moodle and the comments written by the TAs on your graded exams to find weaknesses that you can focus on
- Pick up your exam later today during one of the breaks

Announcements

- Several past final exams now on the Moodle
- There will be two Review sessions in January
  - First one on Thursday, Jan. 9th, from 10h to 12h
  - Second one on Tuesday, Jan. 14th, 10h - 12h
  - Room numbers to be announced on the Moodle
  - You can come and ask any questions that you might have regarding the lectures, the exercises or the exam

Announcements

Swiss Plasma Center

- The final exam will be on Friday January 17th 2025, starting at 9h15 in the SwissTech Convention Center
  - It will be 3.5 hours long in English
  - No calculator or electronic devices allowed
  - Write in blue or black pen
  - You can bring a single cheat sheet (A4, front and back) that must be handwritten by you

Today’s agenda (Serway 10,15; MIT 20)

1. **Course feedback**

2. Damped oscillations and resonance

3. Rolling, circular motion

4. Review

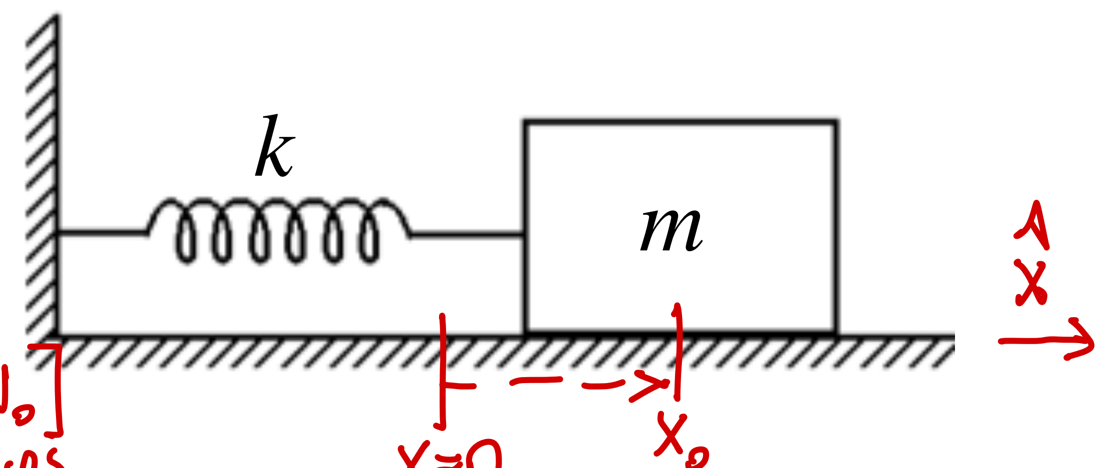
Today's agenda (Serway 10,15; MIT 20)

Swiss Plasma Center

1. Course feedback
2. **Damped oscillations and resonance**
3. Rolling, circular motion
4. Review

Conceptual question

A block of mass m is attached to a spring with spring constant k. It is free to slide along a horizontal frictionless surface. At $t = 0$ the block-spring system is released from rest at a displacement $x_0 > 0$ from the equilibrium position. What is the velocity of the block when it first passes through the equilibrium position?



A. $v = -2x_0\sqrt{k/m}$

B. $v = -x_0\sqrt{k/m}$

C. $v = x_0\sqrt{k/m}$

D. $v = 2x_0\sqrt{k/m}$

$\omega_0^2 = \frac{k}{m}$

$x(t) = A\cos(\omega_0 t + \varphi)$

$v(t) = \dot{x}(t)$

$= A[-\sin(\omega_0 t + \varphi)\omega_0]$

$= -A\omega_0 \sin(\omega_0 t + \varphi)$

$v(t=0) = 0 = -A\omega_0 \sin(\varphi) \Rightarrow \varphi = 0 \text{ or } \pi$

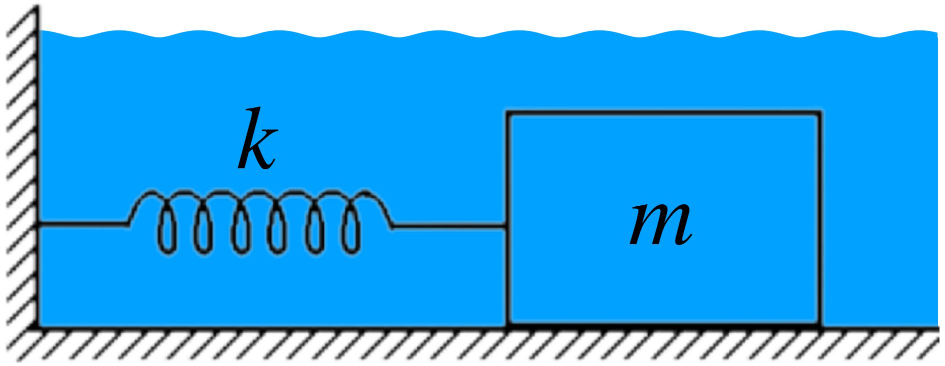
$x(t=0) = x_0 = A\cos(\varphi) \Rightarrow \varphi$ has to be 0 $(\varphi = 0)$ and $A = x_0$

$x_{eq} = 0 = x_0\cos(\omega_0 t_{eq}) \Rightarrow \omega_0 t_{eq} = \frac{\pi}{2} \Rightarrow v_{eq} = v(t = t_{eq}) = -x_0\omega_0 \sin(\omega_0 t_{eq}) = -x_0\omega_0$

Damped oscillation

- Remember viscous drag (lecture 6)?

