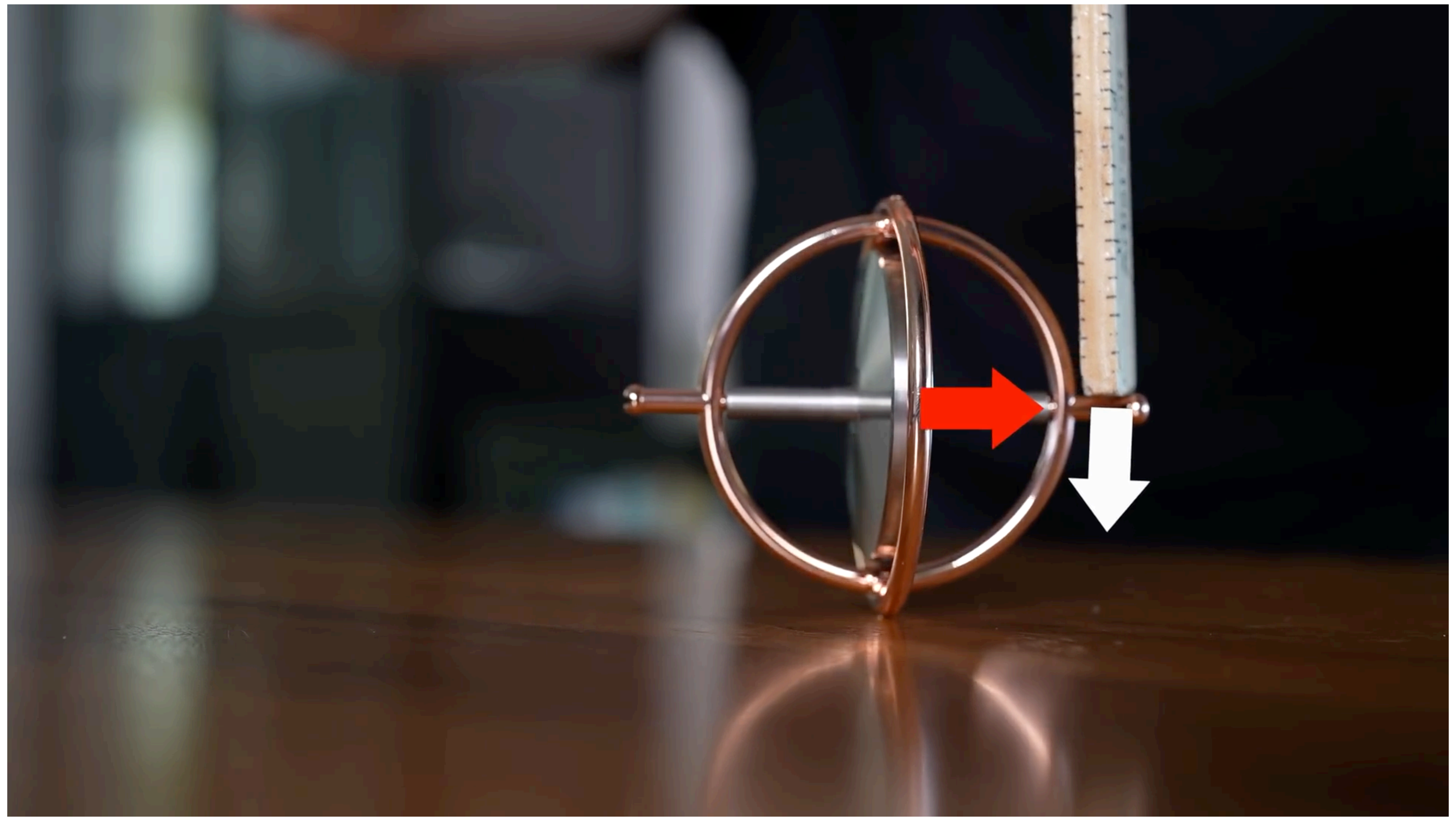
EPFL
Swiss Plasma Center

General Physics: Mechanics



PHYS-101(en)

