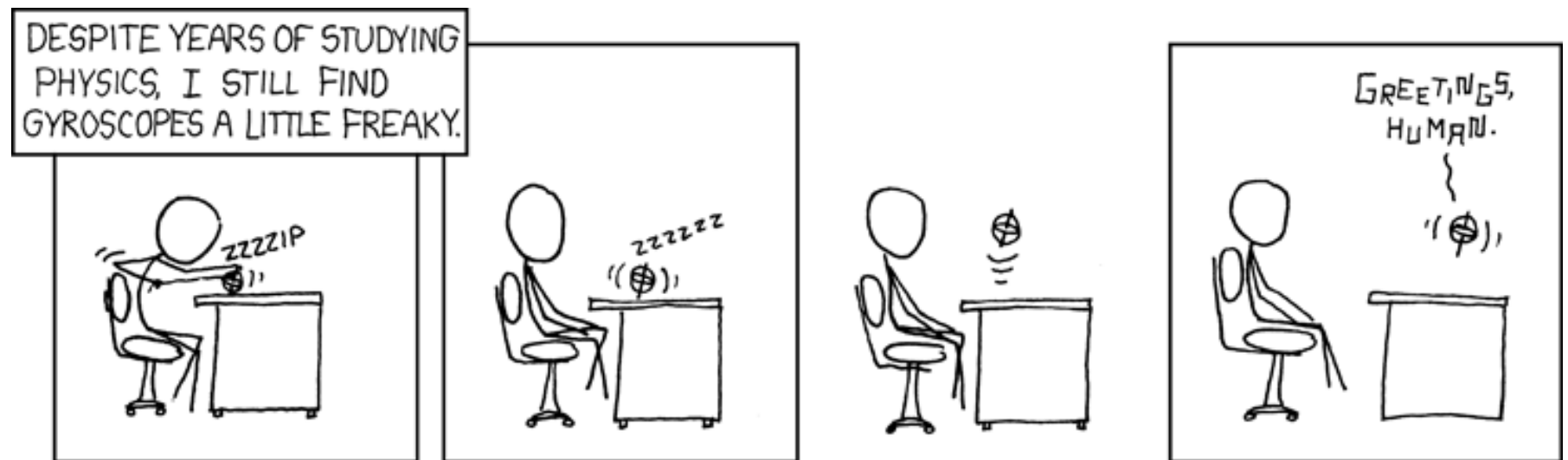


General Physics: Mechanics

PHYS-101(en)

Lecture 13a:
Kepler's laws,
gyroscopes and
harmonic motion



xkcd.com/332

Dr. Marcelo Baquero
marcelo.baquero@epfl.ch
December 9th, 2024

Announcements

- Next Monday (i.e. December 16th) we will start the lecture with written course feedback

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- Final exams of some previous years will be made available in the Moodle for you to see and practice

Today's agenda (Serway 11,13; MIT 22,23)

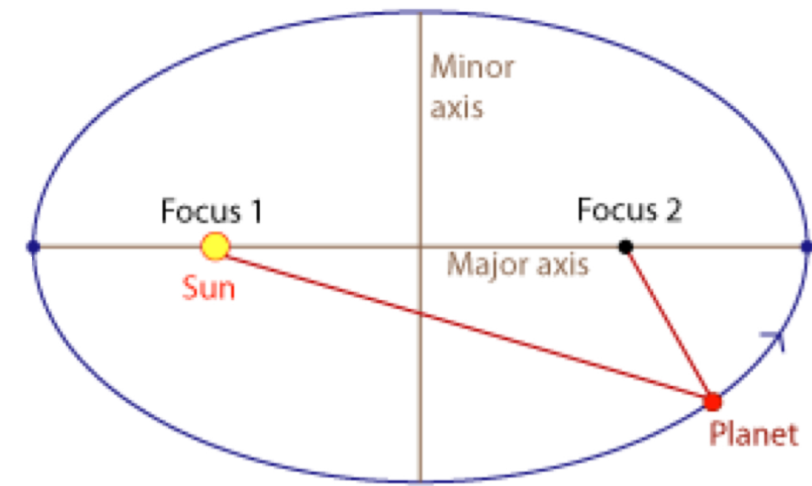
1. **Kepler's laws of planetary motion**
2. Gyroscopes
3. Harmonic motion
 - Simple harmonic motion

Kepler's laws of planetary motion

- From 1610-1619 Johannes Kepler wrote:

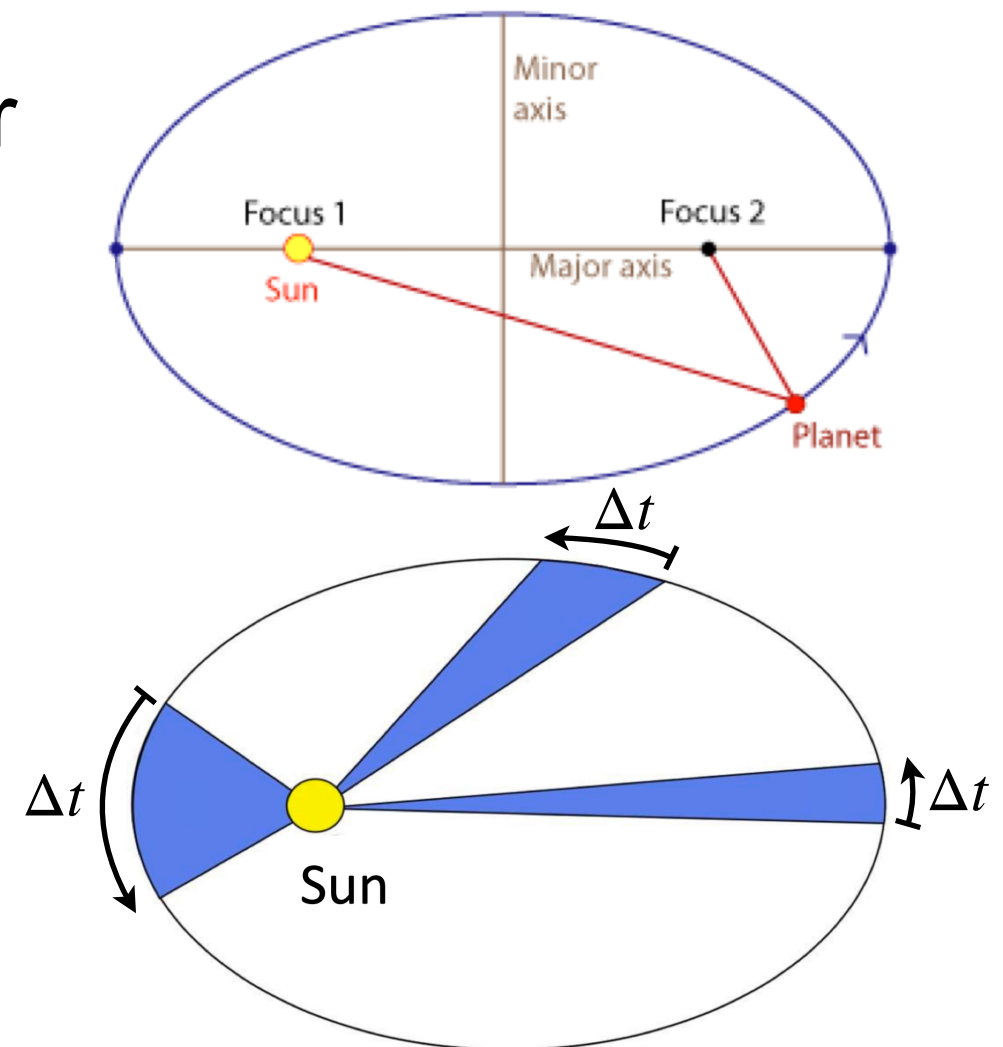
Kepler's laws of planetary motion

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 1. The orbit of each planet is an ellipse, with the Sun at one focus.



