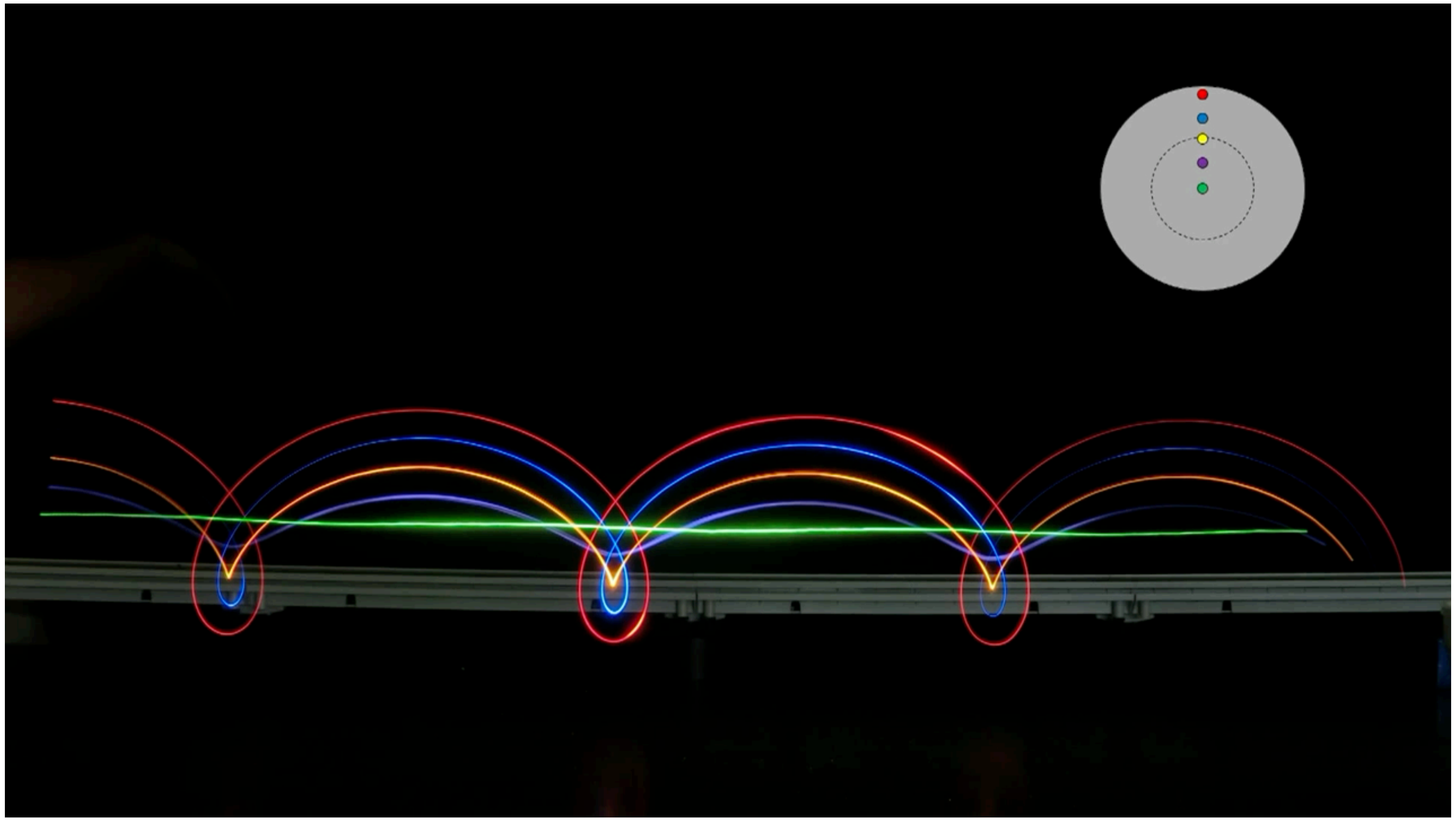
General Physics: Mechanics

Swiss Plasma Center

PHYS-101(en)

Lecture 11a: Rotational motion and static equilibrium



Dr. Marcelo Baquero
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November 25th, 2024

Announcement

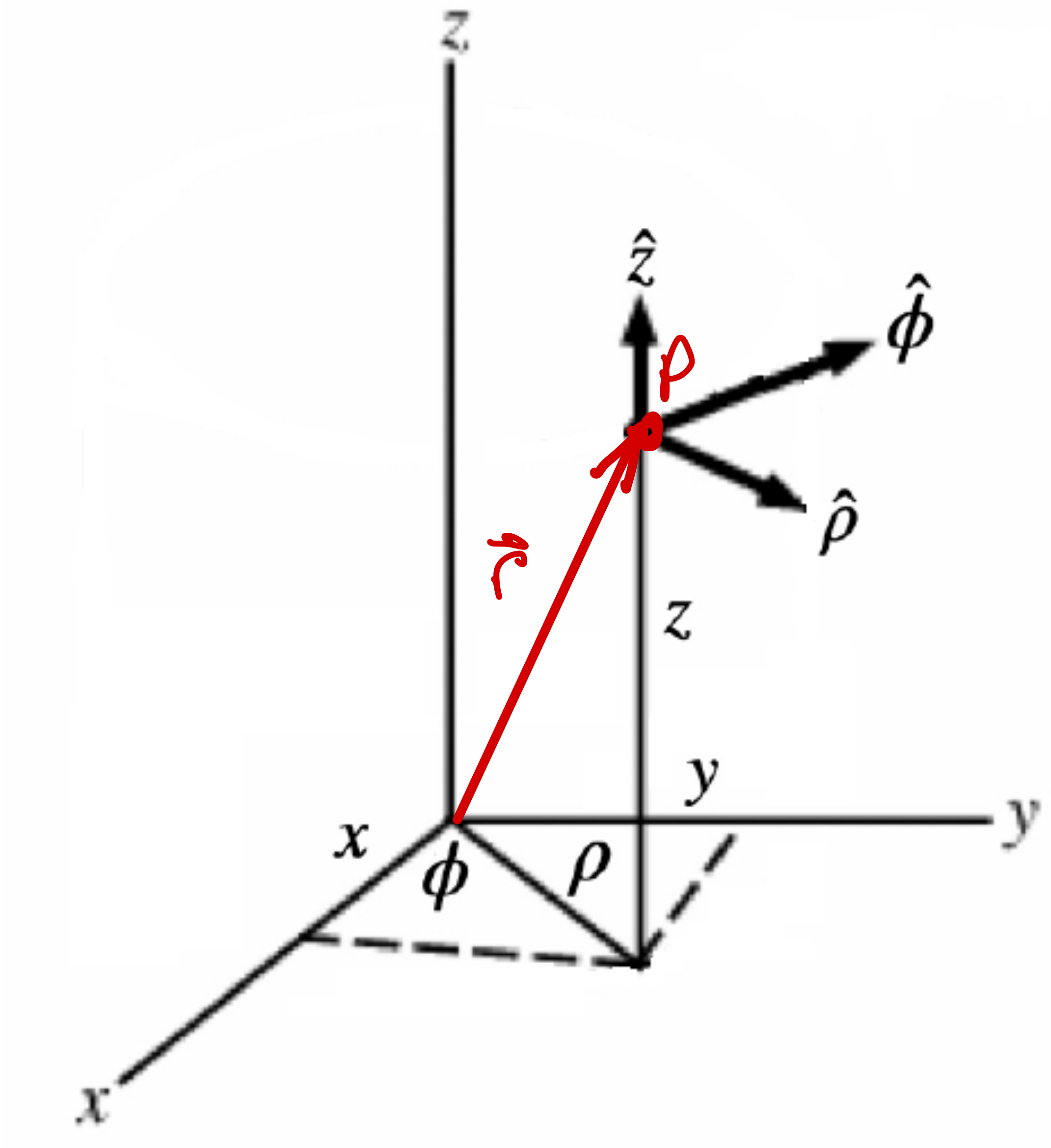
- We’ll hold another ***mini mock exam*** on Tuesday December 3rd
  - In-class (SG1) during normal lecture hours (10:15-11:00)
  - You can bring a “cheat” sheet containing formulas or all of your notes, as you wish
  - Hand exam to me at the end if you want to have it graded (optional)
  - Does **not** matter at all for your final grade
  - Exam solutions will be published

Today’s agenda (Serway 10,12; MIT 16-18)