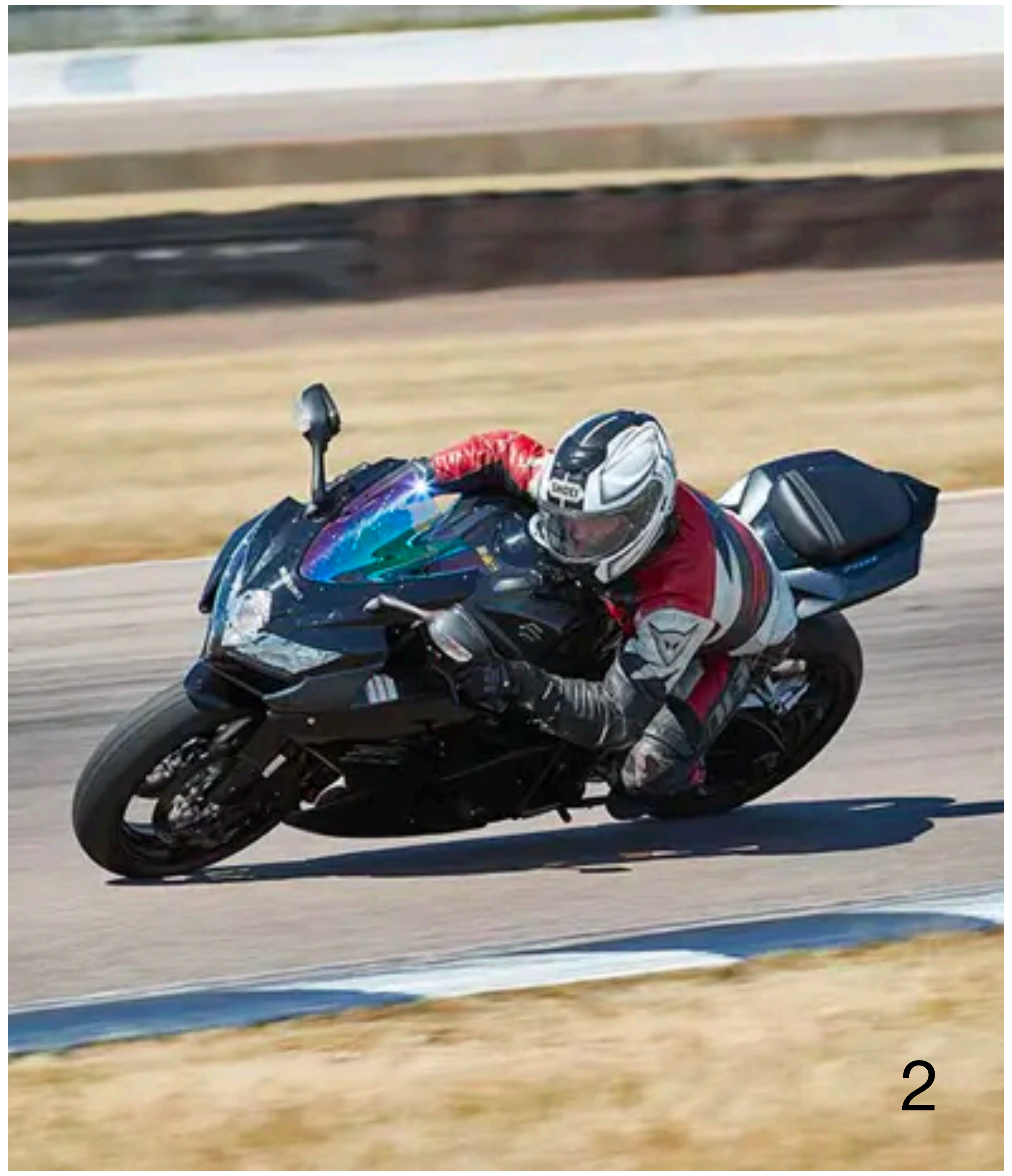
Lecture 13b: Gyroscopes and harmonic motion

Dr. Marcelo Baquero
marcelo.baquero@epfl.ch

December 10th, 2024

Example: Motorcycle steering

You're racing on motorcycle at a constant ~~velocity~~ speed v and are entering a turn. To stay on the road, you must turn the front wheel by an angle $\Delta\phi$ within a distance d. What torque must you apply to the axel of the wheel to make the turn? Treat the motorcycle wheel as a solid cylinder of radius R and mass m. Ignore friction/drag and assume rolling without slipping.