Kepler's laws of planetary motion

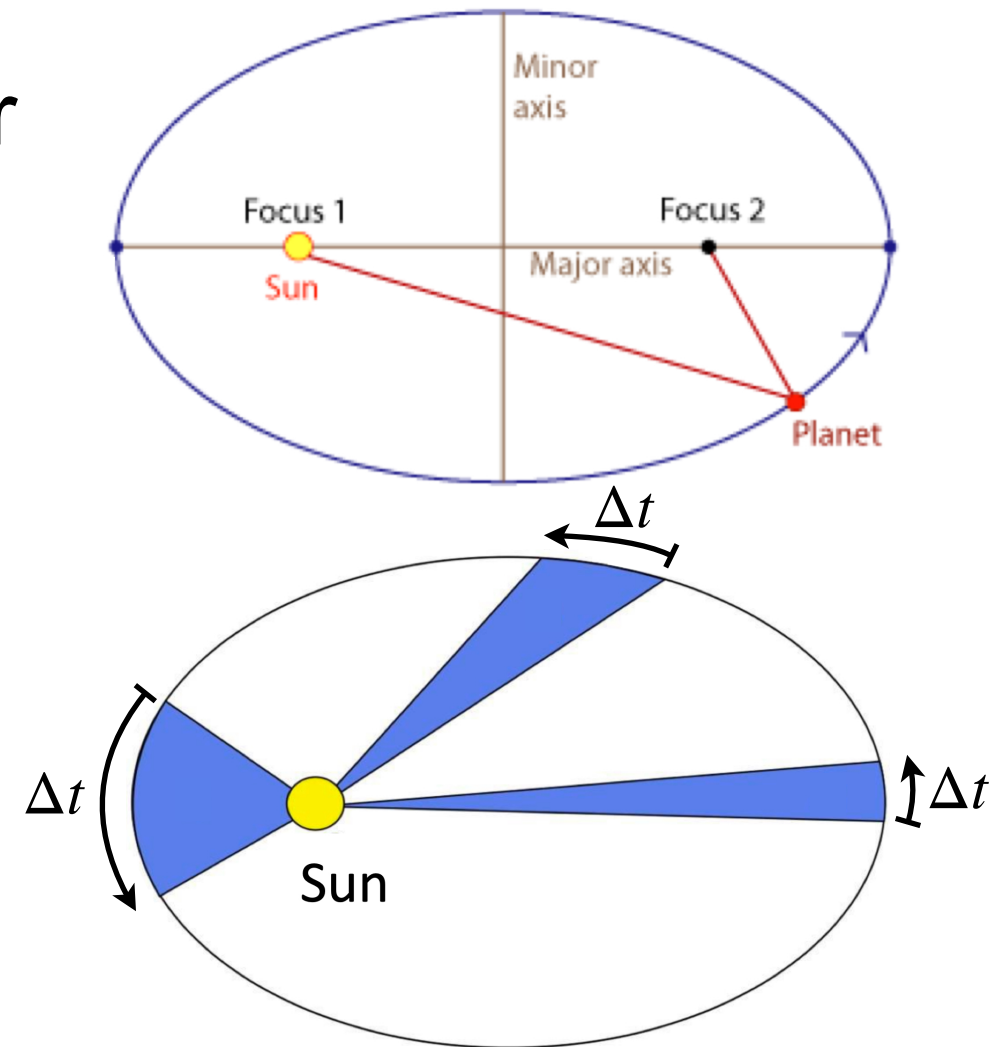
- From 1610-1619 Johannes Kepler wrote:
 - The orbit of each planet is an ellipse, with the Sun at one focus.
 - An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.



Kepler's laws of planetary motion

- From 1610-1619 Johannes Kepler wrote:

1. The orbit of each planet is an ellipse, with the Sun at one focus.
2. An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.
3. The square of a planet's orbital period is proportional to the cube of its mean distance from the Sun.



Planet	Period, T (Earth year)	Avg distance to Sun, r (10^6 km)	T^2/r^3 (10^{-25} yr ² /km ³)
Mercury	0.241	57.9	2.99
Venus	0.615	108.2	2.99
Earth	1	149.6	2.99
Mars	1.88	227.9	2.99
Jupiter	11.86	778.3	2.98
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Kepler's laws of planetary motion

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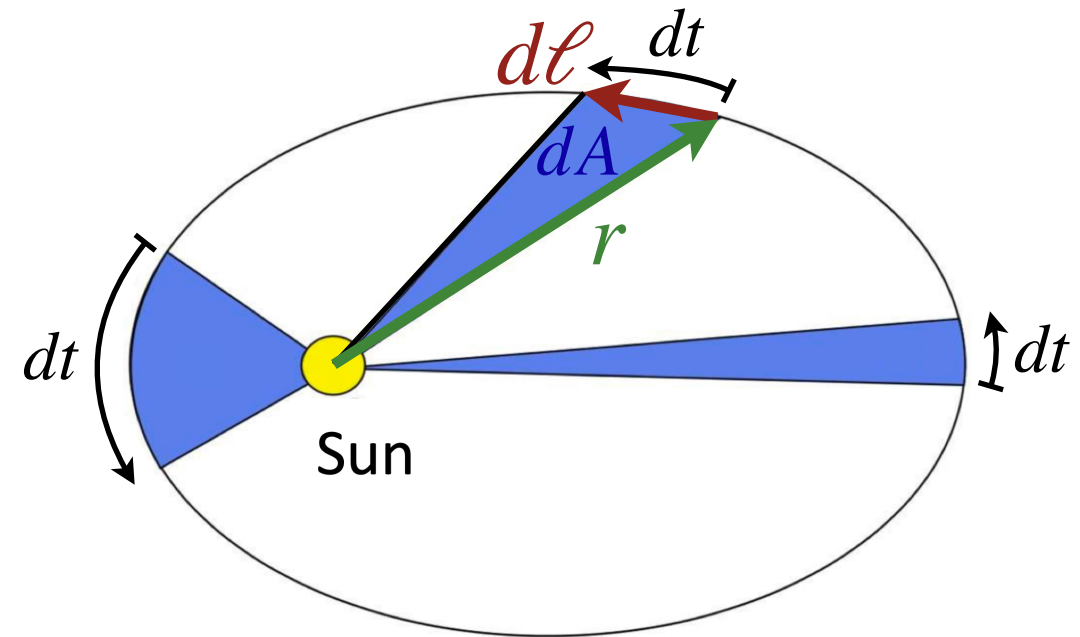
- Approximate orbits as circular and use

$$\vec{F}_G = -G \frac{m_p m_s}{r^2} \hat{r}$$

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Kepler's laws of planetary motion

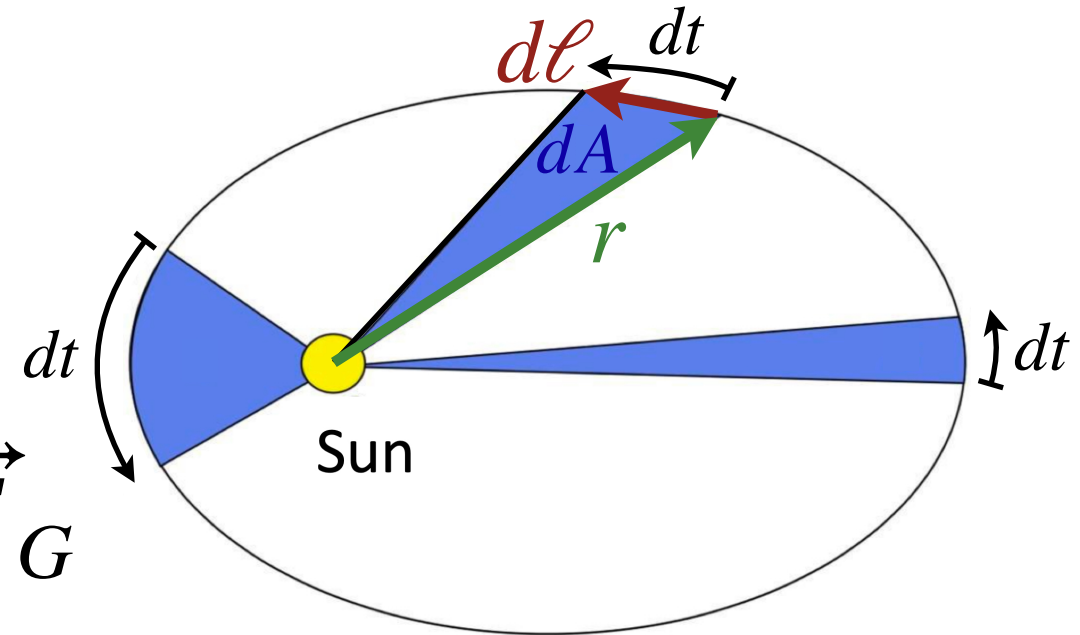
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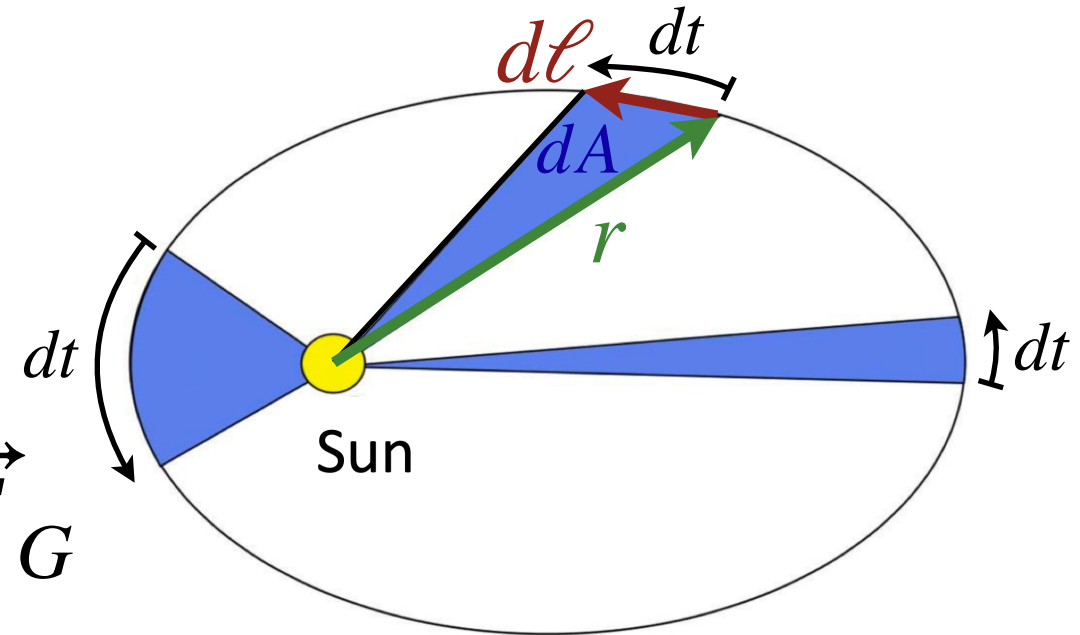
- Gravity is a central force, so $\vec{r} \parallel \vec{F}_G$