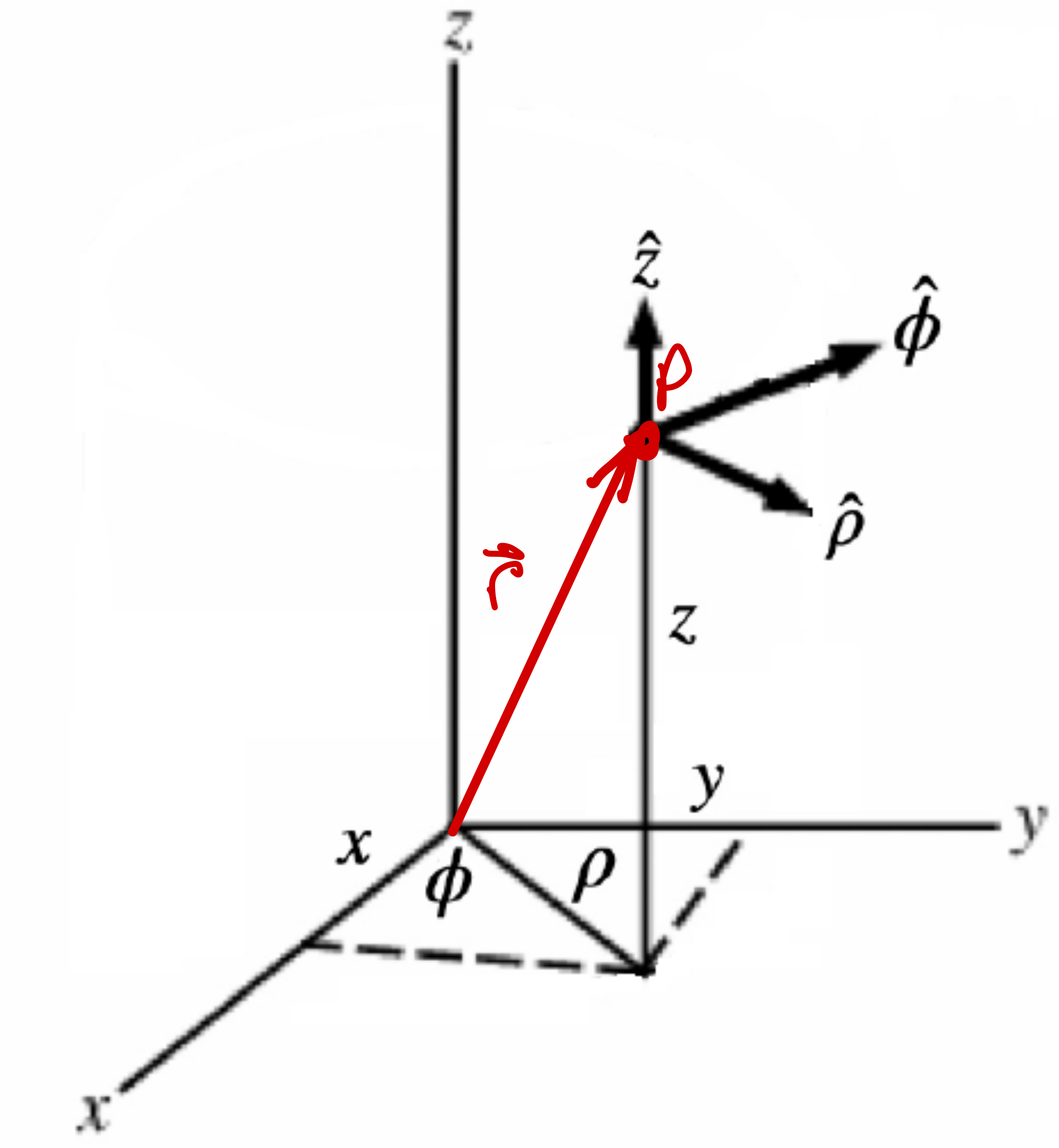
1. Review of circular motion
2. Rotation of rigid objects about a fixed axis
    - Rotational kinetic energy
    - Moment of inertia
    - Torque
3. Static equilibrium

Week 4: Cylindrical coordinates

- Position vector:
  $\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$



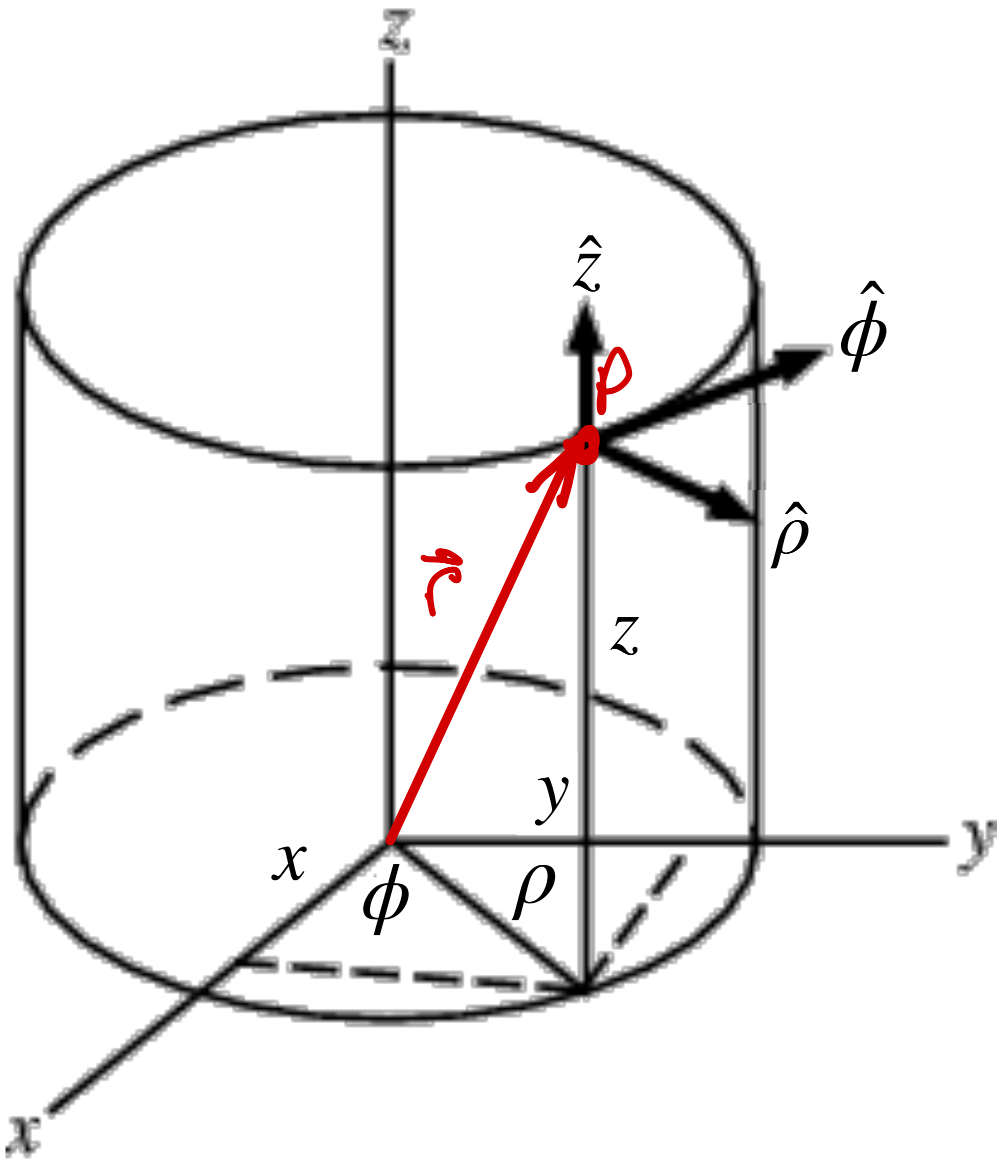
Week 4: Cylindrical coordinates



- Position vector:
$$\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$$
- *Linear* velocity:
$$\vec{v}(t) = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$$
- *Linear* acceleration:
$$\vec{a}(t) = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right)\hat{\phi} + \ddot{z}\hat{z}$$

