EPFL Swiss Plasma Center

General Physics: Mechanics

PHYS-101(en)

Lecture 11b: Rotational motion and static equilibrium

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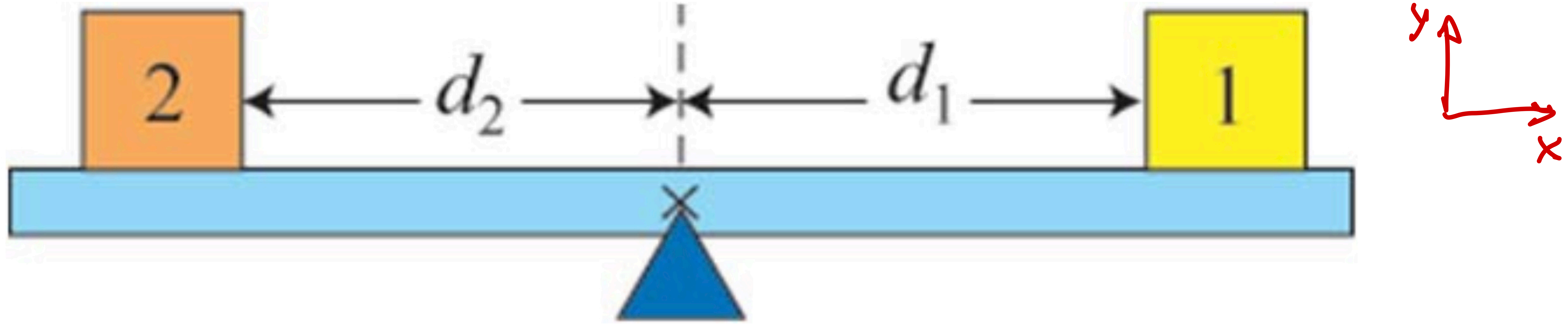
Example: Balance beam



A uniform rigid beam of mass m_B is balanced on a pivot under the center of mass of the beam. We place two point-like objects 1 and 2 of masses m_1 and m_2 on the beam, at distances d_1 and d_2 respectively from the pivot. The beam is in static equilibrium.

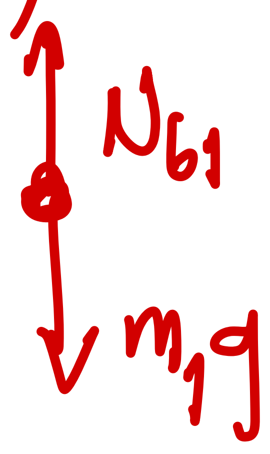
A. What is the magnitude of the force exerted on the pivot point?
B. What is the relationship between d_1 and d_2 for static equilibrium?

$\Sigma \vec{F} = 0$ and $\Sigma \vec{\tau} = 0$

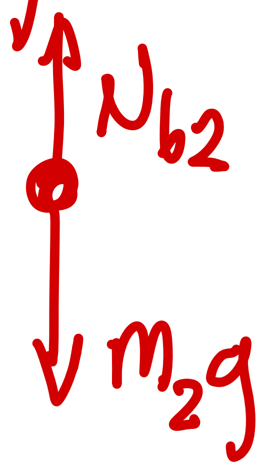
Example: Balance beam

A.

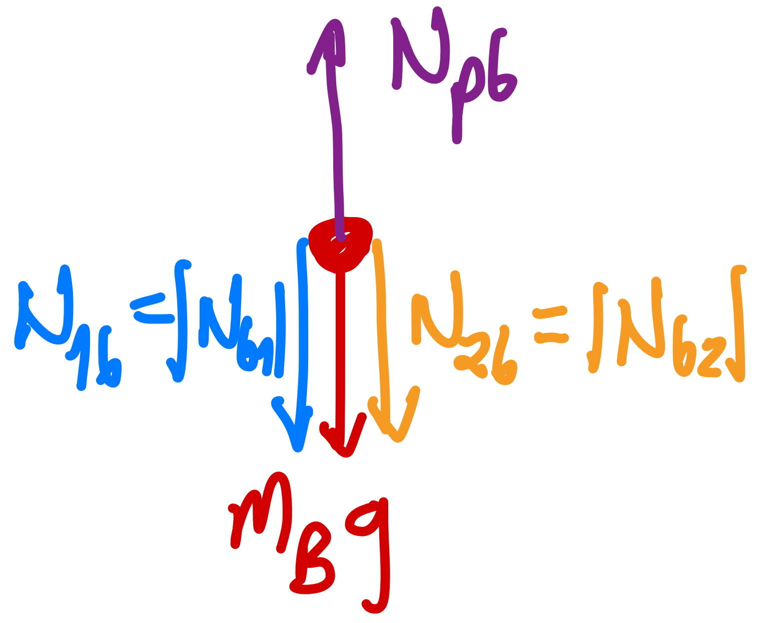
Body 1:



Body 2:



Beam:



Body 1 is not moving,

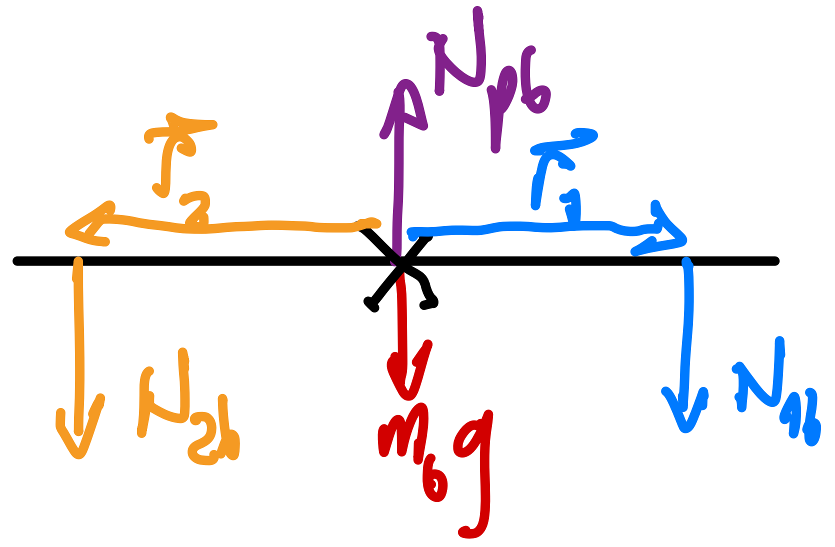
$$\Sigma F: \; N_{b1} - m_1 g = 0 \Rightarrow N_{b1} = m_1 g$$

Same for 2: $N_{b2} = m_2 g$

For beam: $N_{pb} - N_{1b} - N_{2b} - m_B g = 0$

$$N_{pb} = N_{b1} + N_{b2} + m_B g$$
$$= (m_1 + m_2 + m_B) g$$

B.