Kepler's laws of planetary motion

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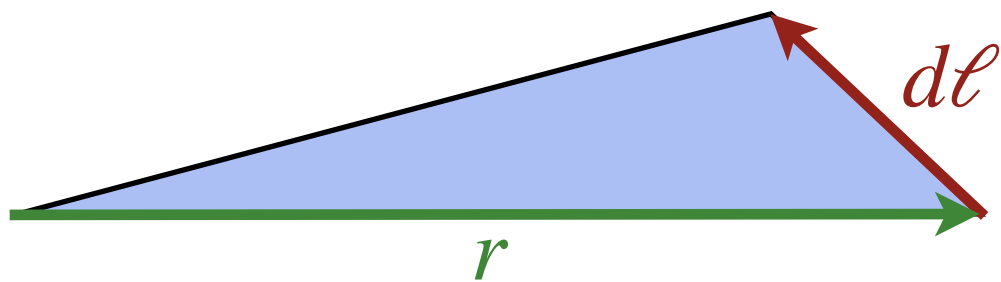
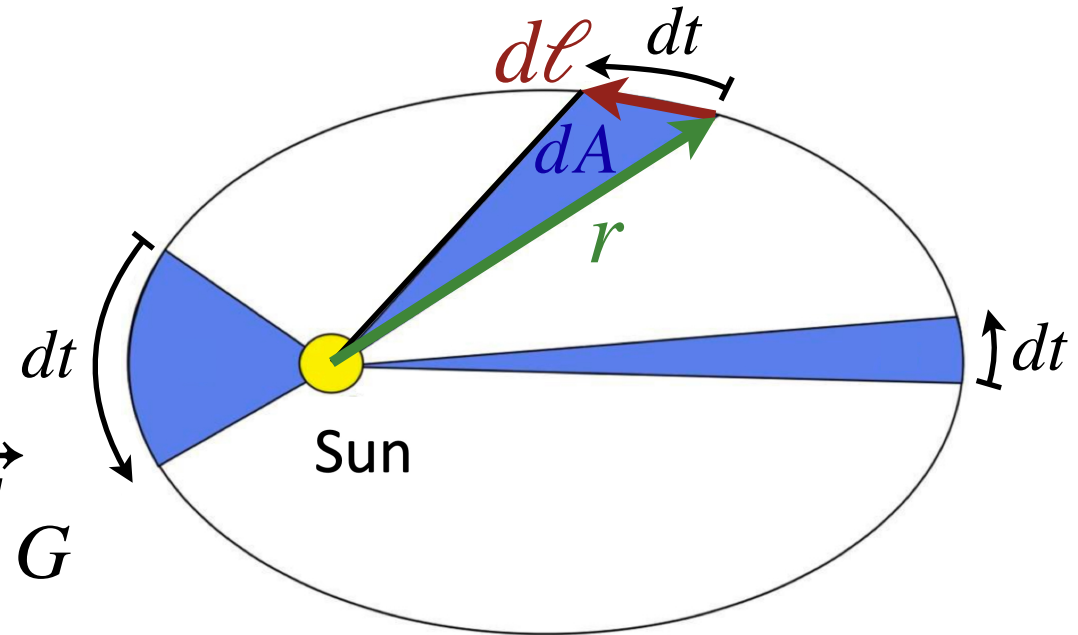
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- Thus, $\vec{\tau}_G = \vec{r} \times \vec{F}_G = 0$, so $\vec{L}_S = \vec{r} \times m_p \vec{v}$ is conserved



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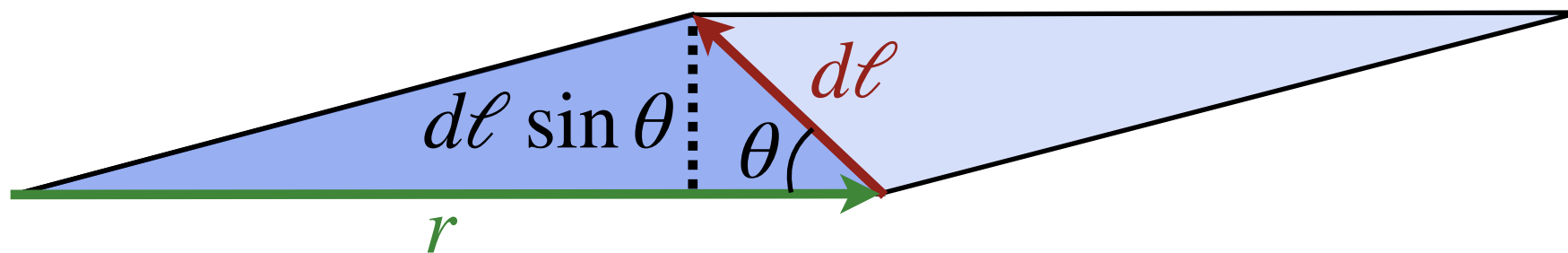
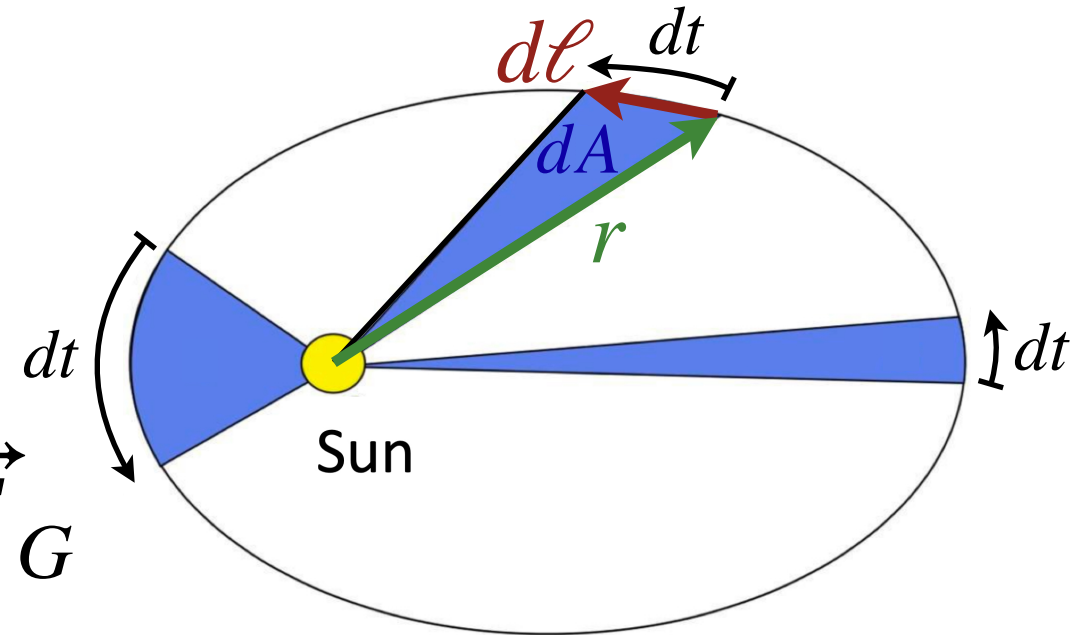
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- How is this related to area?