$$\overrightarrow{F}_{drag} = -\beta v \hat{v}$$

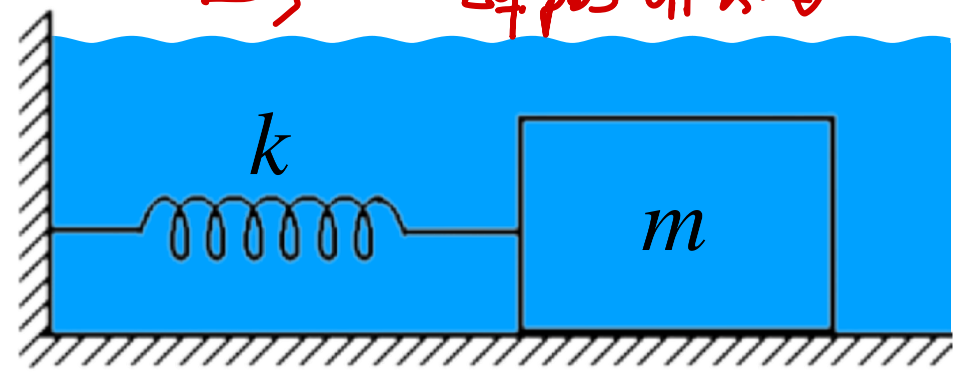


Damped oscillation

- Remember viscous drag (lecture 6)?

$$\vec{F}_{drag} = -\beta v \hat{v}$$

x → Eq pos at $x=0$



- This represents damping, which is realistic for many physical systems

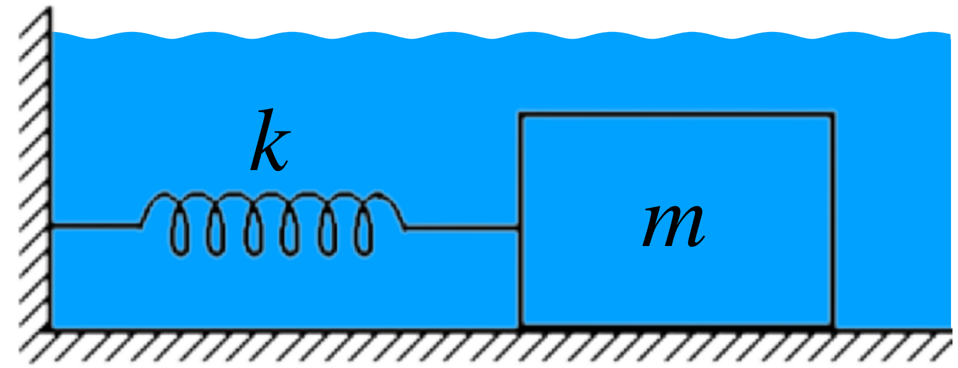
$v = \frac{dx}{dt}$ $\quad a = \frac{d^2x}{dt^2}$

$$\Sigma F_x: \; F_s + F_d = -kx - \beta v = ma$$

$$\Rightarrow -kx - \beta \frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Damped oscillation

- Remember viscous drag (lecture 6)?



$$\vec{F}_{drag} = -\beta v \hat{v}$$

- This represents damping, which is realistic for many physical systems

- Equation of motion becomes

$$-\beta \frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

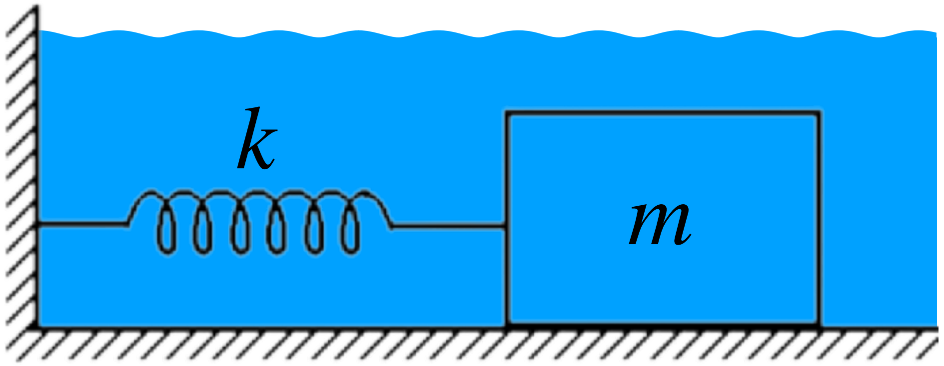
$$\Rightarrow m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0 \quad \Rightarrow \quad \frac{d^2x}{dt^2} + \underbrace{\frac{\beta}{m}}_{\equiv 2\lambda}\frac{dx}{dt} + \overbrace{\frac{k}{m}}^{\equiv \omega_0^2} x = 0$$

$\lambda = \frac{1}{2}\frac{\beta}{m}$ (definition of λ)

Damped oscillation

- Remember viscous drag (lecture 6)?