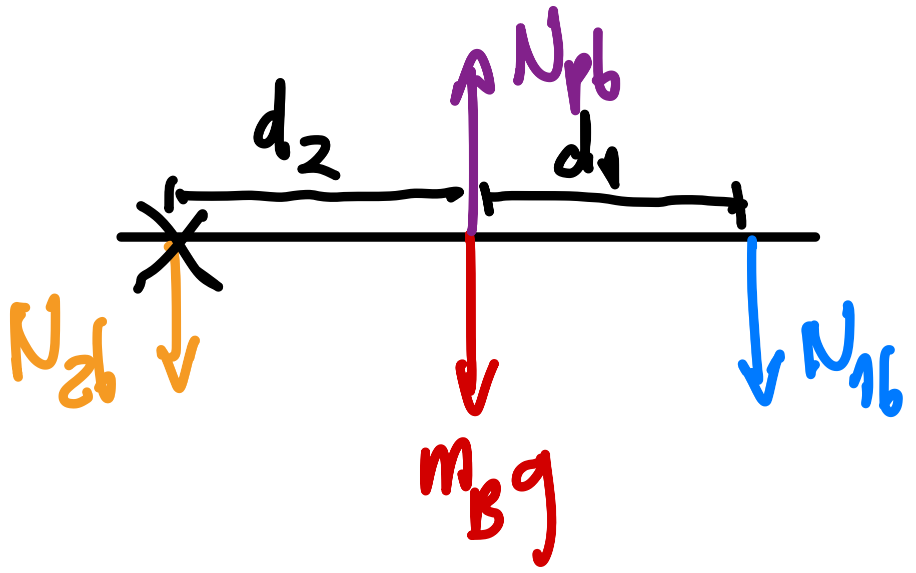


$$\vec{\tau}_1 = \vec{r}_1 \times \vec{N}_{1b} = (d_1 \hat{x}) \times (-m_1 g \hat{y}) = -m_1 d_1 g \, \hat{x} \times \hat{y} = -m_1 d_1 g \, \hat{z}$$
$$\vec{\tau}_2 = \vec{r}_2 \times \vec{N}_{2b} = (\cancel{-} d_2 \hat{x}) \times (\cancel{-} m_2 g \hat{y}) = m_2 d_2 g \, \hat{z}$$
$$\vec{\tau}_p = 0 \times (\vec{N}_{pb} - m_B g \hat{y}) = 0$$

Stat. eq.

$$0 = \sum \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_p = -m_1 d_1 g \hat{z} + m_2 d_2 g \hat{z} + 0 = (-m_1 d_1 + m_2 d_2) g \hat{z} \Rightarrow \boxed{m_1 d_1 = m_2 d_2}$$

Example: Balance beam



For this new choice of pivot point,

$$\vec{\tau}_1 = (d_1+d_2)\hat{x} \times \vec{N}_{1b} = (d_1+d_2)\hat{x} \times (-m_1 g \hat{y}) = -m_1(d_1+d_2) g \hat{z}$$

$$\vec{\tau}_2 = 0 \times \vec{N}_{2b} = 0$$

$$\vec{\tau}_p = d_2 \hat{x} \times (\vec{N}_{pb} - m_B g \hat{y}) = d_2 \hat{x} \times \vec{N}_{pb} - m_B d_2 g \hat{z}$$

$$= d_2 \hat{x} \times [(m_1+m_2+m_B) g \hat{y}] - m_B d_2 g \hat{z}$$

$$= m_1 d_2 g \hat{z} + m_2 d_2 g \hat{z} + \cancel{m_B d_2 g \hat{z}} - \cancel{m_B d_2 g \hat{z}} = (m_1+m_2) d_2 g \hat{z}$$

For stat. eq.: $\sum \vec{\tau} = 0 = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_p = -m_1(d_1+\cancel{d_2}) g \hat{z} + 0 + (\cancel{m_1}+m_2) d_2 g \hat{z}$