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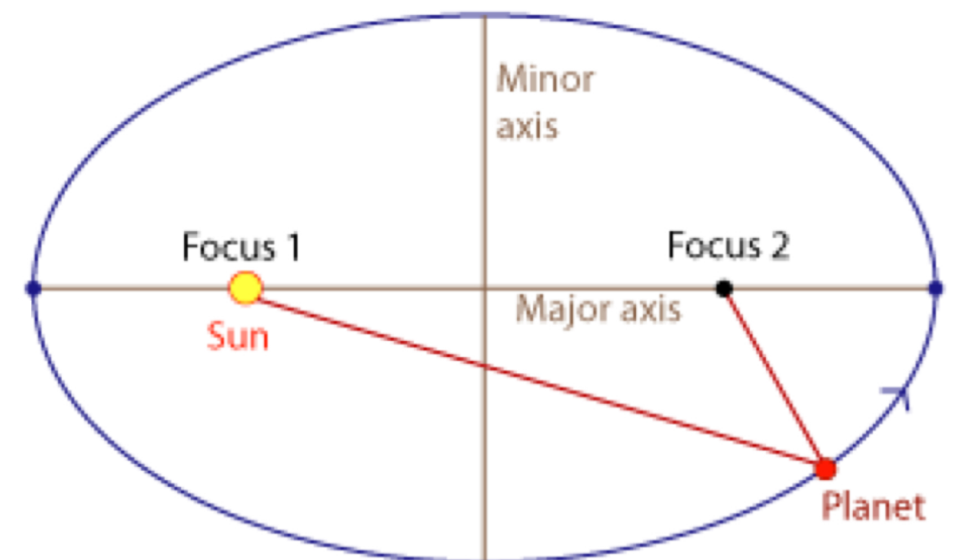
Kepler's laws of planetary motion

1. The orbit of each planet is an ellipse, with the Sun at one focus.

- Need to know the universal gravitational potential energy

$$U_G = -G \frac{m_p m_s}{r}$$

- Apply mechanical energy conservation:
- Apply conservation of angular momentum:
- Considerable mathematical magic



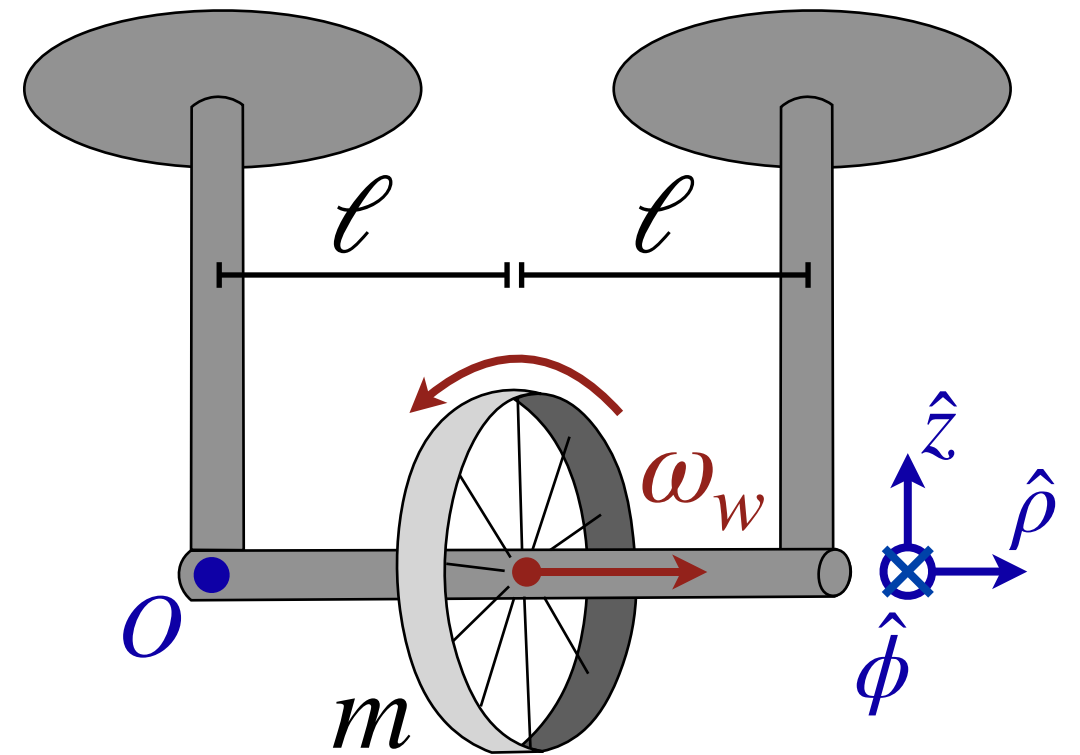
Today's agenda (Serway 11,13; MIT 22,23)

1. Kepler's laws of planetary motion
- 2. Gyroscopes**
3. Harmonic motion
 - Simple harmonic motion

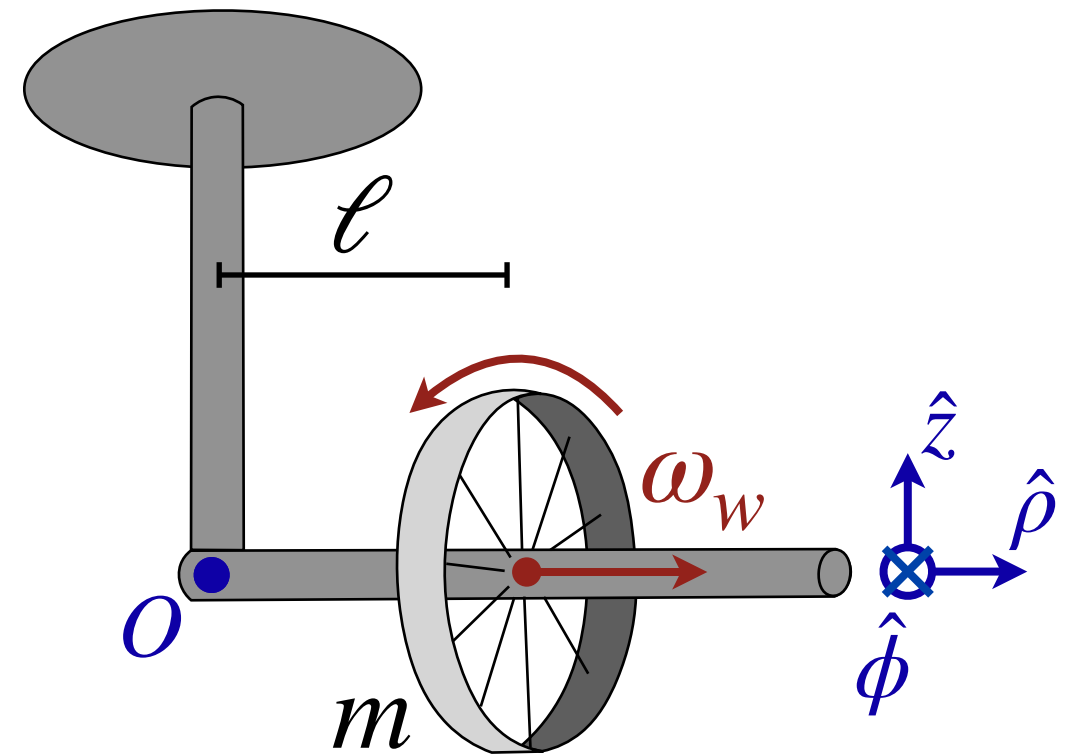
DEMO (50, 48)