Example: Motorcycle steering

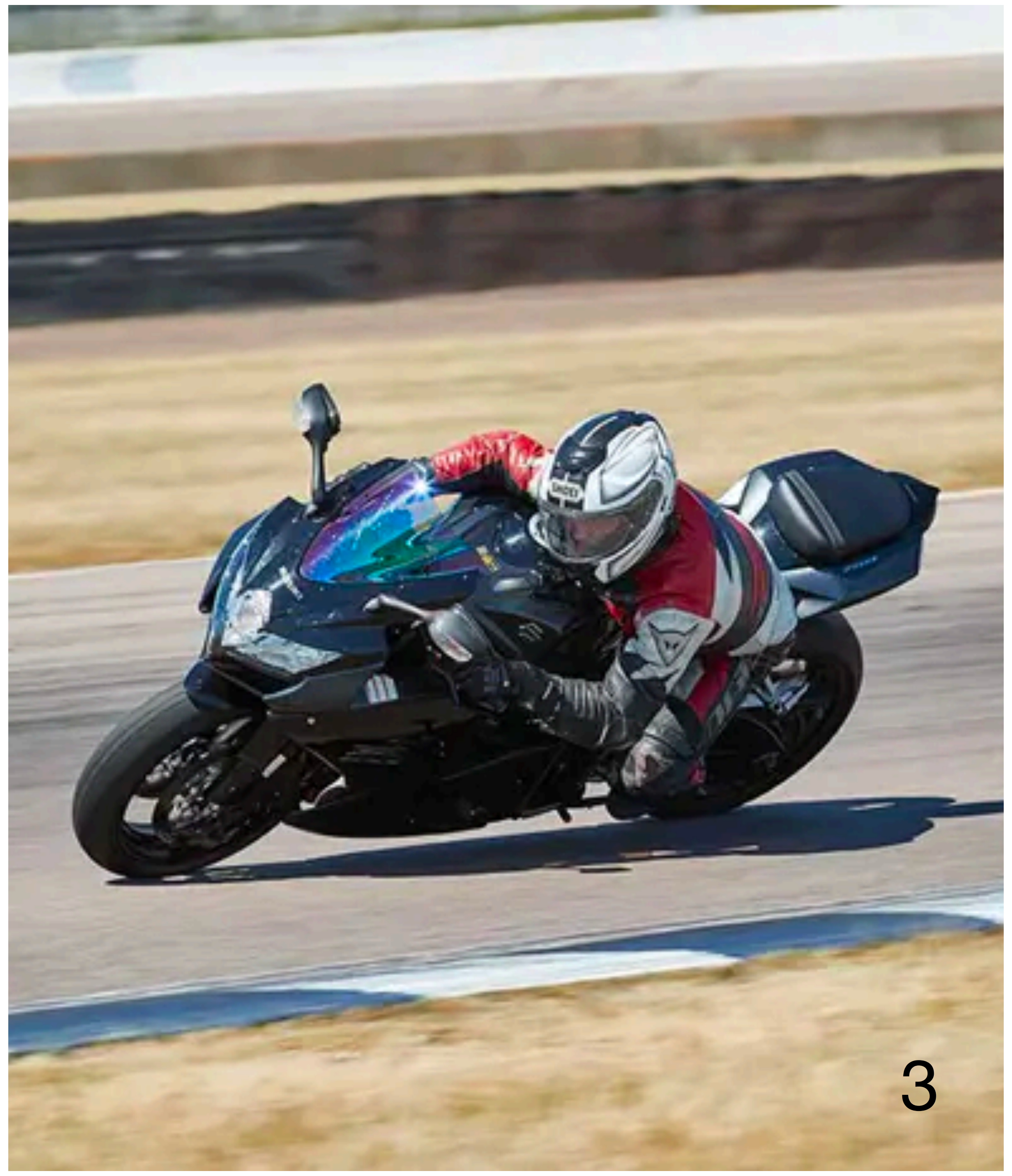
You're racing on motorcycle at a constant ~~velocity~~ speed v and are entering a turn. To stay on the road, you must turn the front wheel by an angle $\Delta\phi$ within a distance d. What torque must you apply to the axel of the wheel to make the turn? Treat the motorcycle wheel as a solid cylinder of radius R and mass m. Ignore friction/drag and assume rolling without slipping.

A. Find an expression for the angular speed of the wheel ω_w.

B. Find an expression for the constant angular speed ω_t with which you need to turn the wheel in order to stay on the road.

C. Find the total angular momentum of the wheel in terms of the moments of inertia, I_w and I_t, for the two types of rotation. (Let the center of the **axel's rotation** be the origin of a cylindrical coordinate system.)