Week 4: Circular motion

- In circular motion $\rho(t) = \rho_0$ where ρ_0 is constant



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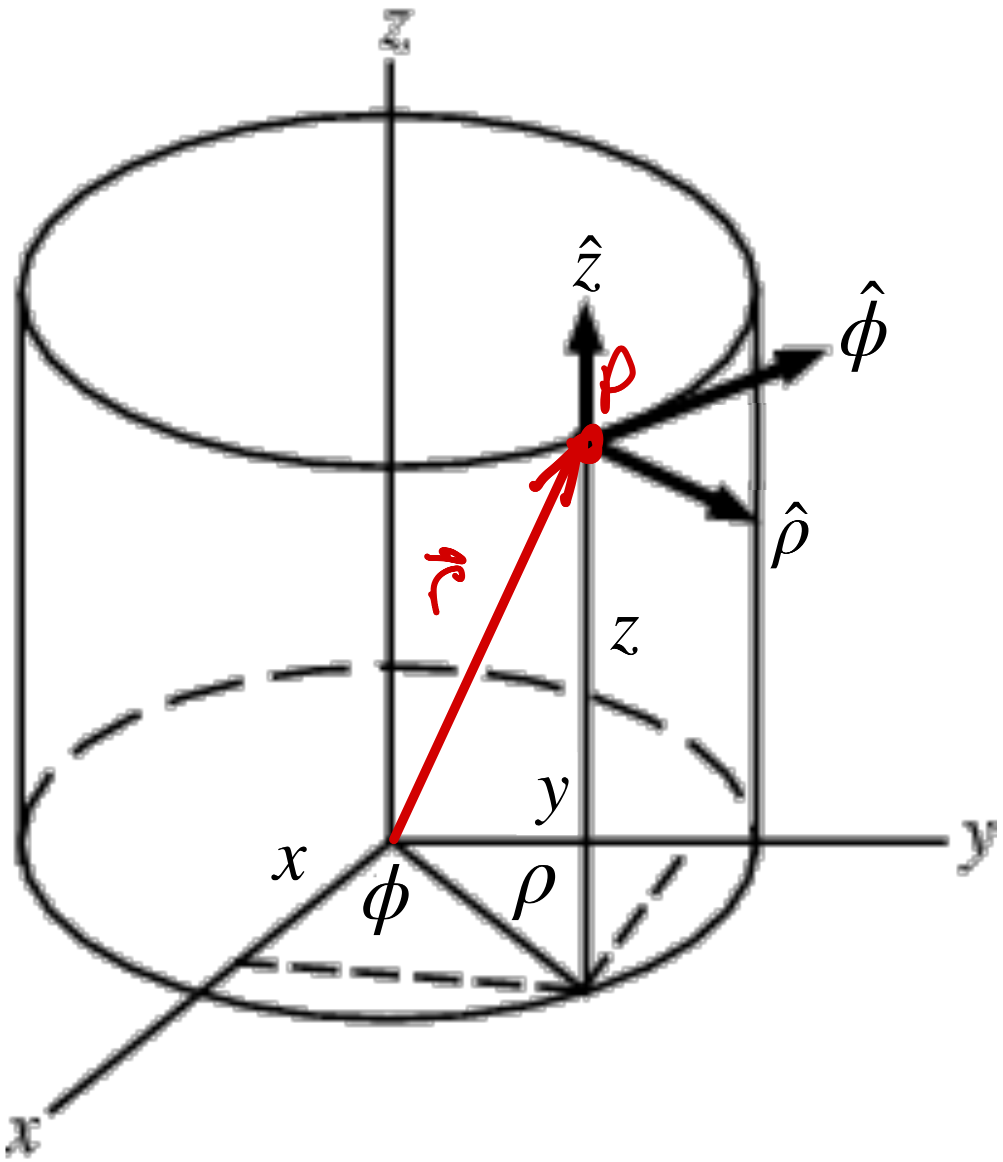
- In circular motion $\rho(t) = \rho_0$ where ρ_0 is constant
- Also $z(t) = z_0$ where z_0 is a constant (which we typically choose to be 0 with an appropriate reference frame)



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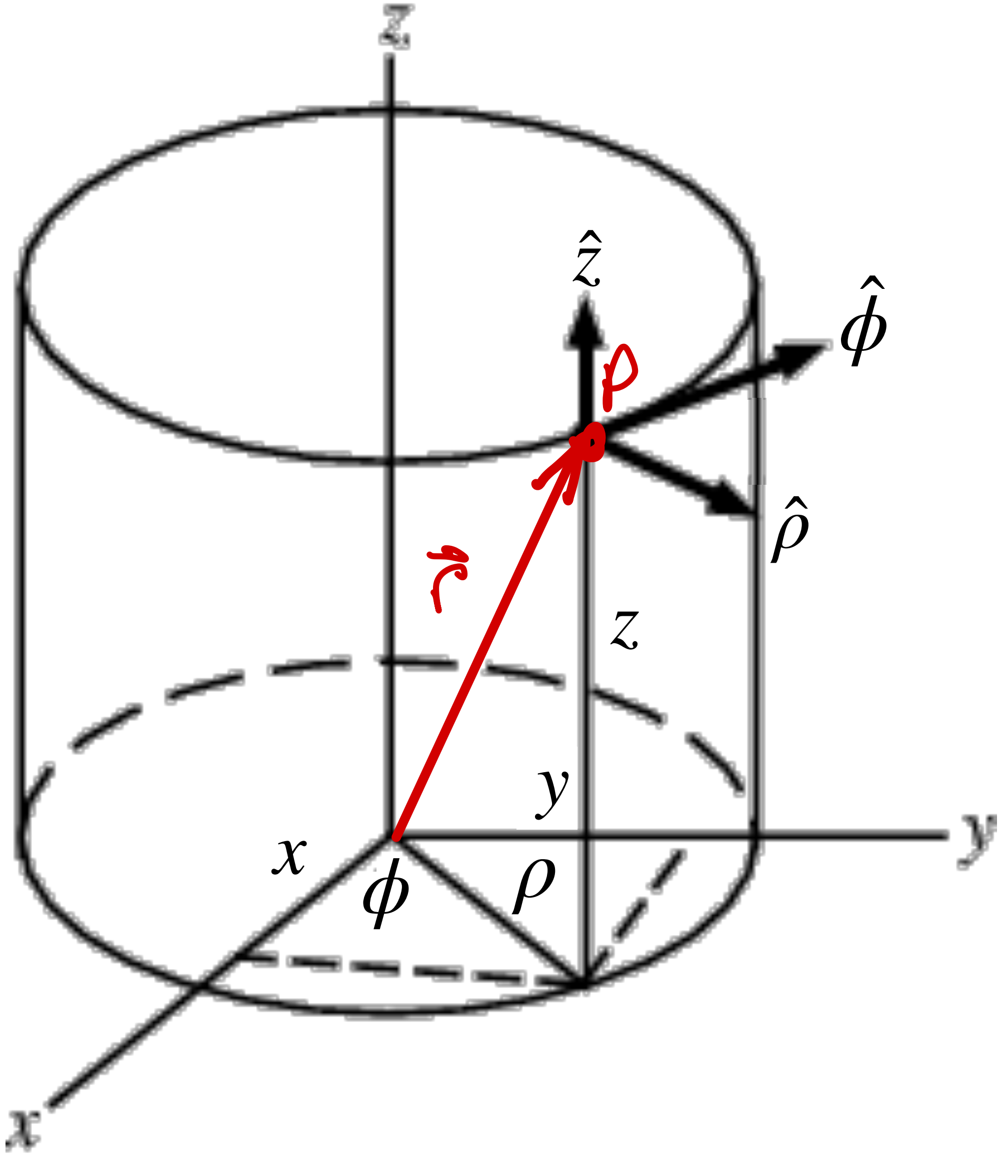
- In circular motion $\rho(t) = \rho_0$ where ρ_0 is constant
- Also $z(t) = z_0$ where z_0 is a constant (which we typically choose to be 0 with an appropriate reference frame)
- Position vector: $\vec{r}(t) = \underset{\rho_0}{\cancel{\rho(t)}}\,\hat{\rho} + \underset{z_0=0}{\cancel{z(t)}}\,\hat{z}$ for following slides
- *Linear* velocity: $\vec{v}(t) = \cancelto{0}{\dot{\rho}}\hat{\rho} + \underline{\rho\dot{\phi}\hat{\phi}} + \cancelto{0}{\dot{z}}\hat{z}$
- *Linear* acceleration: $\vec{a}(t) = \left(\cancelto{0}{\ddot{\rho}} - \underset{\rho_0}{\cancel{\rho}}\dot{\phi}^2\right)\hat{\rho} + \left(2\cancelto{0}{\dot{\rho}}\dot{\phi} + \underset{\rho_0}{\rho}\ddot{\phi}\right)\hat{\phi} + \cancelto{0}{\ddot{z}}\hat{z} = -\rho_0\dot{\phi}^2\hat{\rho} + \rho_0\ddot{\phi}\hat{\phi}$



Week 4: Circular motion

- Angular speed: $\omega(t) = \frac{d\phi}{dt} = \dot{\phi}$
- Angular acceleration: $\alpha(t) = \dot{\omega} = \ddot{\phi}$