Bicycle wheel

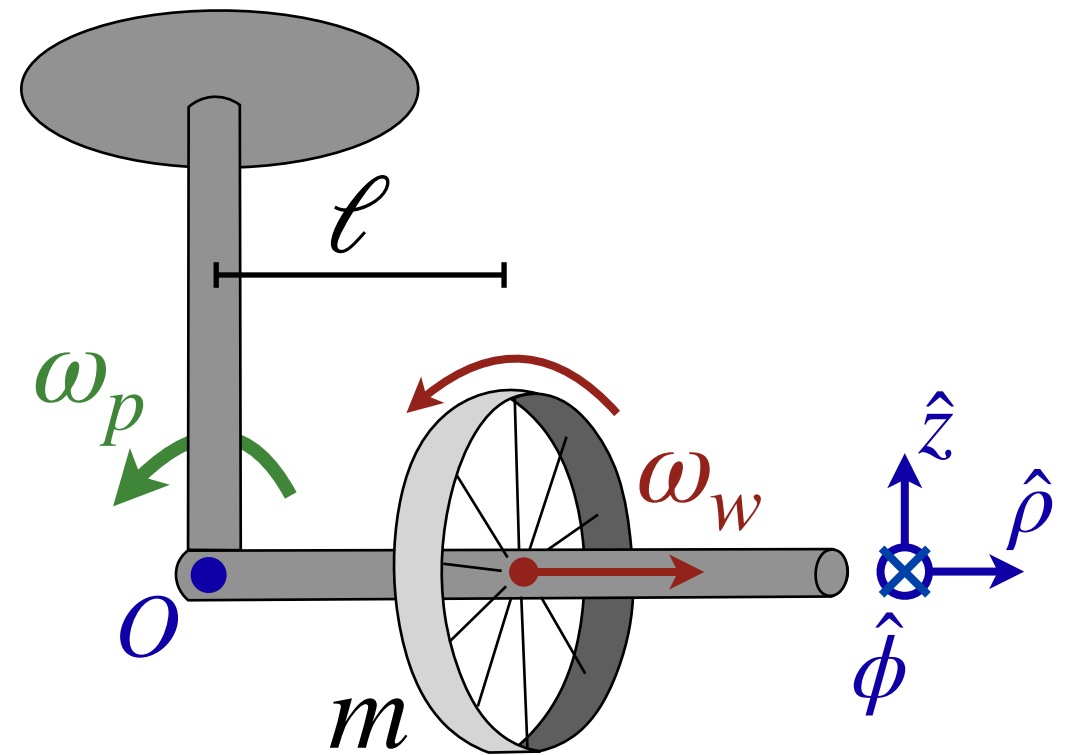
Analyzing fixed axis rotation



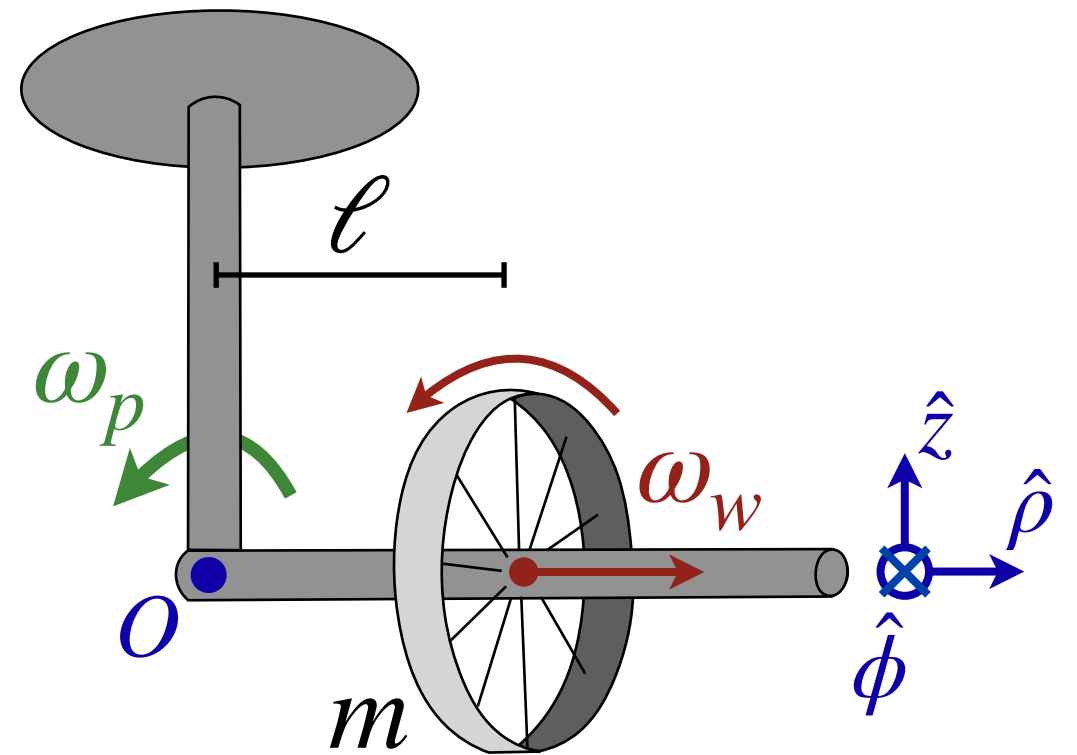
DEMO (50): A gyroscope



Analyzing a gyroscope



Analyzing a gyroscope



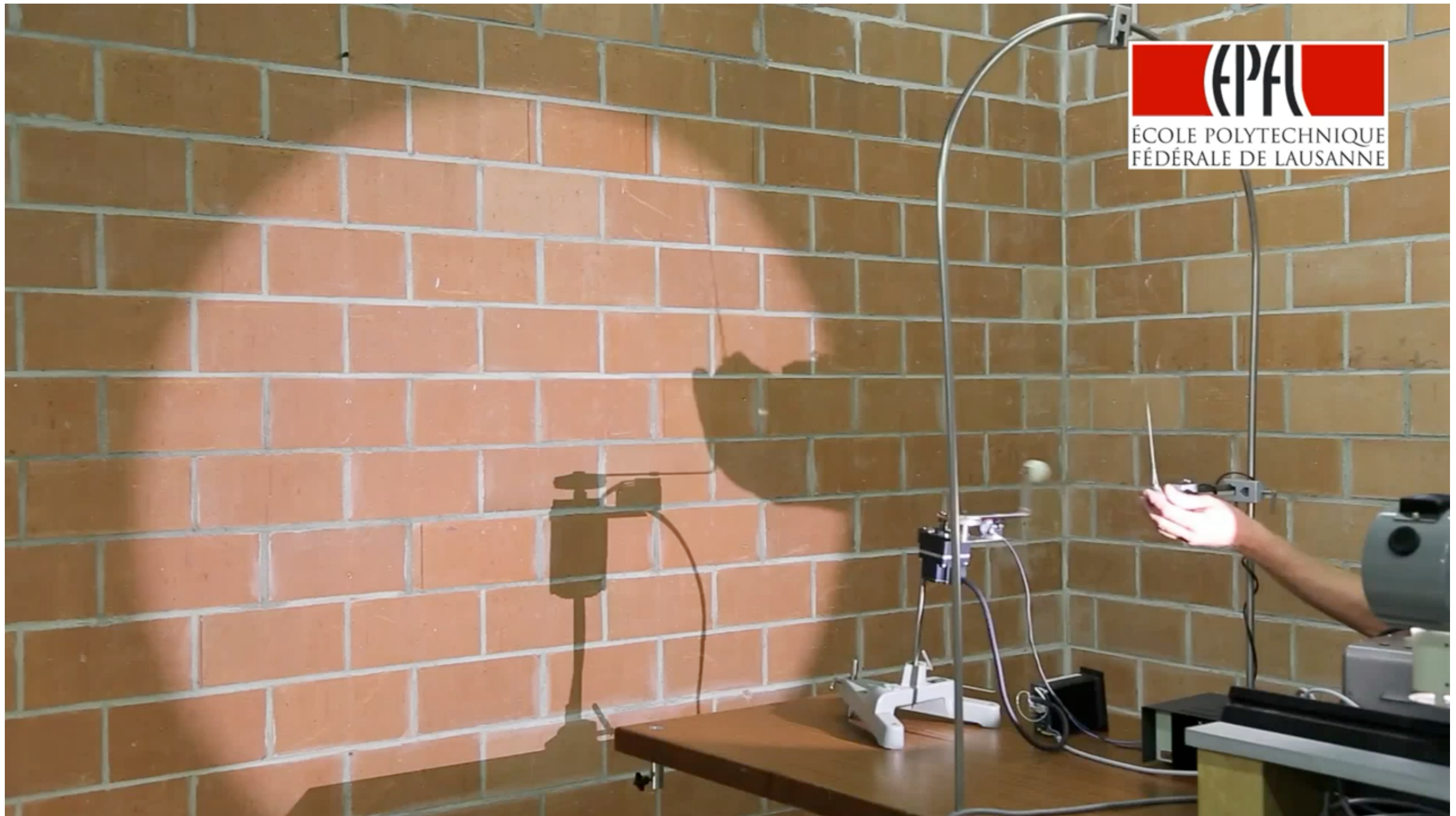
DEMO (501, 40)