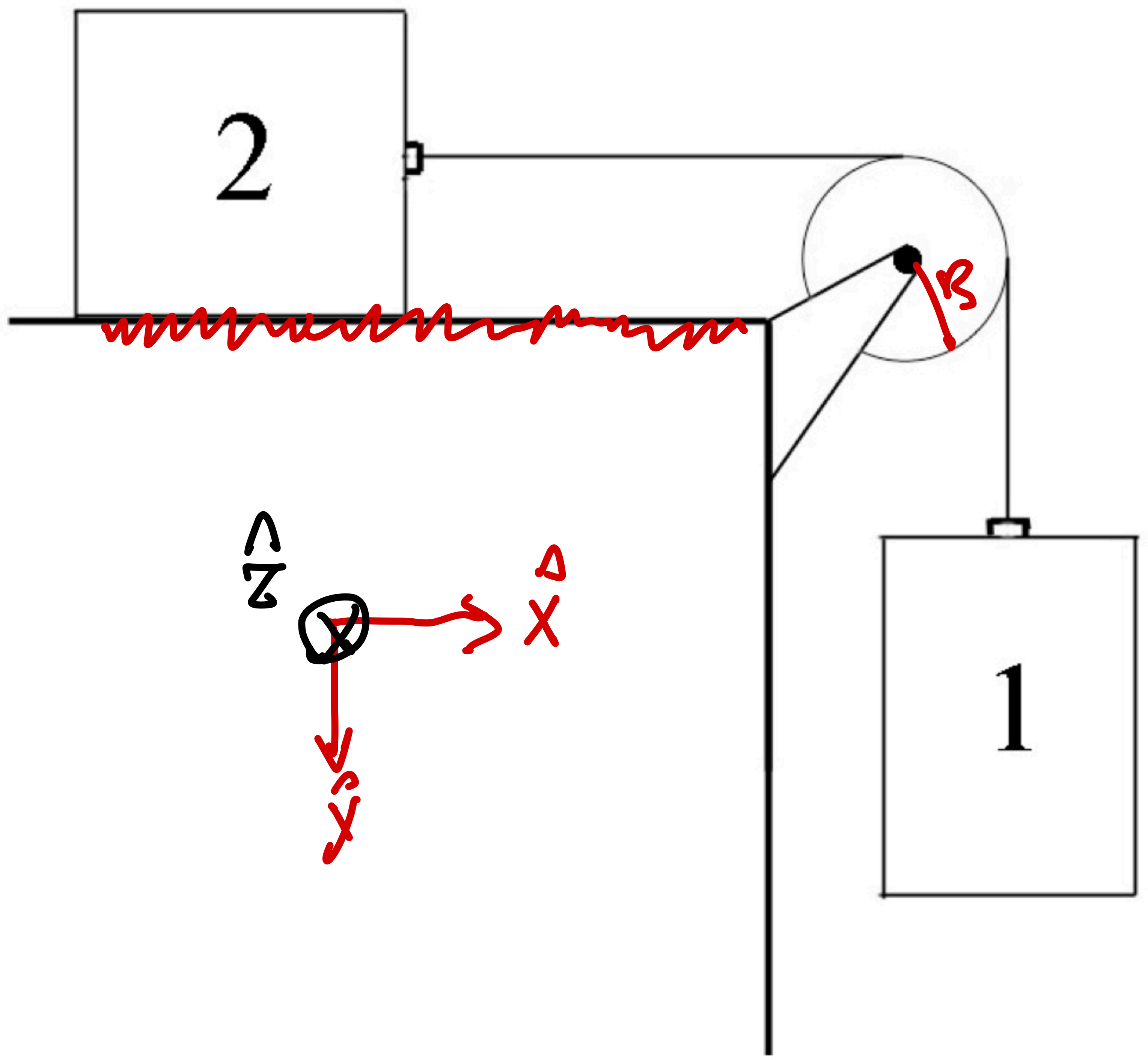
$$= (-m_1 d_1 + m_2 d_2) g \hat{z}$$

$$\Rightarrow \boxed{m_1 d_1 = m_2 d_2}$$

Example: Massive pulley

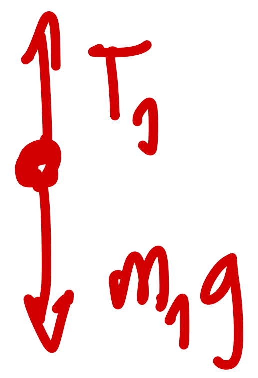
A pulley (with radius R and moment of inertia about its center of mass I) is attached to the edge of a table. A massless string connects two blocks as shown. Block 1 has mass m_1 and hangs off the edge of the table. Block 2 has mass m_2 and can slide along a table with a coefficient of kinetic friction of μ. Note that $m_1 > \mu m_2$. The blocks are released from rest and the string does not slip around the pulley.

Find the magnitude of the acceleration of each block. Express your answer in terms of R, I, m_1, m_2, and μ as needed.