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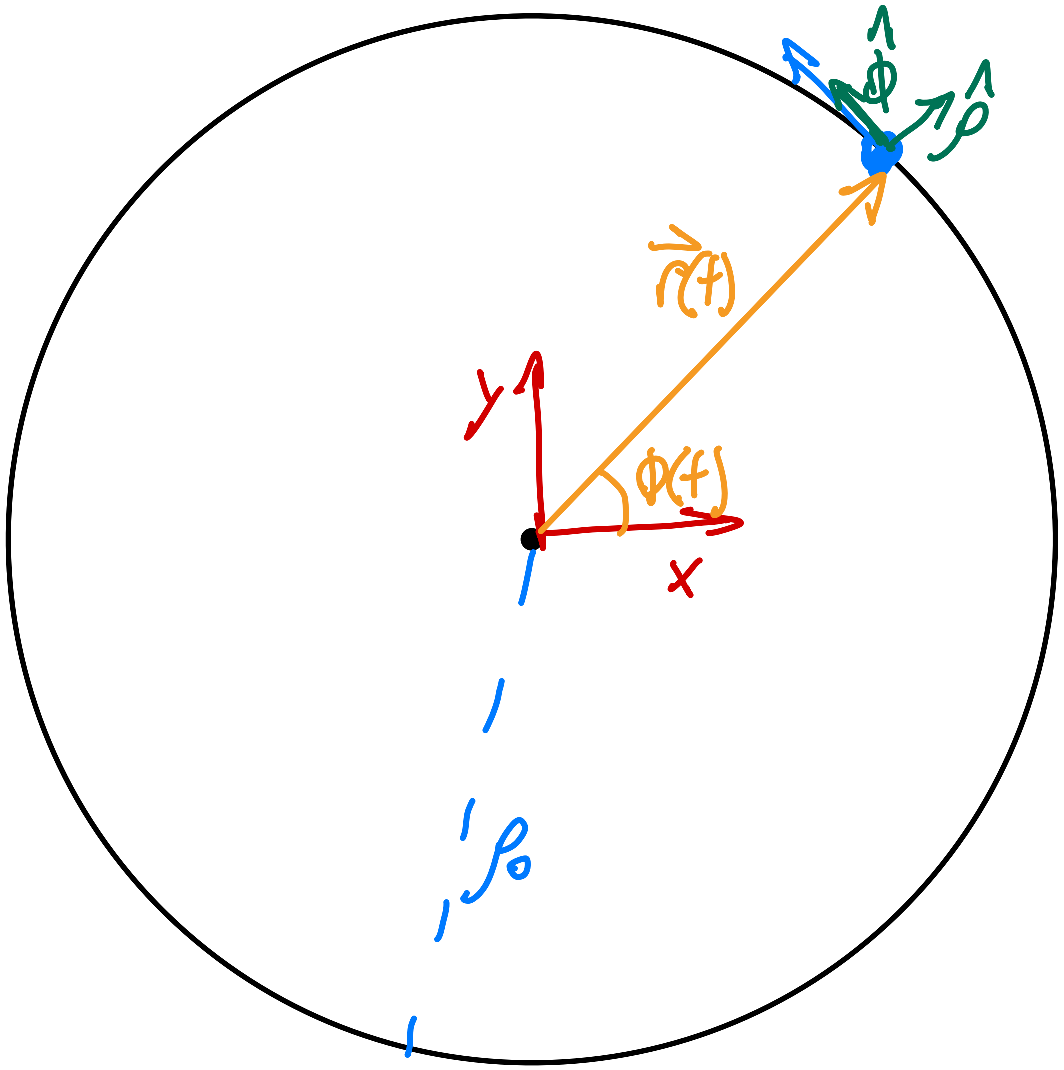
- Position: $\vec{r}(t) = \rho_0 \hat{\rho}(t)$
- Velocity: $\vec{v}(t) = \rho_0 \omega(t) \hat{\phi}(t)$
- Acceleration:
  $$\vec{a}(t) = \underbrace{-\,\rho_0[\omega(t)]^2\,\hat{\rho}}_{a_c} + \underbrace{\rho_0 \alpha(t)\,\hat{\phi}}_{a_\phi}$$



Week 4: Circular motion

- Angular speed: $\omega(t) = \dfrac{d\phi}{dt} = \dot{\phi}$
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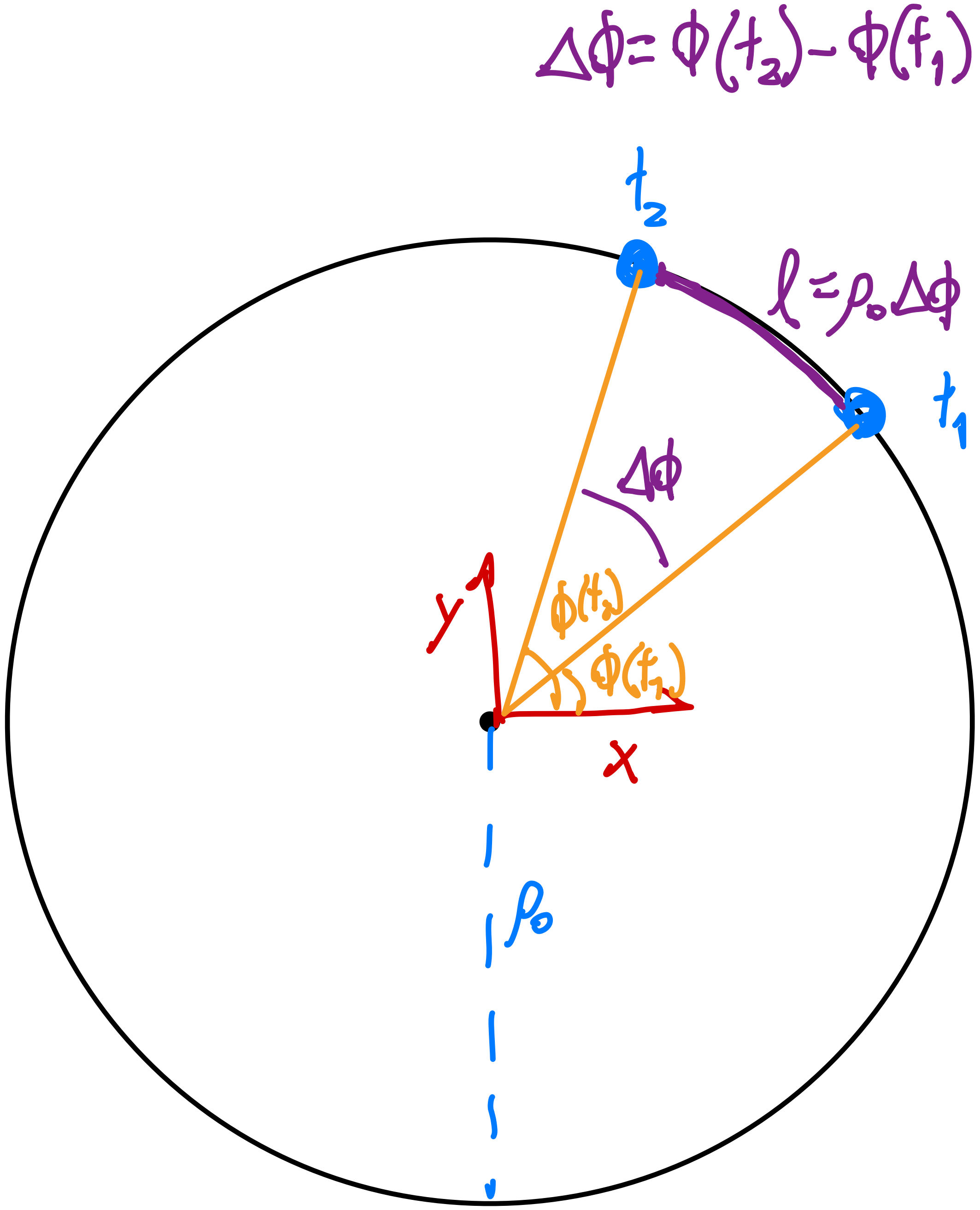
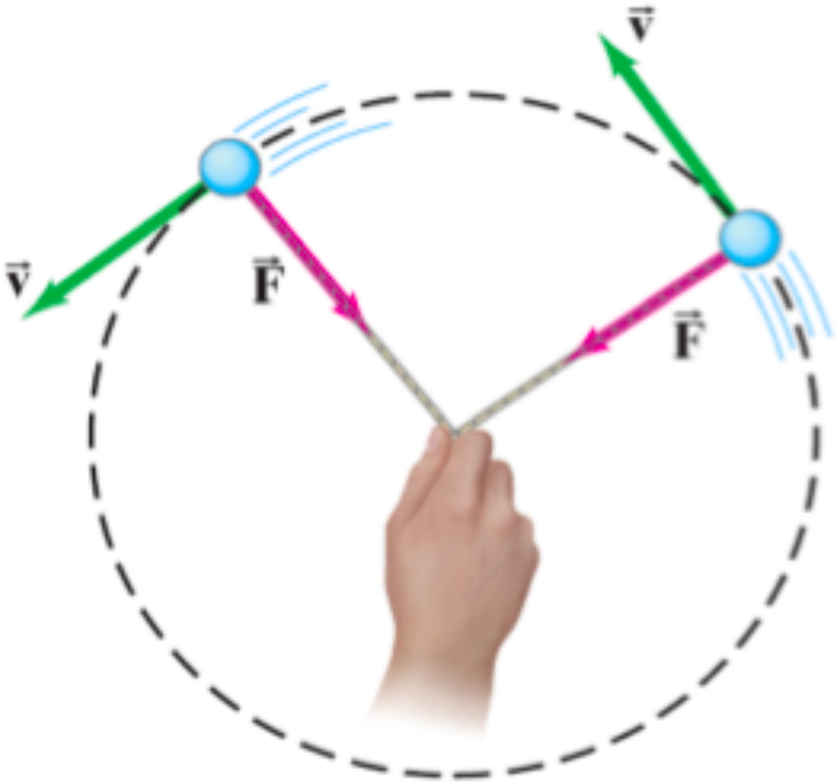
- Acceleration:
  $\vec{a}(t) = -\rho_0 [\omega(t)]^2 \, \hat{\rho} + \rho_0 \alpha(t) \, \hat{\phi}$