$$\vec{F}_{drag} = -\beta v \hat{v}$$



- This represents damping, which is realistic for many physical systems

- Equation of motion becomes

$$-\beta \frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = 0$$

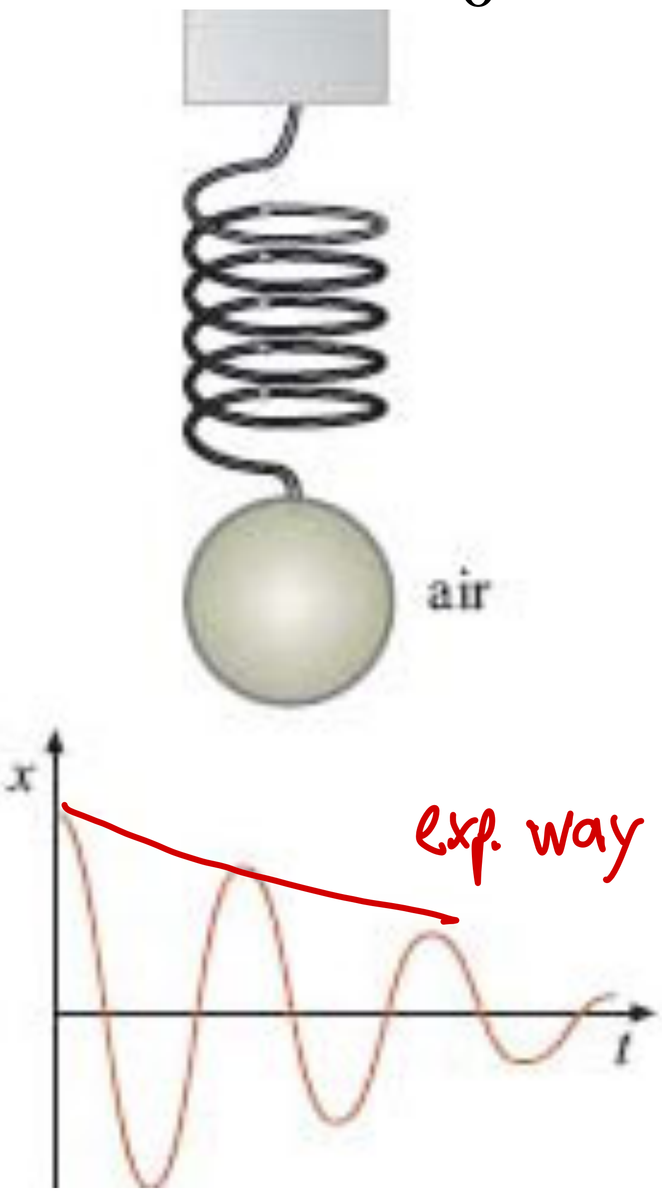
$\omega_0^2 = \frac{k}{m}$

$\lambda = \frac{1}{2}\frac{\beta}{m}$

Three cases of damped oscillation

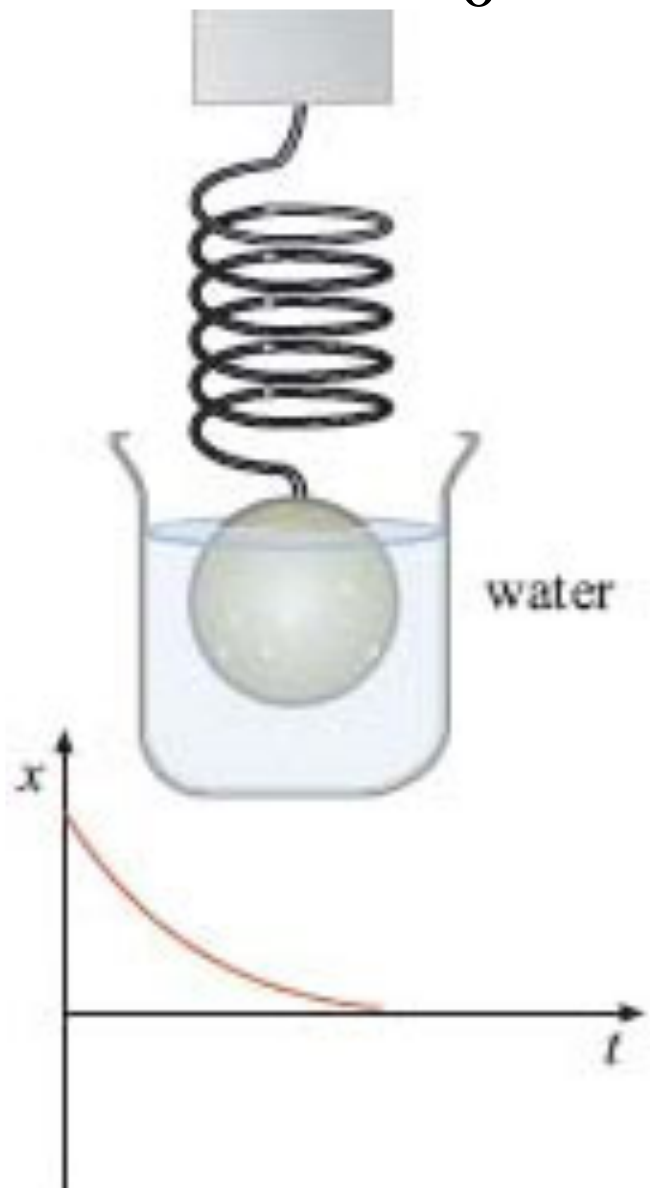
Weak damping

$$\lambda^2 < \omega_0^2$$



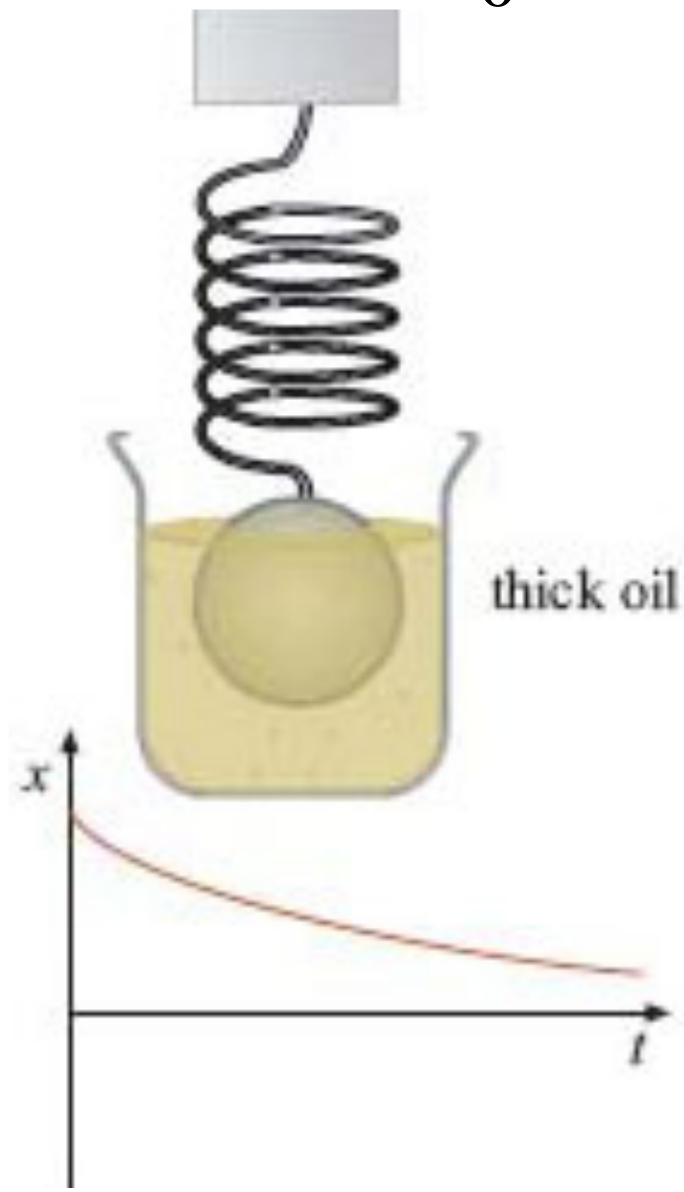
Critical damping

$$\lambda^2 = \omega_0^2$$

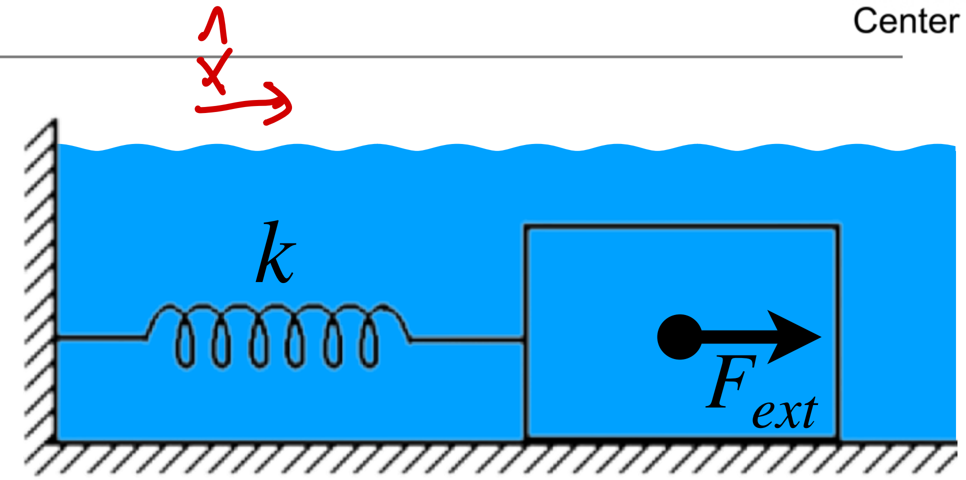


Strong damping

$$\lambda^2 > \omega_0^2$$



Forced oscillation



- Now let's add yet another term, an external driving force

$$F_{ext}(t) = F_d \cos(\omega_d t)$$