Example: Massive pulley

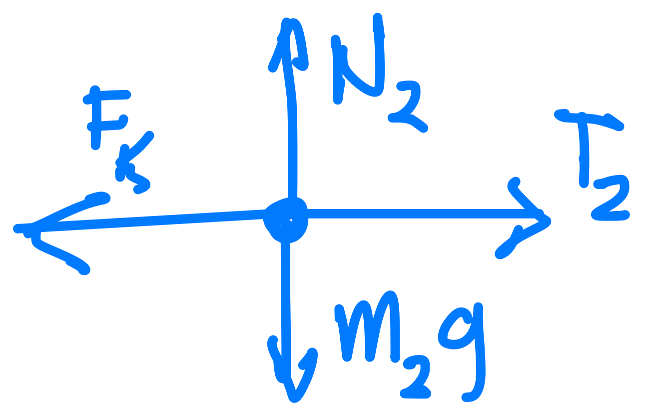
Block 1:



$\sum F_y: m_1 g - T_1 = m_1 a_{1y}$

$\Rightarrow T_1 = m_1 g - m_1 a_{1y}$ (1)

Block 2:

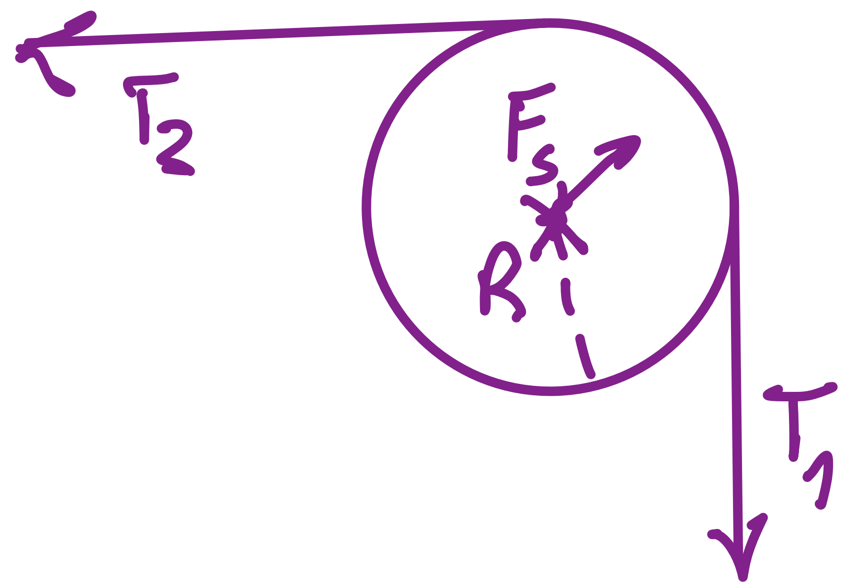


$\sum F_y: N_2 = m_2 g$

$\sum F_x: T_2 - F_k = m_2 a_{2x}$

$T_2 = m_2 a_{2x} + \mu m_2 g$ (2)

Pulley:



$\vec{\tau}_{T_1} = R\hat{x} \times T_1\hat{y} = RT_1\hat{z}$

$\vec{\tau}_{T_2} = \cancel{-}R\hat{y} \times (\cancel{-}T_2\hat{x}) = RT_2\,\hat{y}\times\hat{x} = -RT_2\hat{z}$

$\sum \vec{\tau} = RT_1\hat{z} - RT_2\hat{z} = R(T_1 - T_2)\hat{z}$

$= I\vec{\alpha} = I\alpha\hat{z}$ (3)

We have $a_{1y} = a_{2x} \equiv a$

From yesterday, $\alpha = \frac{a}{R}$ $(a = \alpha R)$

Example: Massive pulley

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From ①, $T_1 = m_1 g - m_1 a_{1y} = m_1 g - m_1 a$

From ②, $T_2 = m_2 a_{2x} + \mu m_2 g = m_2 a + \mu m_2 g$

From ③, $R(T_1 - T_2) = I\alpha = I\frac{a}{R}$

$$\Rightarrow \frac{I}{R^2} a = T_1 - T_2 = m_1(g-a) - m_2(a+\mu g) = \underline{m_1 g} - \underline{m_1 a} - \underline{m_2 a} - \underline{\mu m_2 g}$$

$$= (m_1 - \mu m_2) g - (m_1 + m_2) a$$

$$\Rightarrow (m_1 - \mu m_2) g = \frac{I}{R^2} a + (m_1 + m_2) a = a\left(m_1 + m_2 + \frac{I}{R^2}\right)$$

$$\Rightarrow \boxed{a = \frac{(m_1 - \mu m_2) g}{m_1 + m_2 + I/R^2}}$$