- Angular displacement: $\Delta\phi$
- Distance traveled (arc length): $\ell = \rho_0 \Delta\phi$



Week 4: Circular motion

- Due to Newton's 1st law, an object in circular motion must be experiencing a net radial force (called the centripetal force)



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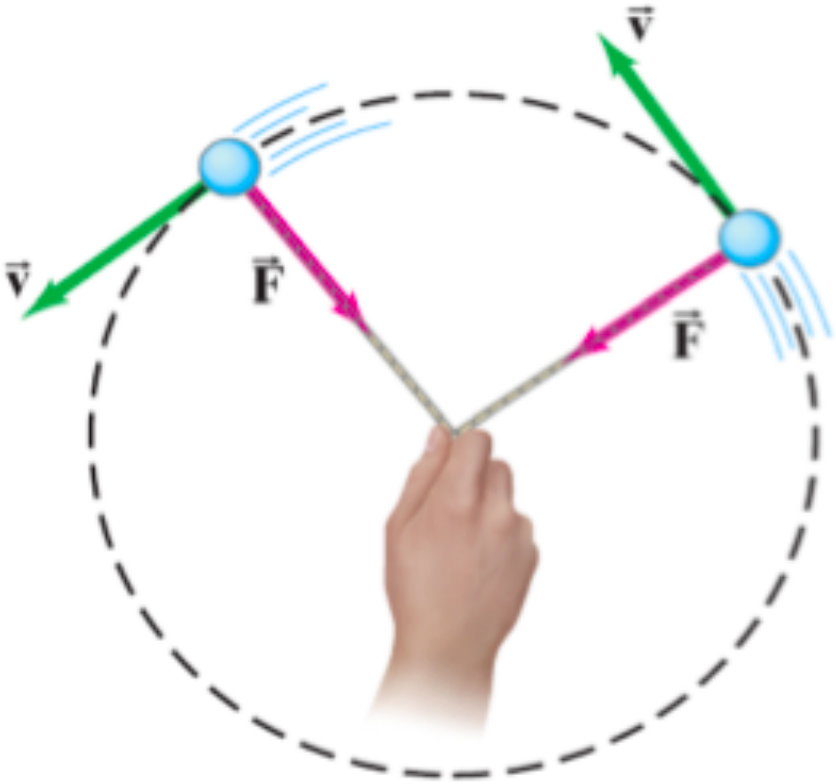
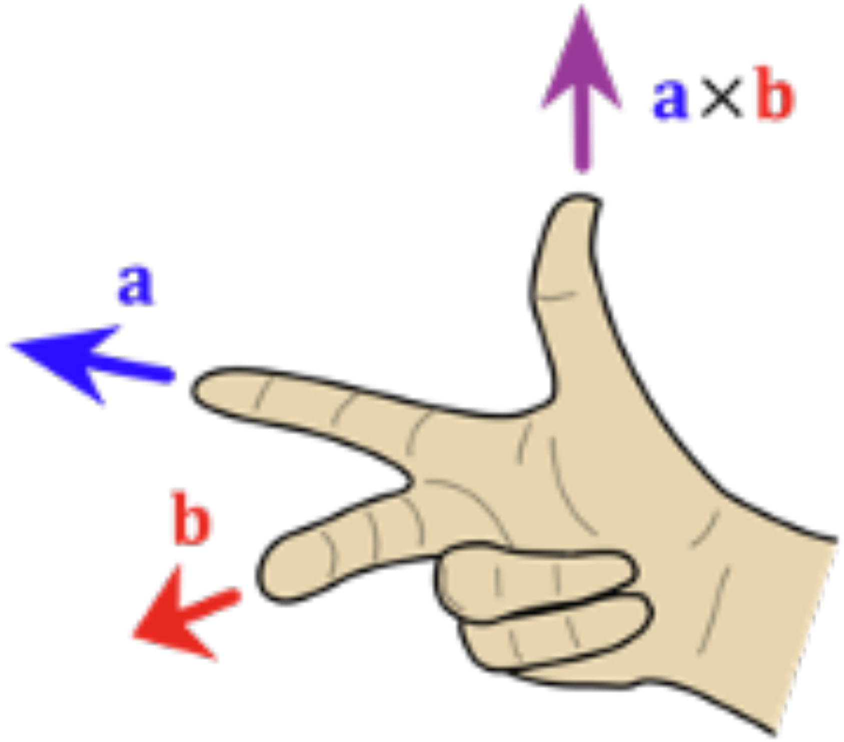


- Determining the identity of this force requires further investigation
- For the case of a ball on a string, this force is provided by tension in the string (possibly in combination with gravity)
- Using $\vec{a} = a_\rho \hat{\rho} + a_\phi \hat{\phi} = -\rho_0 \omega^2 \hat{\rho} + \rho_0 \alpha \hat{\phi}$, we know that

$$\overrightarrow{F}_{net} = m\vec{a} = \underbrace{-m\rho_0\omega^2\hat{\rho}}_{\vec{F}_{cent}} + m\rho_0\alpha\hat{\phi}$$

Week 4: Cross (or vector) product



- Two vectors are multiplied in a cross product to produce another vector

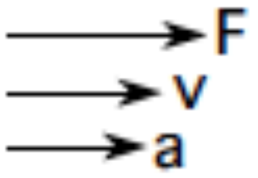
$$\vec{a} \times \vec{b} = \vec{c}$$

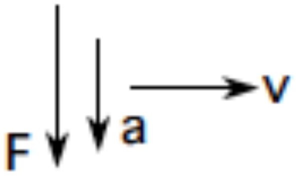
- Magnitude: $|\vec{c}| = c = ab\sin\theta$
- Direction: Use right hand rule
- If $\vec{a} \,||\, \vec{b}$, then $\vec{a} \times \vec{b} = 0$ or if $\vec{a} \perp \vec{b}$, then $|\vec{a} \times \vec{b}| = ab$
- Not commutative: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- Distributive: $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- Derivative product rule: $\dfrac{d}{dt}(\vec{a} \times \vec{b}) = \dfrac{d\vec{a}}{dt} \times \vec{b} + \vec{a} \times \dfrac{d\vec{b}}{dt}$

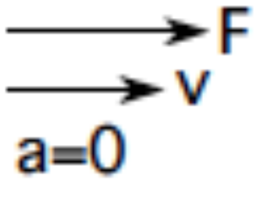
Conceptual question

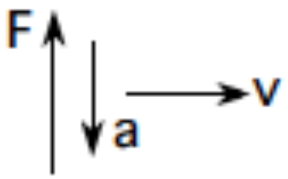
A piece of sushi rests on a circular turntable, rotating about a vertical axis at a constant angular speed ω as illustrated in the diagram below.

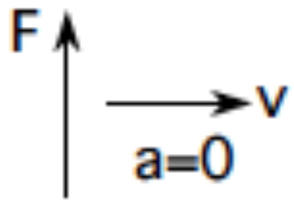
The sushi rotates with the turntable and fortunately does not slip. What are the directions of the velocity, acceleration and net force vectors acting on the sushi at the moment shown in the diagram?