D. Find the torque. How do you create such a torque?

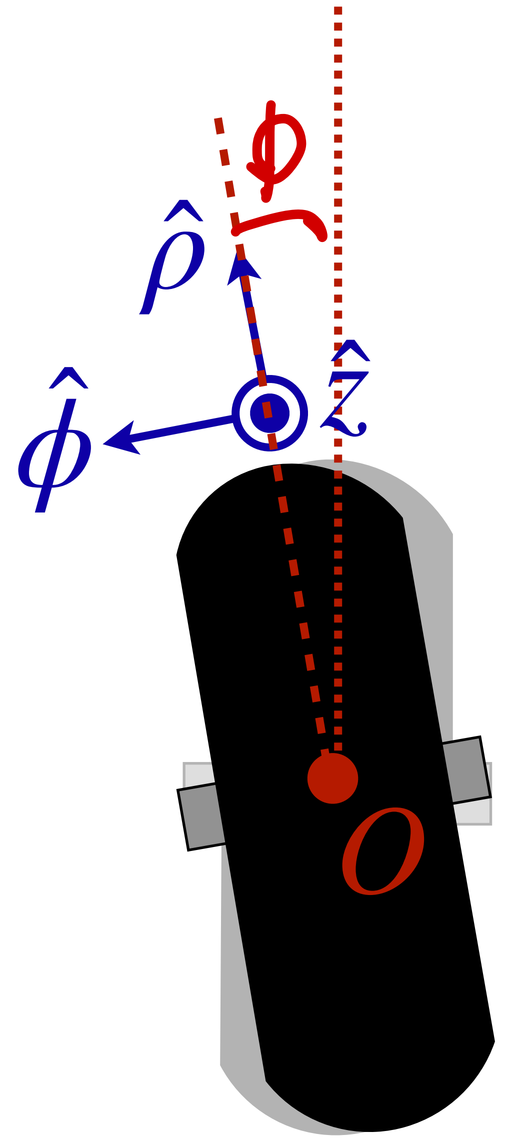


Example: Motorcycle steering

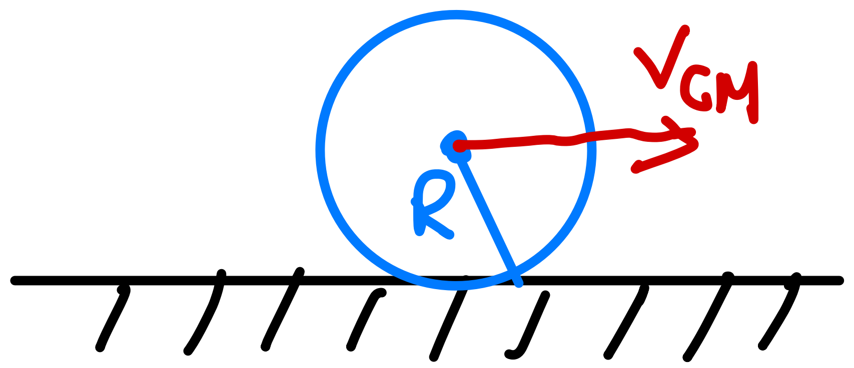
EPFL
Swiss Plasma Center

A. Find an expression for the angular speed of the wheel ω_w.

B. Find an expression for the constant angular speed ω_t with which you need to turn the wheel in order to stay on the road.



A.



No slipping means

$$V = V_{CM} = R\omega_w \Rightarrow V = R\omega_w \Rightarrow \omega_w = \frac{V}{R}$$

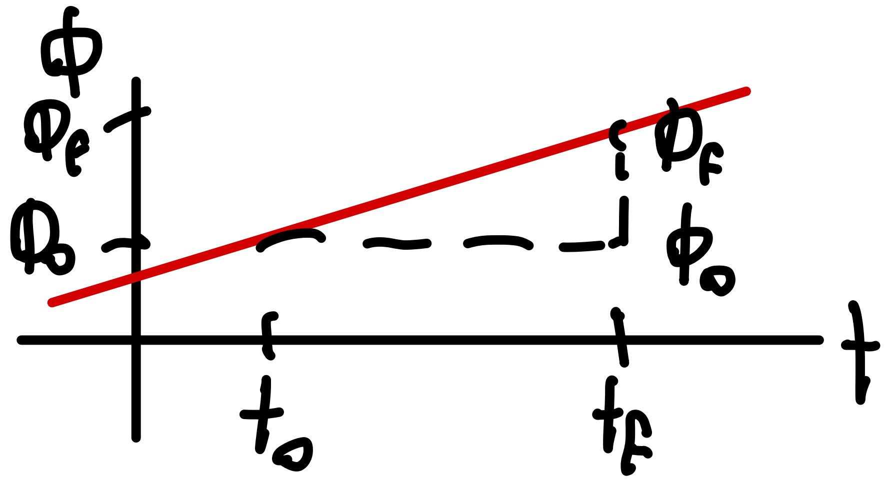
B.