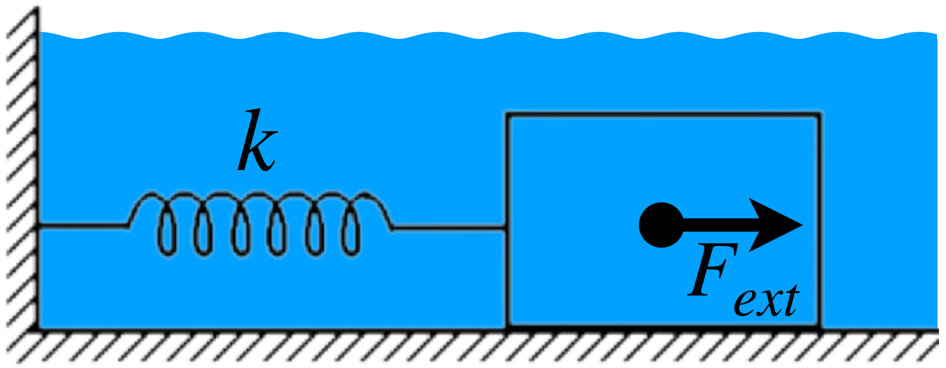
- This represents the influence of an externally applied force with amplitude F_d and angular frequency ω_d

$$\sum F_x = F_s + F_d + F_{ext} = -kx - \beta v + F_{ext} = ma$$

$$\Rightarrow m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F_{ext}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \underbrace{\frac{\beta}{m}}_{2\lambda}\frac{dx}{dt} + \underbrace{\frac{k}{m}}_{\omega_0^2}x = \frac{1}{m}F_{ext} = \frac{F_d}{m}\cos(\omega_d t)$$

Forced oscillation



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$$F_{ext}(t) = F_d \cos(\omega_d t)$$

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- Equation of motion becomes

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = \frac{F_d}{m}\cos(\omega_d t)$$