Gyroscopes

Today's agenda (Serway 11,13; MIT 22,23)

1. Kepler's laws of planetary motion
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 - Simple harmonic motion

DEMO (190): Harmonic motion is like 1D circular motion



Harmonic motion

- Special type of periodic motion caused by forces of the form

$$\vec{F} = -k \Delta \vec{r}$$

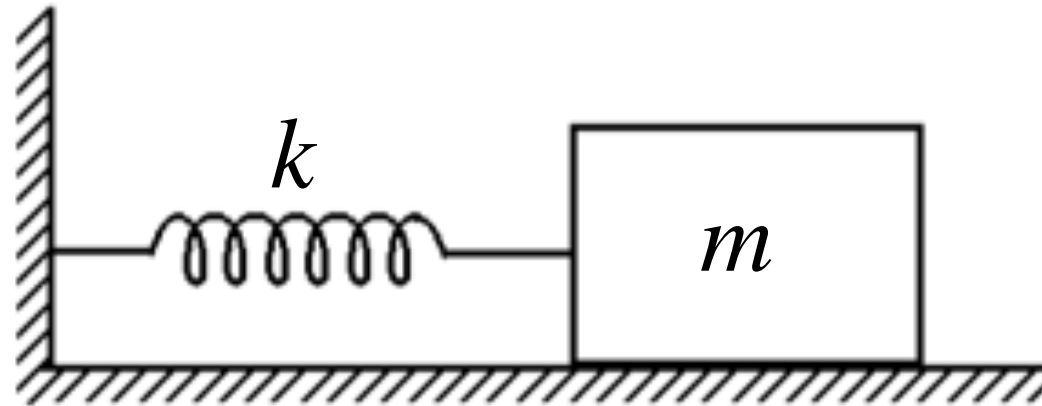
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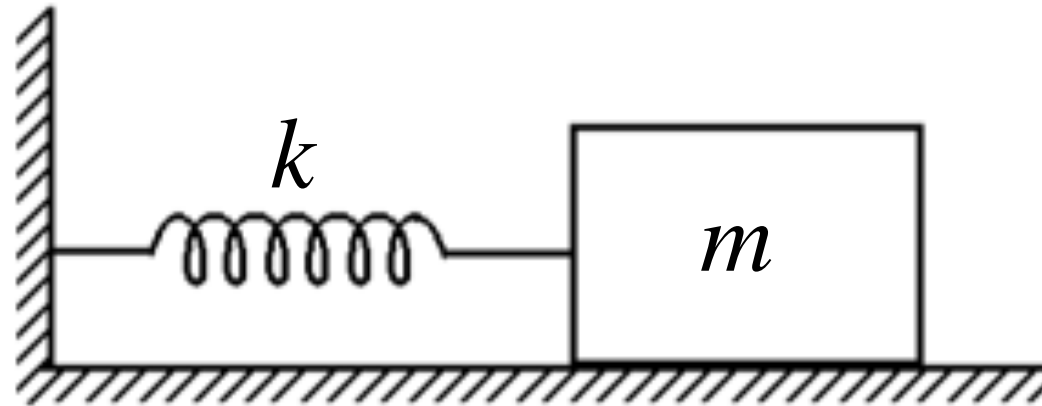
- This has the same form as the *spring force*, which can represent many systems
 - e.g. atoms in crystals, pendulums, balls rolling in bowls

“Simple” harmonic oscillation in 1D



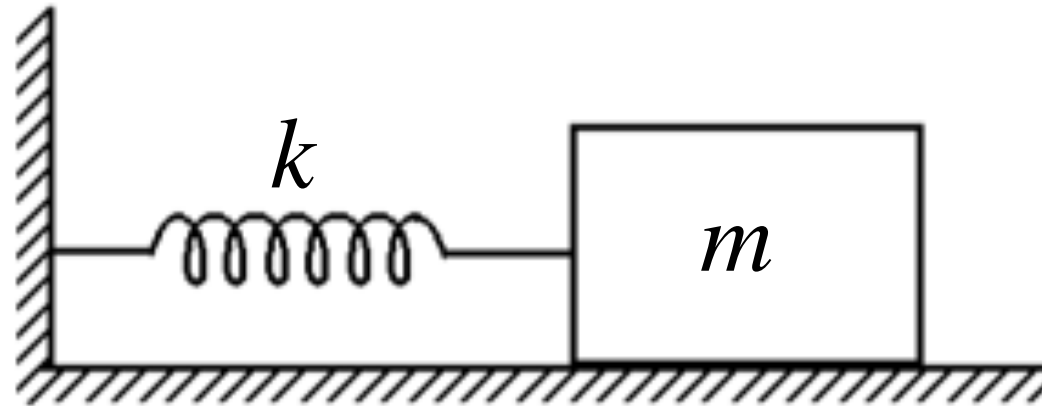
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“Simple” harmonic oscillation in 1D



- Consider a frictionless mass-spring system
- The spring has equilibrium position $x_0 = 0$

“Simple” harmonic oscillation in 1D



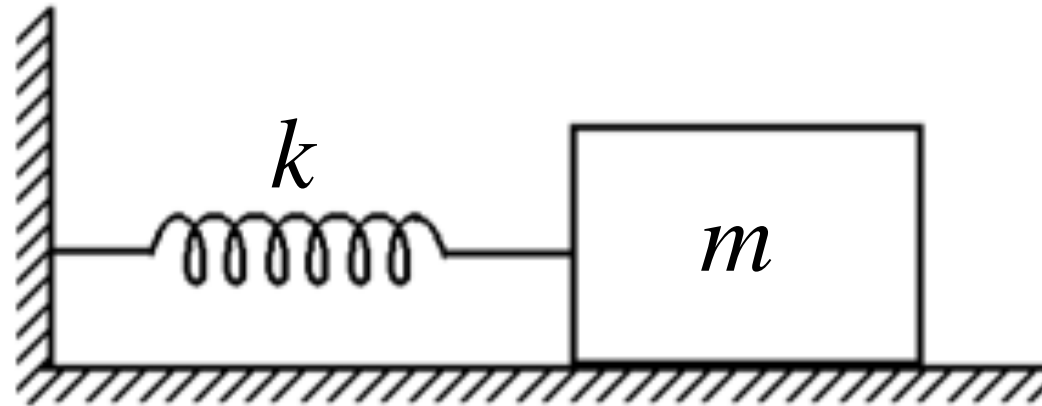
- Consider a frictionless mass-spring system
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$$F = -kx$$

- Newton's 2nd law, $F = ma$, then yields

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which leads to the differential equation

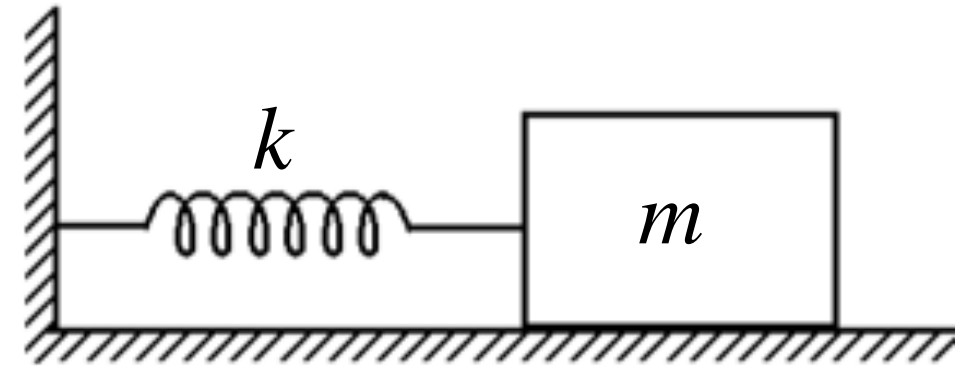
$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