Because ω_t = const

$$\omega_t = \frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t}$$

$$\Delta t = \frac{d}{V}$$

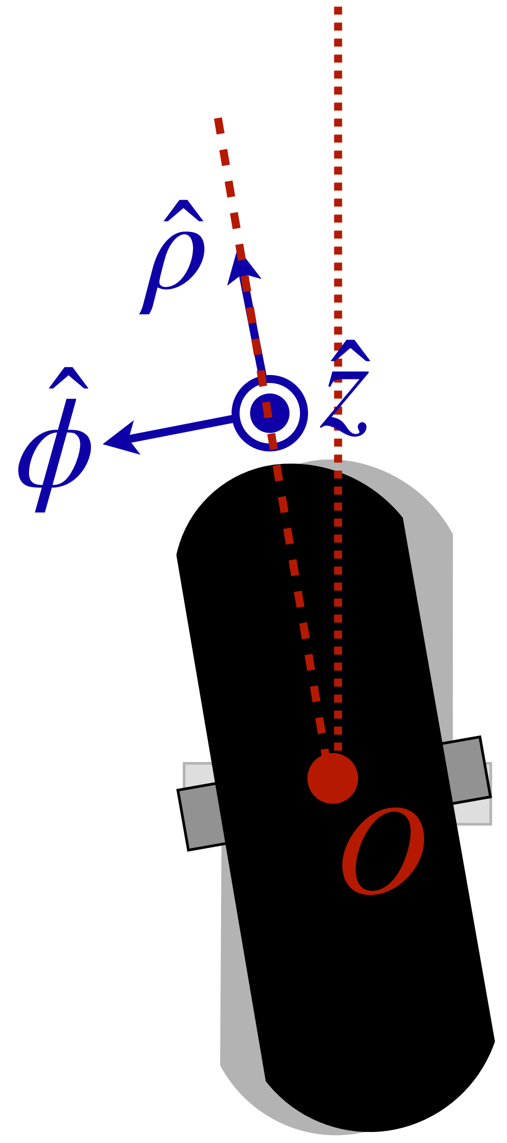
$$\omega_t = \frac{\Delta\phi}{d/V} = \frac{V}{d}\Delta\phi$$



$$\frac{\Delta\phi}{\Delta t} = \frac{\phi_f - \phi_0}{t_f - t_0}$$

Example: Motorcycle steering

C. Find the total angular momentum of the wheel in terms of the moments of inertia, I_w and I_t, for the two types of rotation. (Let the center of the **axel's rotation** be the origin of a cylindrical coordinate system.)



$$\vec{L}_{tot} = \vec{L}_w + \vec{L}_t$$

$$\vec{L}_w = I_w \vec{\omega}_w = I_w \omega_w \hat{\phi}$$

$$\vec{L}_t = I_t \vec{\omega}_t = I_t \omega_t \hat{z}$$

$$\vec{L}_{tot} = I_w \omega_w \hat{\phi} + I_t \omega_t \hat{z}$$

Example: Motorcycle steering

D. Find the torque. How do you create such a torque?

$$\vec{\tau}_{net} = \frac{d\vec{L}_{tot}}{dt}$$

$$= \frac{d}{dt}\left[I_w \omega_w \hat{\phi} + I_f \omega_f \hat{z} \right] = I_w \frac{d}{dt}(\omega_w \hat{\phi}) + I_f \frac{d}{dt}(\omega_f \hat{z})$$

$$= I_w\left(\cancelto{0}{\frac{d\omega_w}{dt}} \hat{\phi} + \omega_w \underbrace{\frac{d\hat{\phi}}{dt}} \right) + I_f\left(\cancelto{0}{\frac{d\omega_f}{dt}} \hat{z} + \omega_f \cancelto{0}{\frac{d\hat{z}}{dt}} \right)$$

$$= I_w \omega_w (-\omega_f \hat{\rho}) = -I_w \omega_w \omega_f \hat{\rho}$$

$$= -\frac{1}{2} m R^2 \left(\frac{v}{R}\right)\left(\frac{v}{d}\Delta\phi\right)\hat{\rho}$$

$$\frac{d\hat{\phi}}{dt} = -\dot{\phi}\hat{\rho} = -\omega_f \hat{\rho} \quad \text{(lecture 4)}$$

$$I_w = \frac{1}{2} m R^2 \quad \text{(lecture 11)}$$

$$\boxed{\vec{\tau}_{net} = -\frac{1}{2} m R \frac{v^2}{d} \Delta\phi \, \hat{\rho}}$$

Conceptual question

An object can execute harmonic motion (i.e. oscillate) about…

A. any point.
B. any equilibrium point.
C. any stable equilibrium point.
D. any point, provided the forces exerted on the object obey Hooke's law.