Forced oscillation

- The solution is

Decays to zero after waiting long enough

$$x(t) = \left(\begin{array}{c} \text{homogeneous} \\ \text{solution} \end{array} \right) + A_d \cos(\omega_d t + \varphi_d)$$

Forced oscillation

- The solution is

$$x(t) = \begin{pmatrix}\text{homogeneous}\\ \text{solution}\end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

$$A_d = A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

Forced oscillation

- The solution is

$$x(t) = \left(\text{homogeneous solution}\right) + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

$$A_d = A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

and the *forced* phase is

$$\varphi_d = \varphi_d(\omega_d) = \tan^{-1}\left(\frac{2\lambda\omega_d}{\omega_d^2 - \omega_0^2}\right)$$

Resonance

- When the driving frequency and natural frequency are very close, $\omega_d \approx \omega_0$, weird stuff can happen

$$A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

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3. **Rolling, circular motion**
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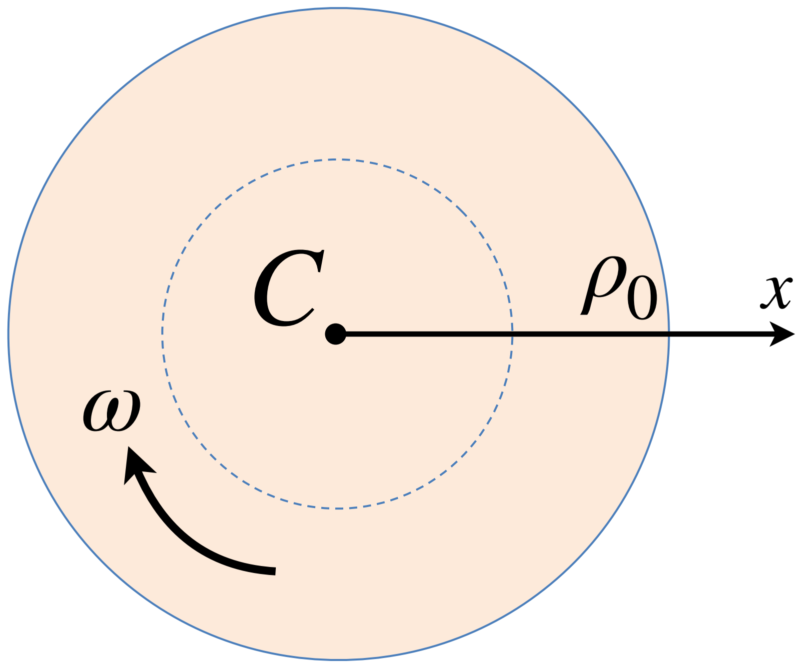
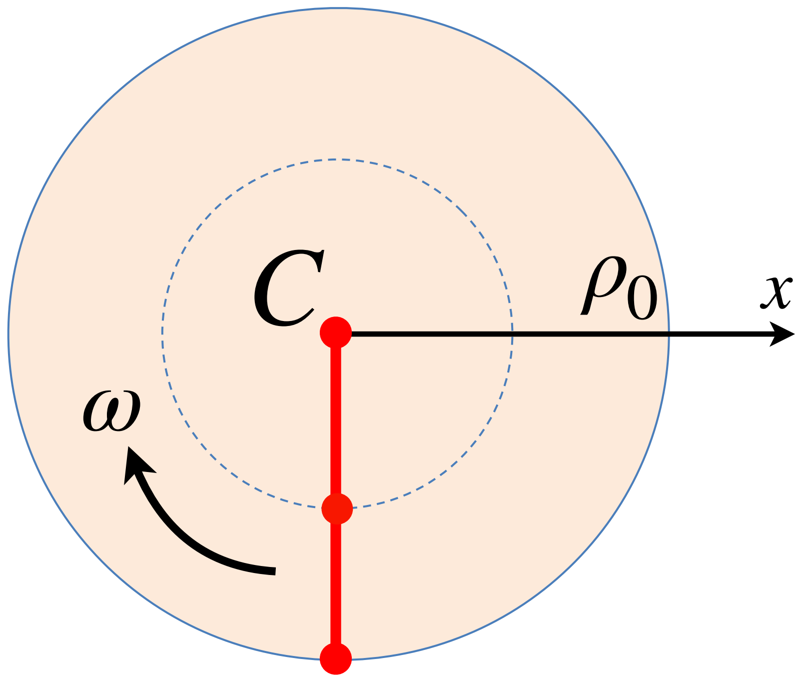
Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C



Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt



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Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α