“Simple” harmonic oscillation in 1D

- The solution of

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

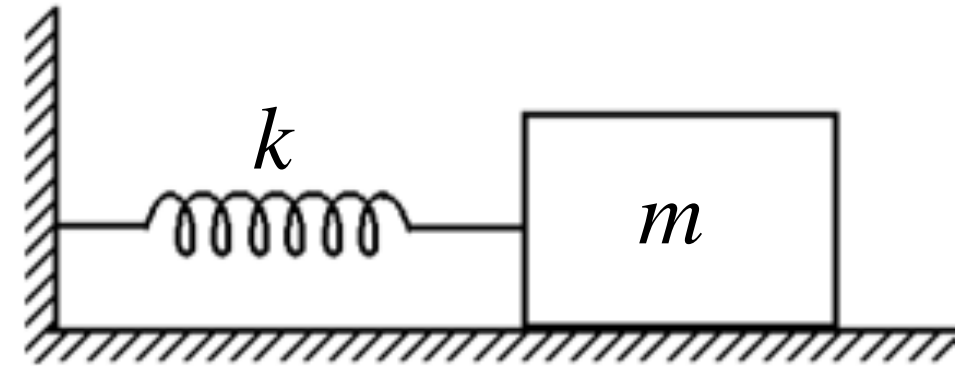
is $x(t) = A_0 \cos(\omega_0 t) + B_0 \sin(\omega_0 t)$



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is $x(t) = A_0 \cos(\omega_0 t) + B_0 \sin(\omega_0 t)$

- Using trigonometric identities, this is equivalent to

$$x(t) = A \cos(\omega_0 t + \varphi)$$

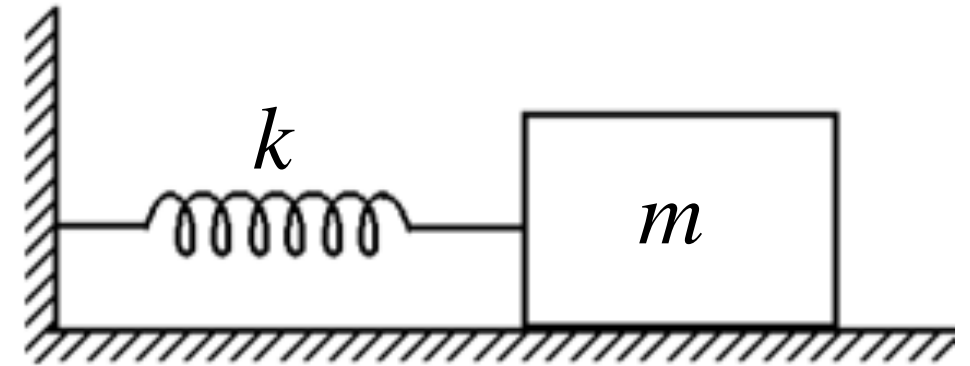
where

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“Simple” harmonic oscillation in 1D

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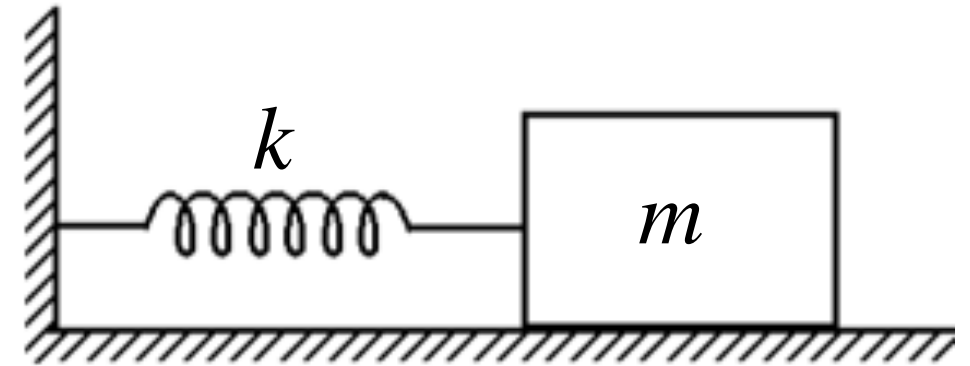
where

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“Simple” harmonic oscillation in 1D

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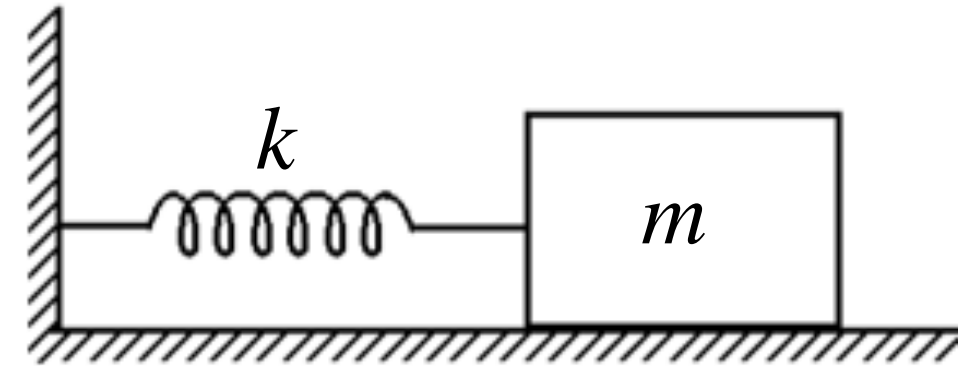
where

- A is the amplitude of the oscillation
- φ is the initial phase
- $\omega_0 = \sqrt{k/m} = 2\pi/T_0$ where T_0 is the period of the oscillation. ω_0 is called the *angular frequency*.

“Simple” harmonic oscillation in 1D

- The solution of

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$



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Kinetic energy of oscillation