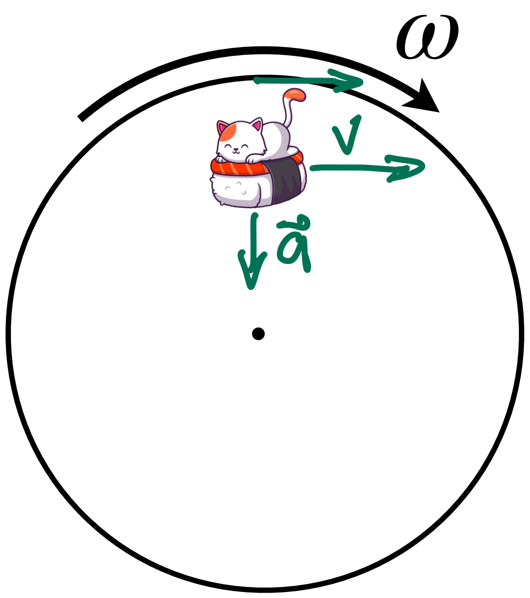
A.


[✓ D.]


B.


E.


C.




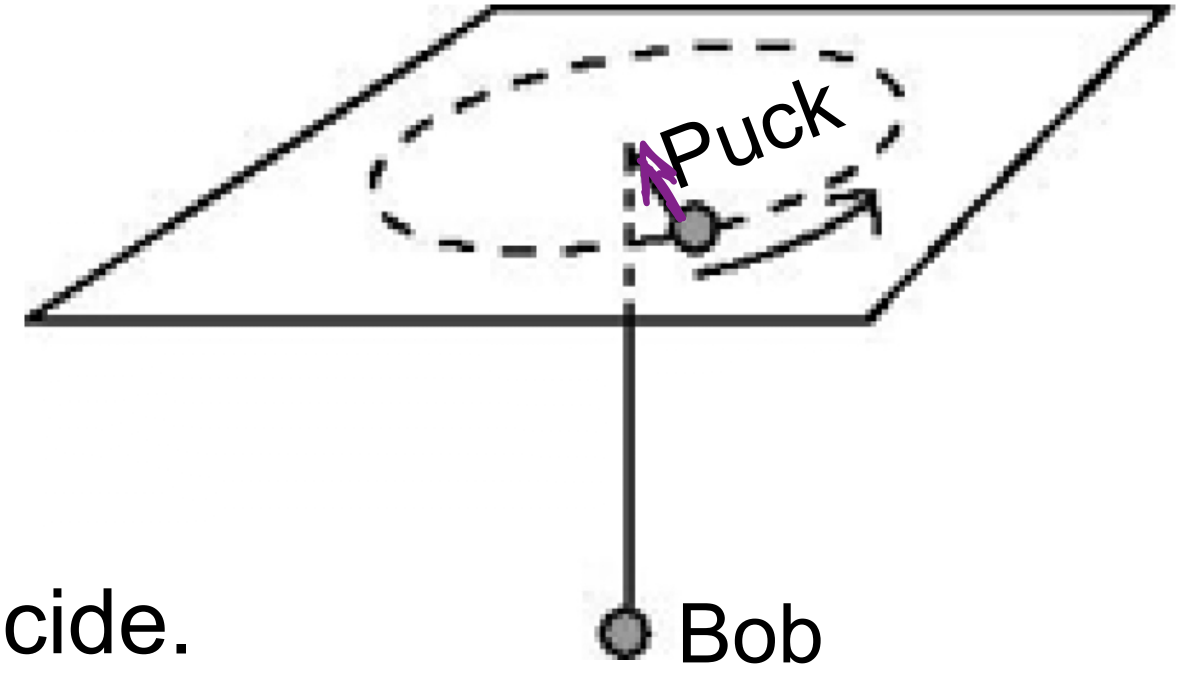
View of turntable from above

Conceptual question

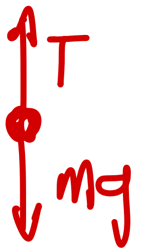
$\omega = \text{const}, \quad \alpha = \dot{\omega} = 0$

A puck of mass m is moving in a circle at constant speed on a frictionless table. The puck is connected by a massless string to a suspended bob, also of mass m, which is at rest below the table. What is the magnitude of the centripetal acceleration of the moving puck? Ignore friction.

A. Greater than g
[✓ B.] Equal to g
C. Less than g
D. 0
E. Not enough information to decide.

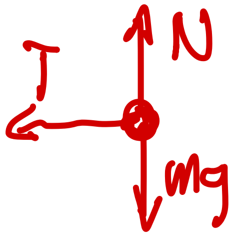


Bob:



$T = mg$

Puck



$T = ma_c$

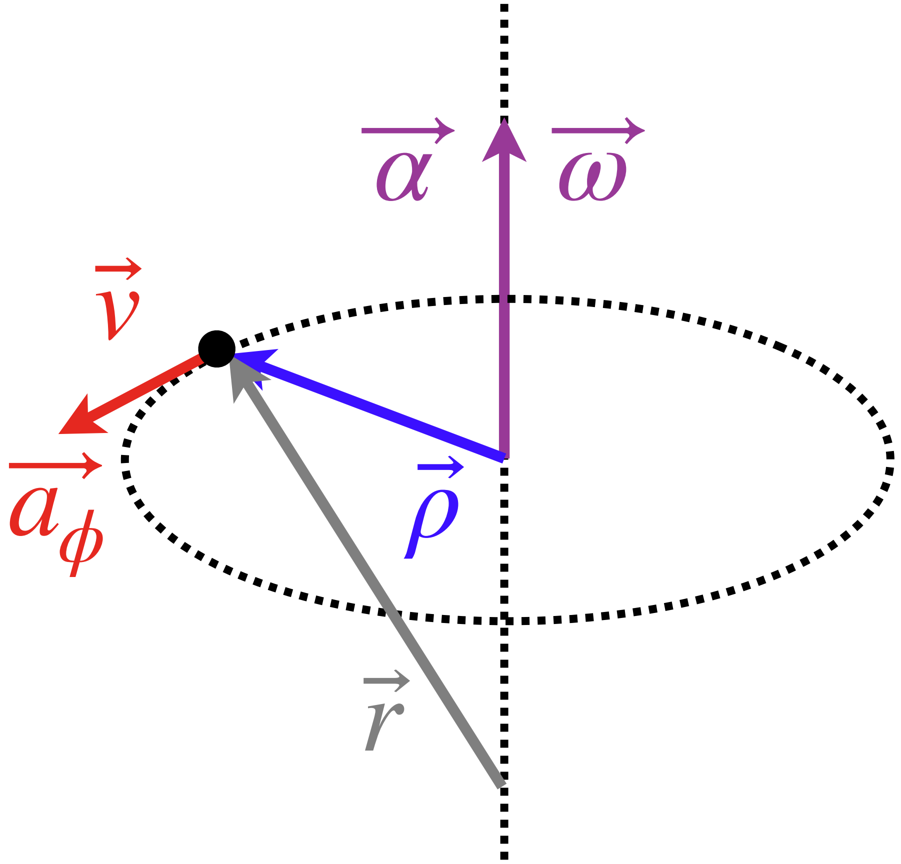
$mg = ma_c$

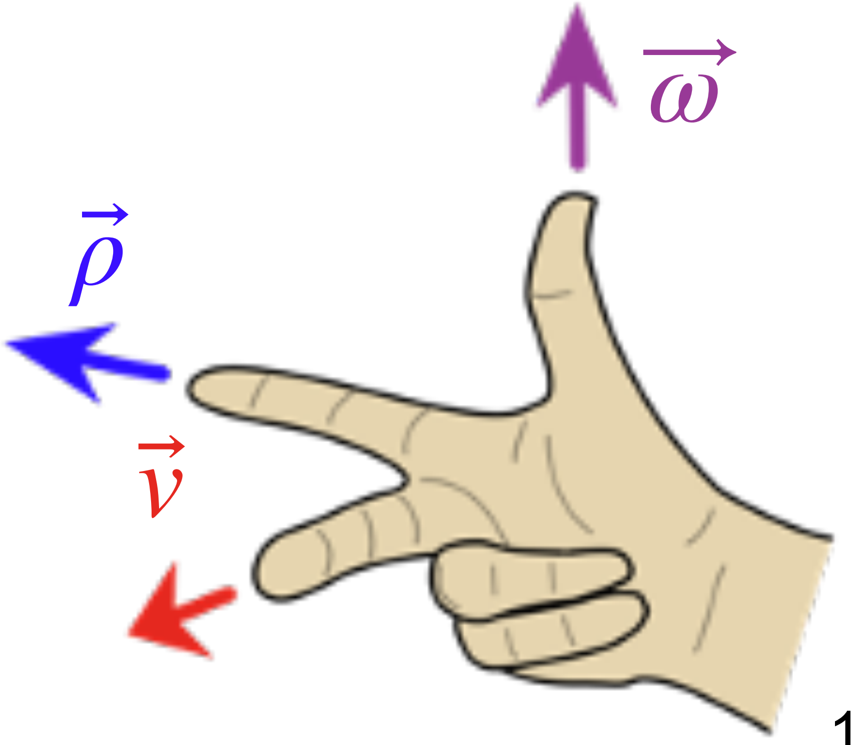
$\Rightarrow a_c = g$

Angular velocity vector in circular motion

- Angular velocity **vector** points along the axis of rotation according to the right-hand rule

$$\vec{v} = \overrightarrow{\omega} \times \vec{\rho} \quad \text{and} \quad \overrightarrow{\omega} = \frac{\vec{\rho} \times \vec{v}}{\rho^2}$$