$\frac{d\phi}{dt}$ $\frac{d\omega}{dt} = \frac{d^2\phi}{dt^2}$

Pure rotation of a rigid body

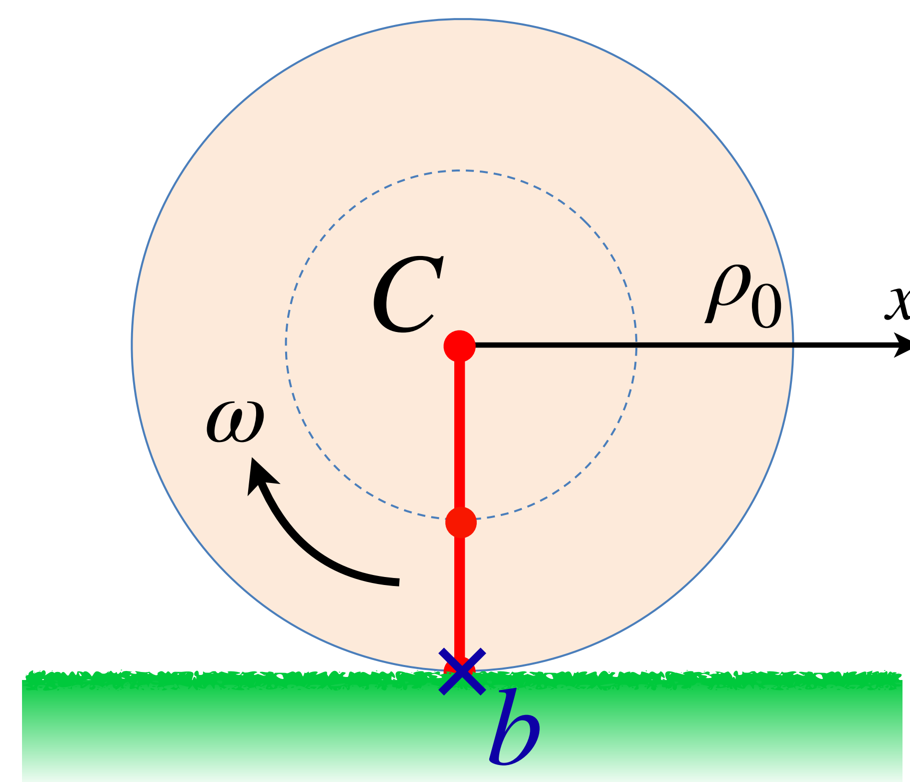
- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α
- The distance they move is the arc length $\Delta\ell(\rho) = \rho\Delta\phi$, so any point p has $\vec{v}_{Cp} = \dfrac{\Delta\ell}{\Delta t}\hat{\phi} = \dfrac{\rho\Delta\phi}{\Delta t}\hat{\phi} = \rho\omega\,\hat{\phi}$ in the CM frame

Rolling without slipping

- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

$$\vec{v}_{gb} = 0$$

<u>in the ground frame of reference</u>



Rolling without slipping

- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

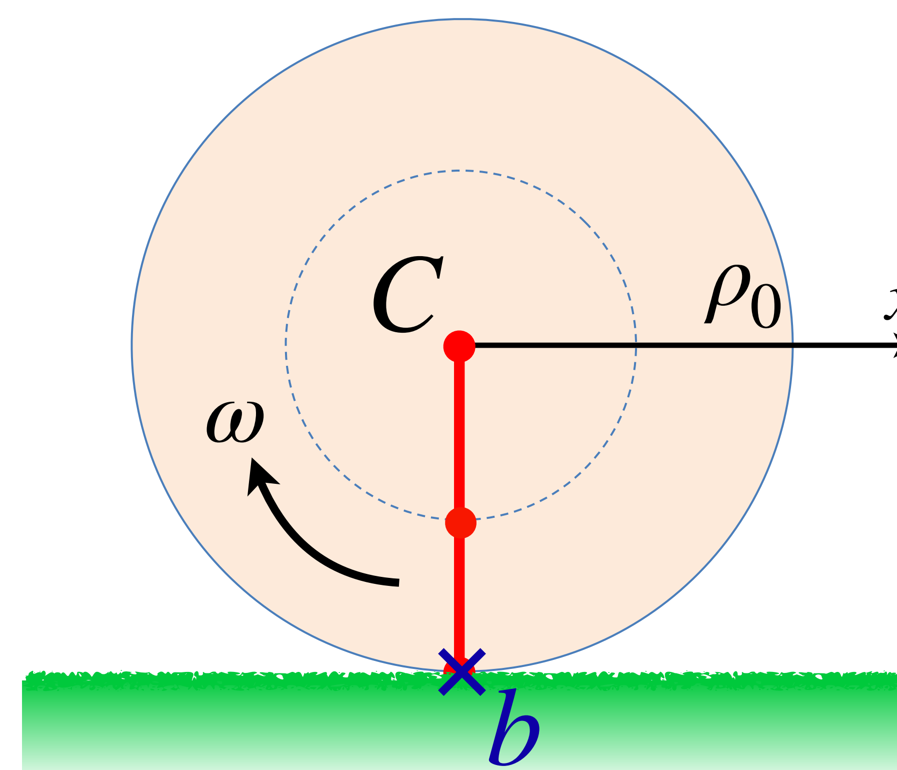
$$\vec{v}_{gb} = 0$$

<u>in the ground frame of reference</u>

- This means the friction is static, points in the opposite direction to translational acceleration, and does no work

$$W_s = \int \vec{F}_s \cdot \vec{dl} = 0$$

because $\vec{F}_s \perp \vec{dl}$ <u>always</u>



Rolling velocity and switching reference frames

- In the CM frame, all points on the rim have $\vec{v}_{Cp} = \rho_0 \omega \, \hat{\phi}$
- In the frame of reference of the ground, the point touching the ground b has $\vec{v}_{gb} = 0$