- Kinetic energy is still $K = \frac{m}{2}v^2$

Total energy of oscillation

- Potential energy is still

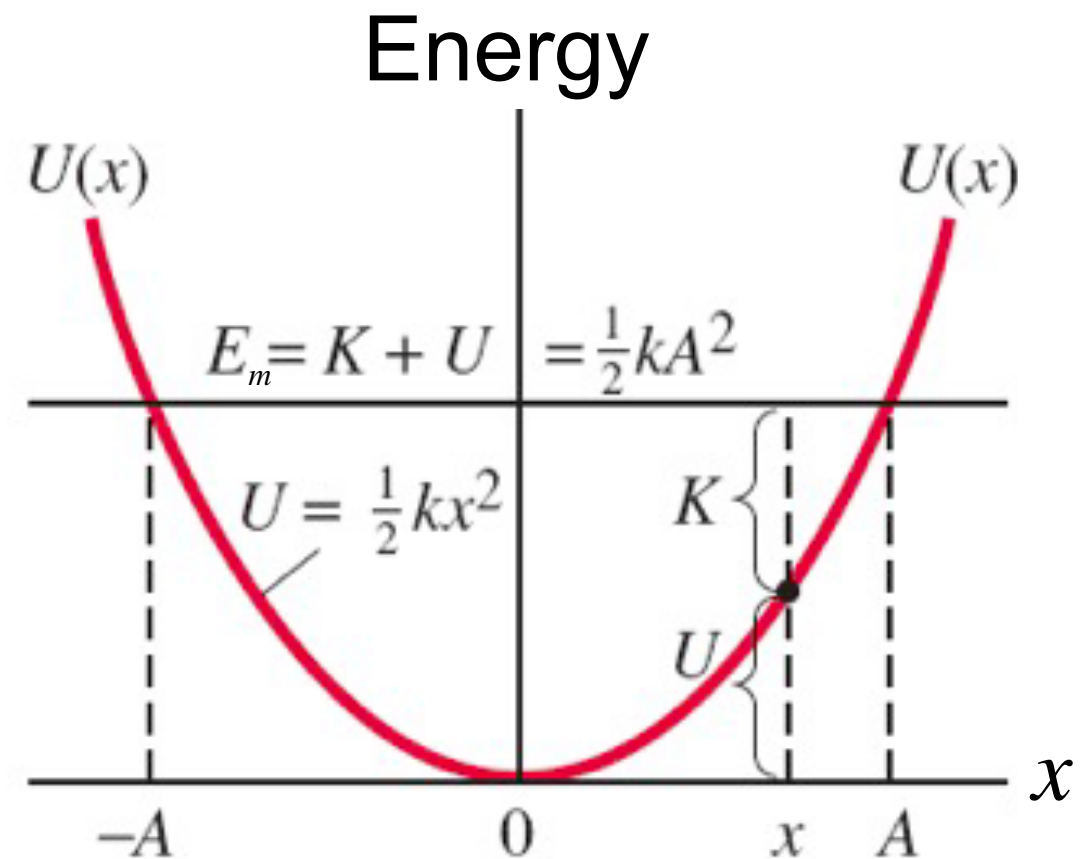
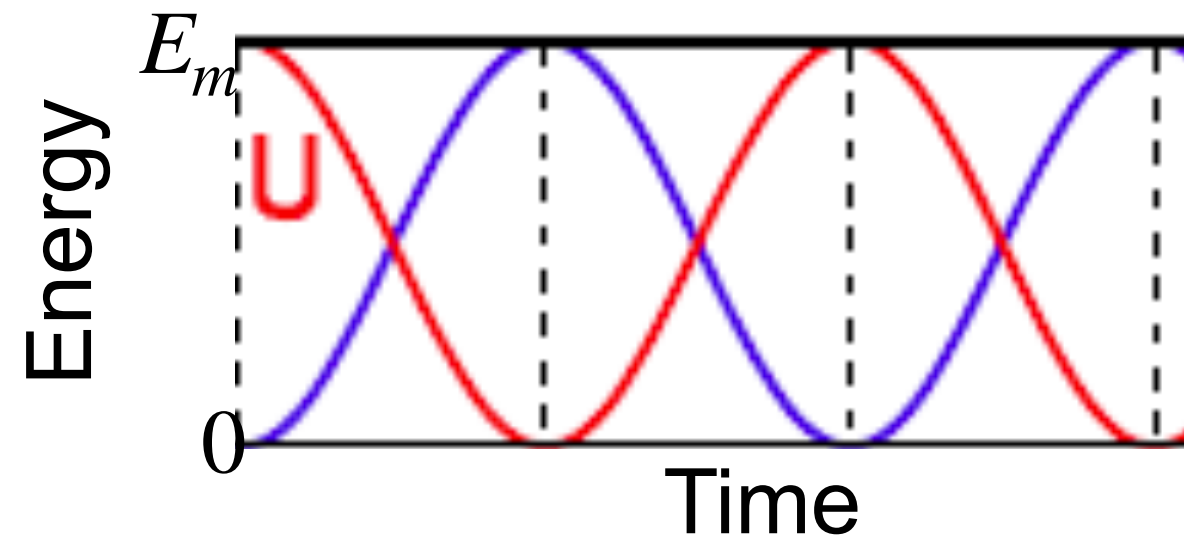
$$U = \frac{k}{2} x^2$$

Total energy of oscillation

- Potential energy is still

$$U = \frac{k}{2} x^2$$

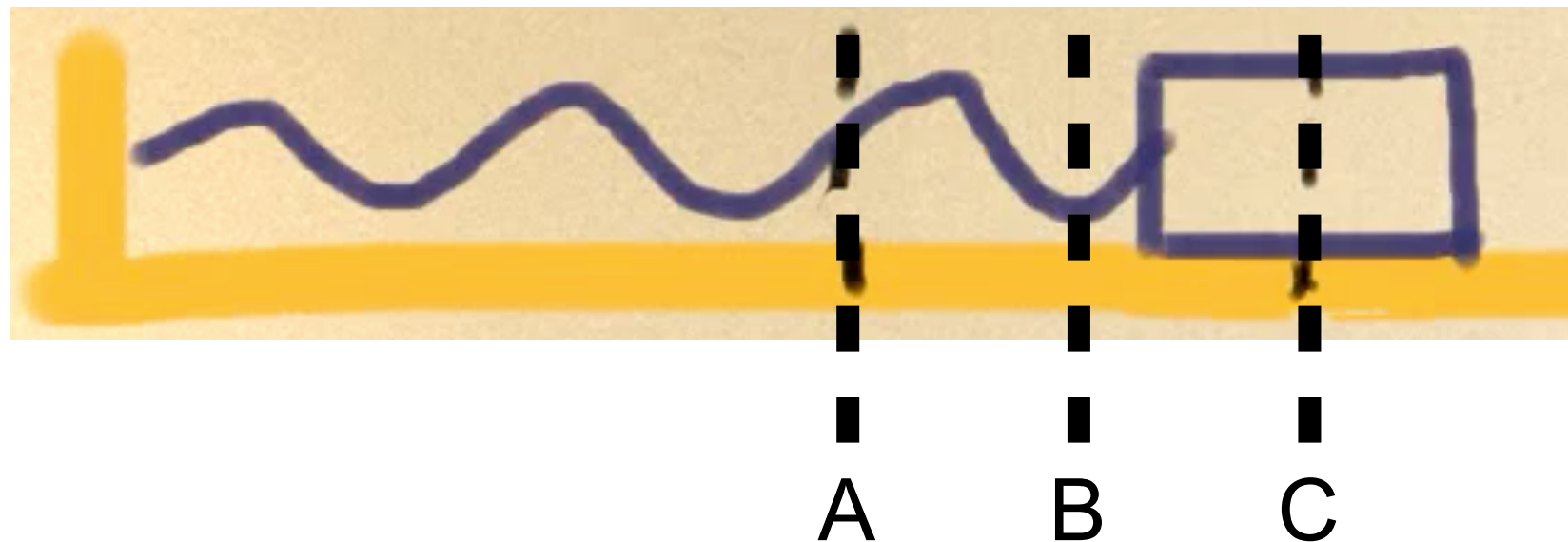
- Total energy is $E_m = K + U$



Conceptual question

A mass is oscillating back and forth on a spring about point A as shown. Point A is the equilibrium (unstretched) position of the mass. At which position is the magnitude of its acceleration the largest?

- A. Point A
- B. Point B
- C. Point C



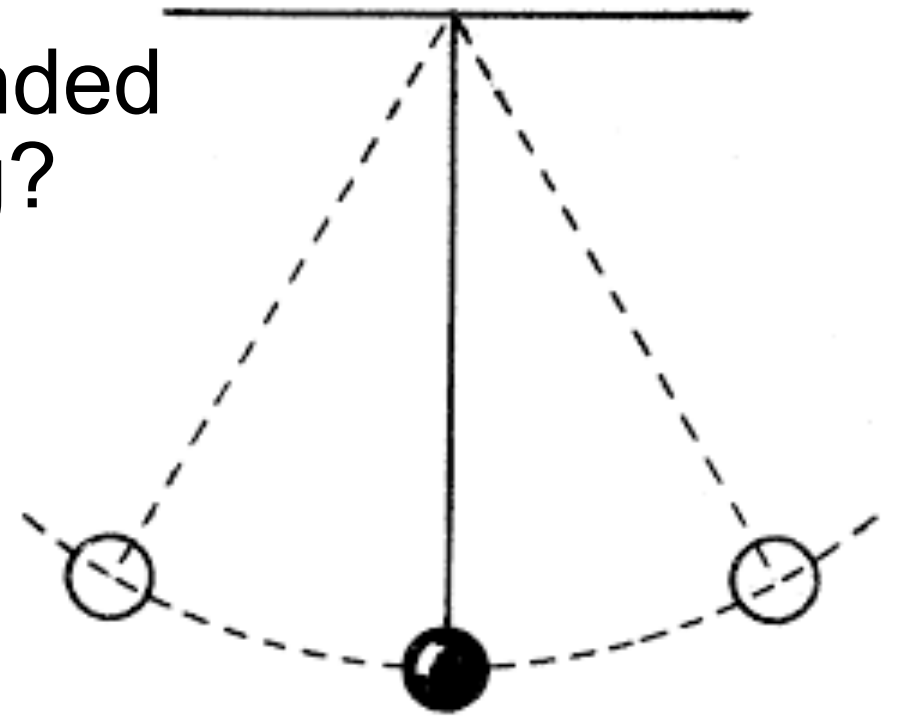
Conceptual question

An object can execute harmonic motion (i.e. oscillate) about...

- A. any point.
- B. any equilibrium point.
- C. any stable equilibrium point.
- D. any point, provided the forces exerted on the object obey Hooke's law.

Aside: Oscillation of a simple pendulum

- What is the motion of a mass suspended from a weightless, inextensible string?



Mass and frequency of a pendulum

See you tomorrow!