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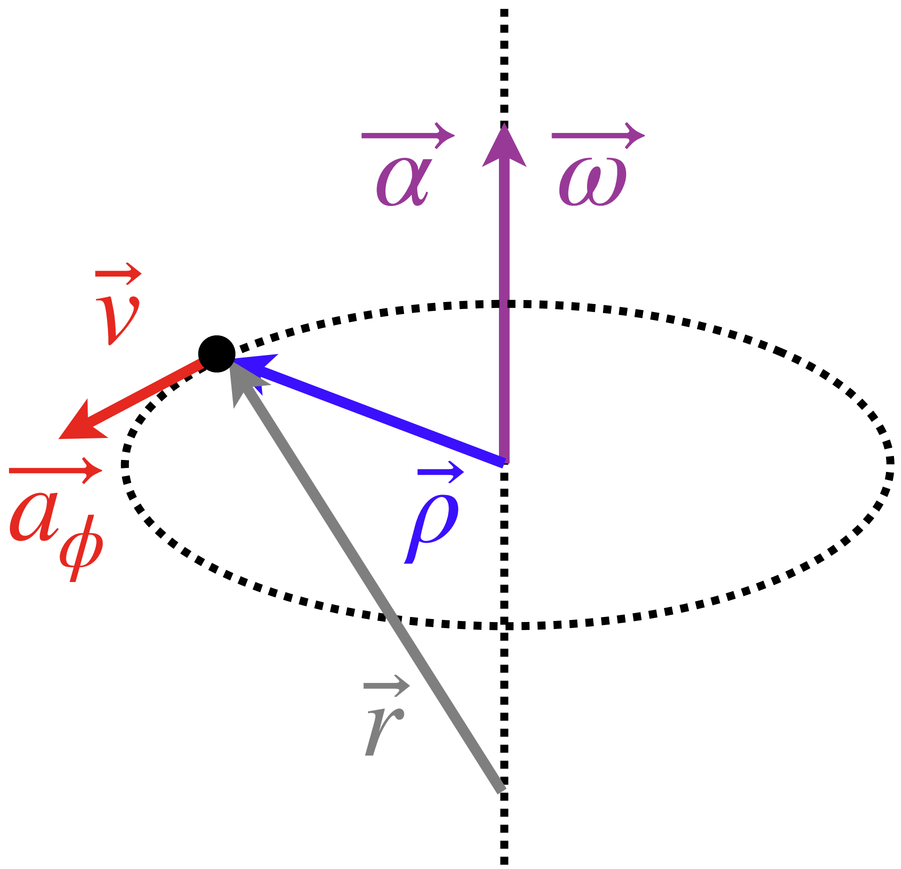
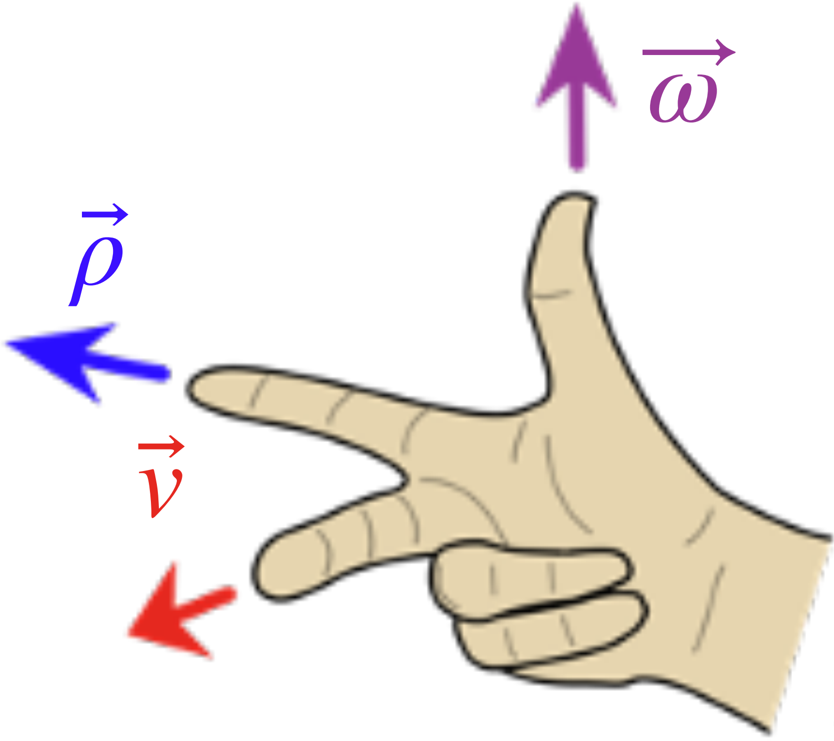


- The angular acceleration vector is

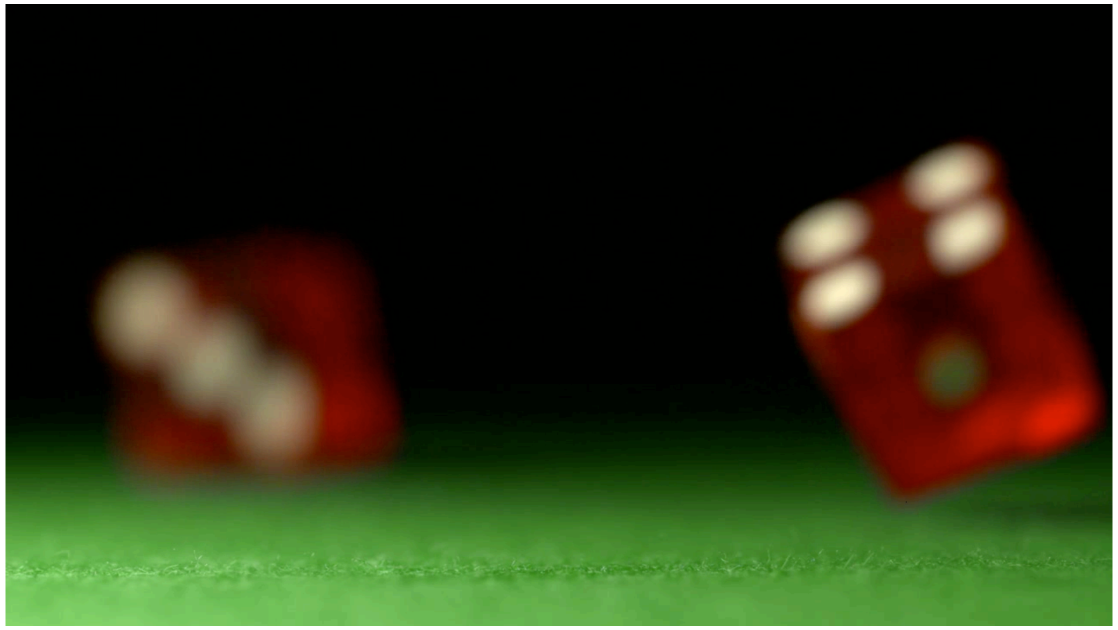
$$\vec{a_\phi} = \vec{\alpha} \times \vec{\rho} \quad \text{and} \quad \vec{\alpha} = \frac{\vec{\rho} \times \vec{a_\phi}}{\rho^2}$$

- If the direction of the rotation axis does not change, the angular acceleration vector points along it



Rigid body

- In a **rigid body** all points move together without deformation, i.e. the distance between any two points is fixed in time