$$\vec{r}_{gb} = \vec{r}_{gc} + \vec{r}_{cb}$$

$$\frac{d}{dt}(\vec{r}_{gb}) = \frac{d}{dt}(\vec{r}_{gc} + \vec{r}_{cb})$$

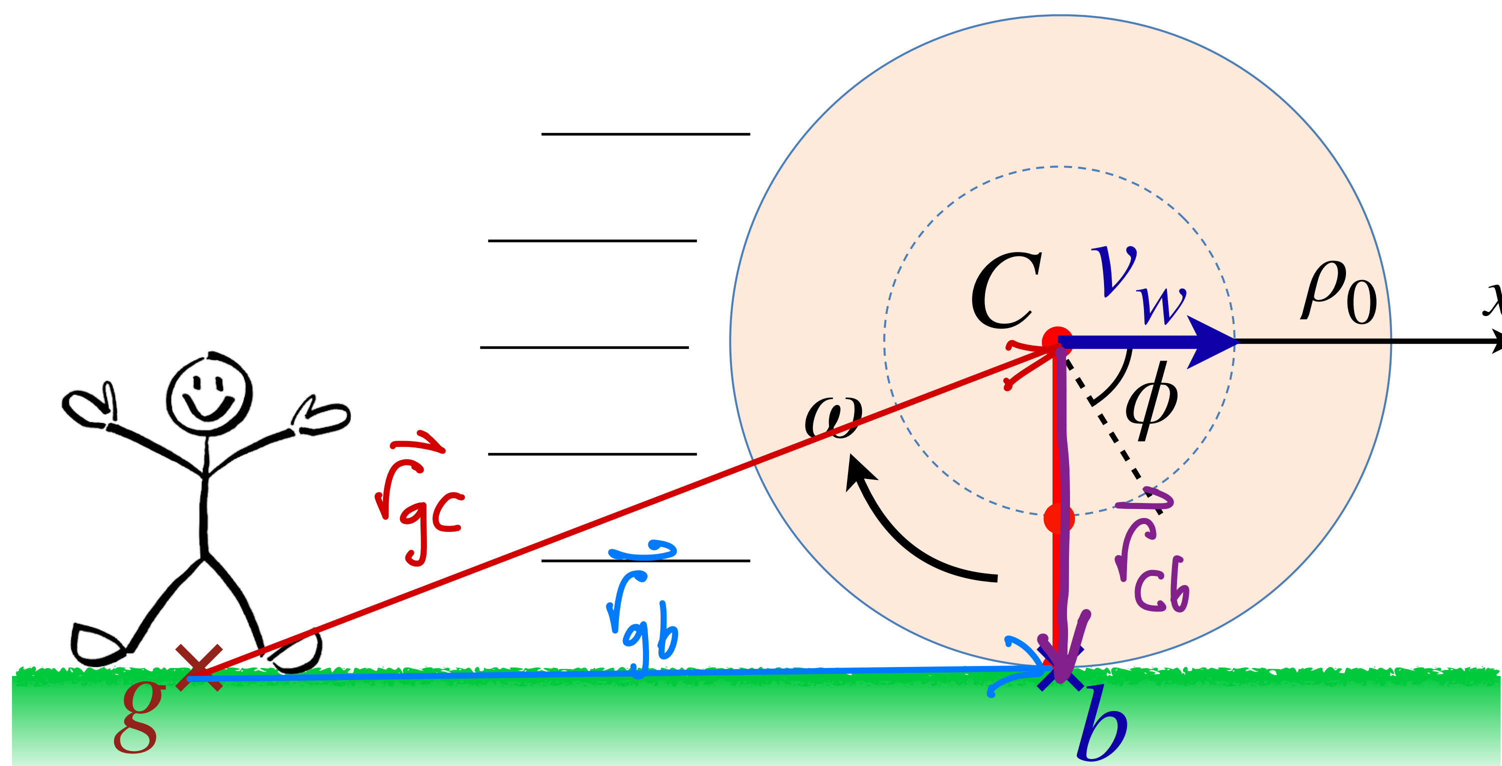
$$\vec{v}_{gb} \rightarrow 0 = \underbrace{\vec{v}_{gc}}_{=\vec{v}_w} + \underbrace{\vec{v}_{cb}}_{=\rho_0 \omega \hat{\phi}_{cb}}$$

$$0 = \vec{v}_w + \rho_0 \omega \hat{\phi}_{cb}$$

$$\Rightarrow \vec{v}_w = -\rho_0 \omega \hat{\phi}_{cb}$$

$$= -\rho_0 \omega (-\hat{x})$$

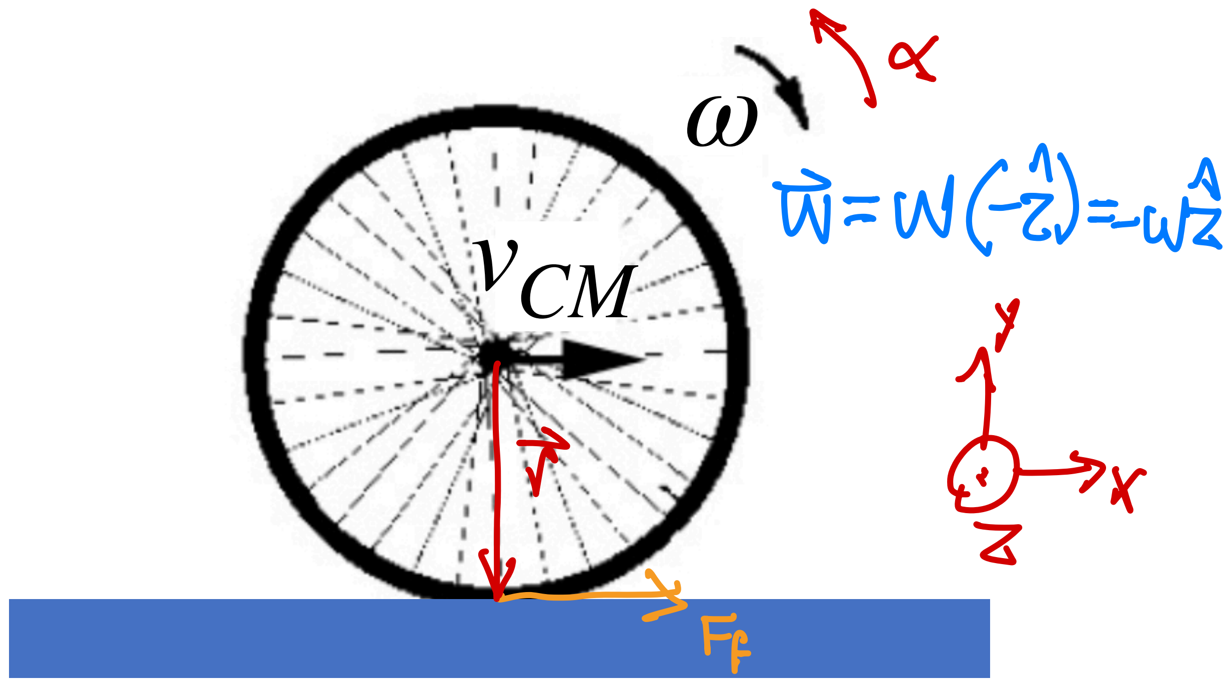
$$\boxed{\vec{v}_w = \rho_0 \omega \hat{x}}$$



Conceptual question

A bicycle wheel is initially spinning in the air and then is put into contact with a rough surface (see figure). It slips against the surface. What is the direction of the kinetic friction force acting on the wheel?

A. Points to the right
B. Points to the left
C. Points up
D. Points down



$\vec{\tau}_{net} = I\vec{\alpha}$

$\vec{\tau}_f = \vec{r}\times\vec{F}_f = rF_f\hat{z} \Rightarrow \vec{\tau}_{net} = \sum\vec{\tau} = \vec{\tau}_f = rF_f\hat{z} = I\vec{\alpha}$

$\vec{\omega} = -\omega\hat{z}$

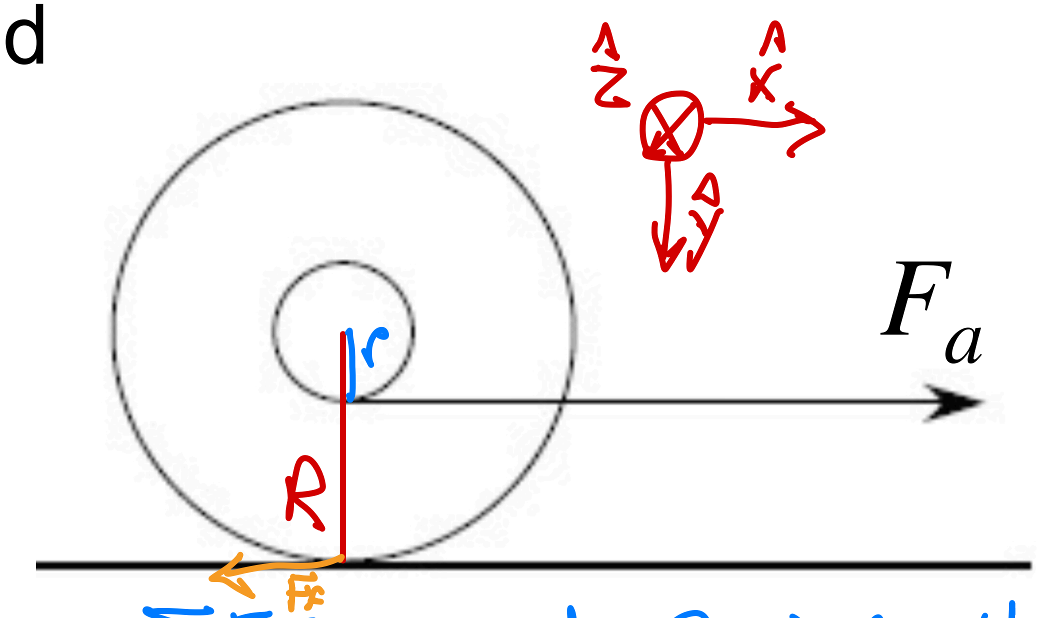
$\vec{\alpha} = \frac{rF_f}{I}\hat{z}$

} $\vec{\alpha}$ has direction opposite to $\vec{\omega}$

Example: Yo-yo

mass m

A yo-yo is placed on a rough surface and rolls without slipping. It is composed of two disks separated by a spindle with a smaller diameter. A string is wound around the spindle and pulled with a force F_a. In which direction does it move? To the right, winding up the string, or to the left, unwinding the string?



$\sum F_y$: $mg - N = 0 \Rightarrow mg = N$

$\sum F_x$: $F_a - F_f = m a_{cM}$

$F_f = F_a - m a_{cM}$

$\vec{\tau}_{net} = I\vec{\alpha}$ $\qquad$ $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau}_{net} = \sum \vec{\tau} = \vec{\tau}_{F_a} + \vec{\tau}_{F_f} = (r\hat{y}) \times (F_a\hat{x}) + (R\hat{y}) \times (-F_f\hat{x}) = rF_a(\underbrace{\hat{y}\times\hat{x}}_{=-\hat{z}}) - RF_f(\hat{y}\times\hat{x})$$

$$= (rF_a - RF_f)(-\hat{z}) = (RF_f - rF_a)\hat{z} = I\vec{\alpha} = I\left(\frac{a_{cM}}{R}\right)\hat{z}$$

$\alpha = \frac{a_{cM}}{R}$ for no slip

$$\frac{I}{R}a_{cM} = RF_f - rF_a = R[F_a - m a_{cM}] - rF_a = \underbrace{RF_a - rF_a}_{(R-r)F_a} - mRa_{cM}$$

$$\frac{I}{R}a_{cM} + mRa_{cM} = \left(\frac{I}{R} + mR\right)a_{cM} = (R-r)F_a \Rightarrow \boxed{a_{cM} = (R-r)\left(\frac{I}{R} + mR\right)^{-1} F_a > 0}$$

DEMO (45)

Rolling with slipping