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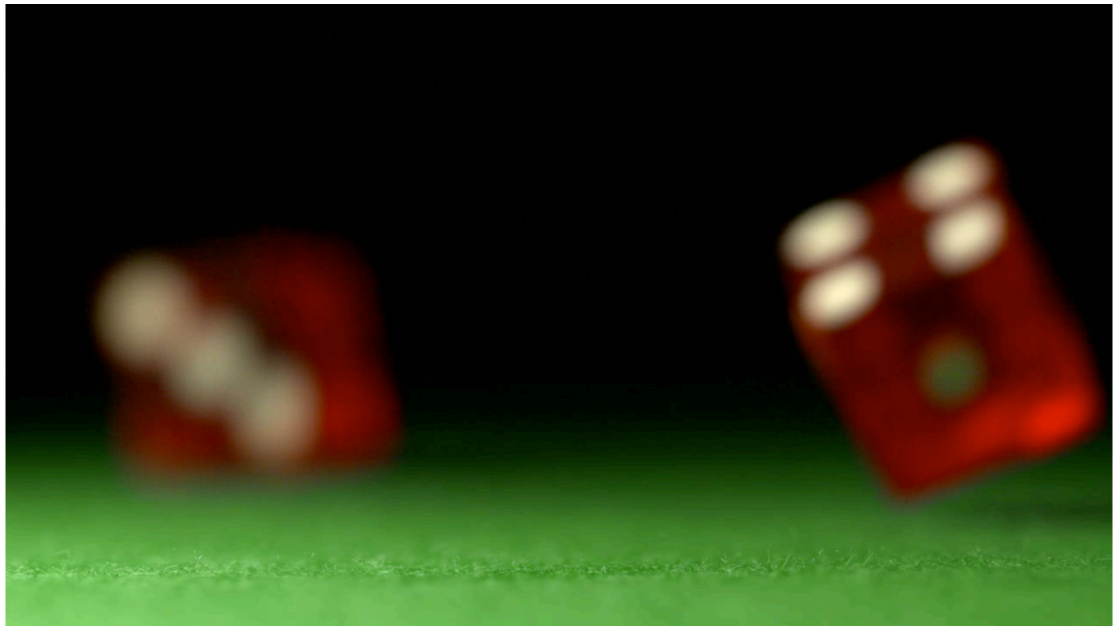
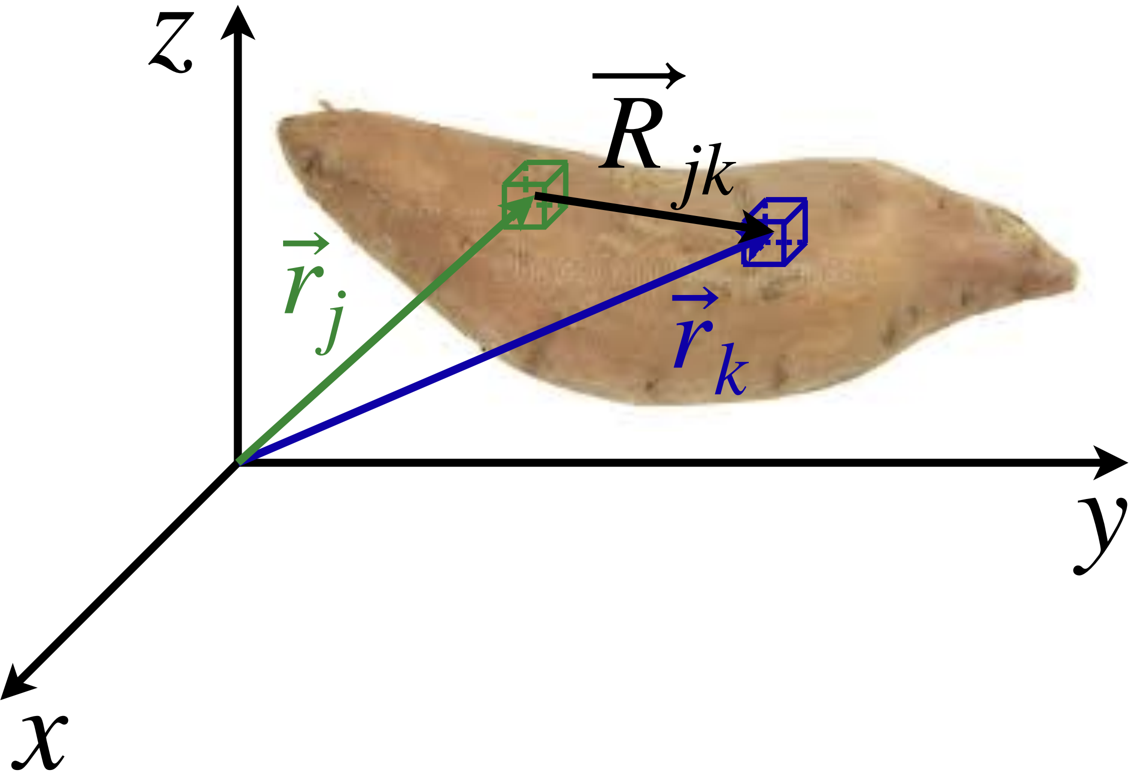


- Imagine an object is composed of a huge number of tiny differential elements, labeled $i = 1, 2, 3, \ldots$ with positions $\vec{r}_i$
- A body is rigid if, for all pairs of differential elements j and k, the distance $R_{jk} = |\vec{r}_k - \vec{r}_j|$ is constant in time



Pure translation of rigid bodies

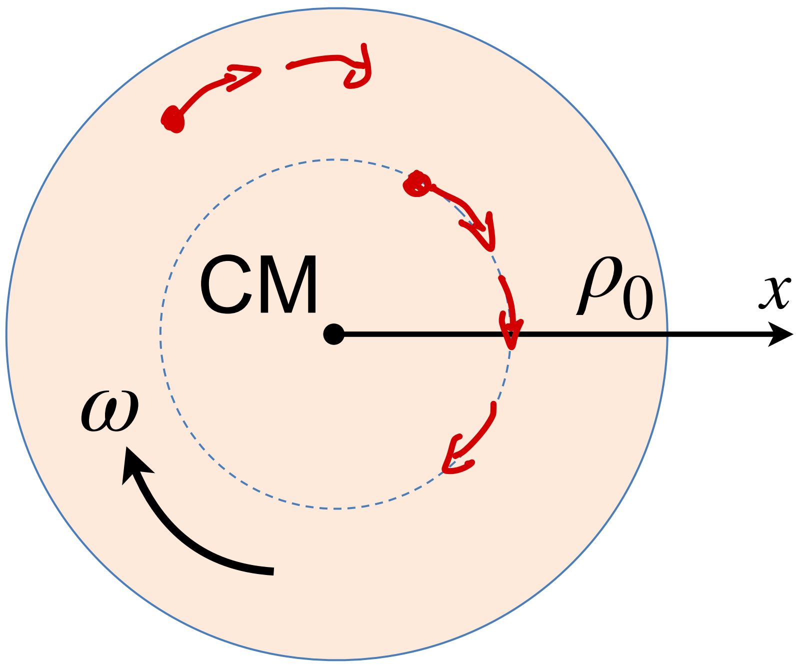
- We already know how to handle pure translation of a rigid body
  - Represent the entire object as a point at the object's Center of Mass (CM), as discussed in lecture 6
  - Then apply all net external forces to the CM and calculate its motion

Pure translation of rigid bodies

- We already know how to handle pure translation of a rigid body
  - Represent the entire object as a point at the object’s Center of Mass (CM), as discussed in lecture 6
  - Then apply all net external forces to the CM and calculate its motion
- What about if the object is also rotating?
  - Can decompose motion into pure translation of CM (treated as above) and pure rotation around CM (will study now)

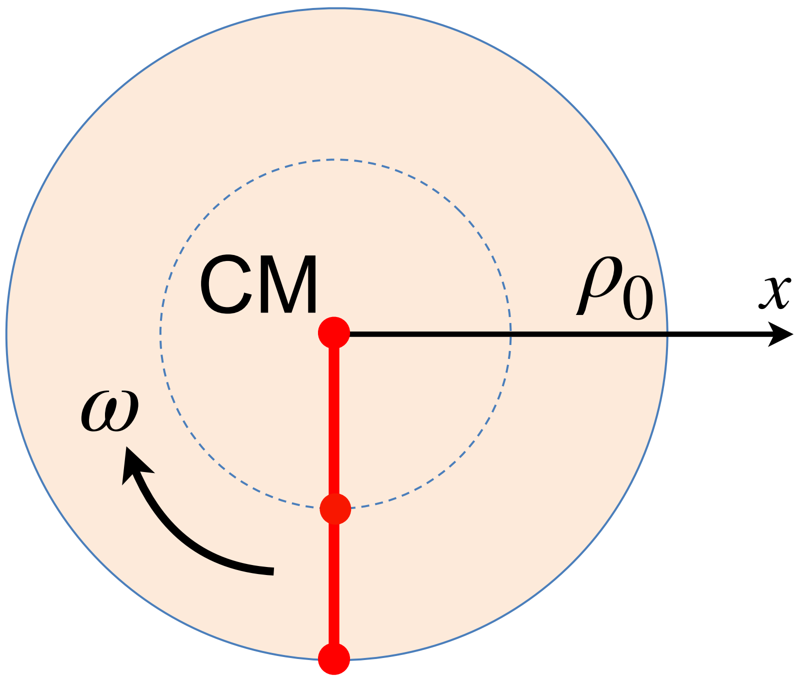
Pure rotation of a rigid body

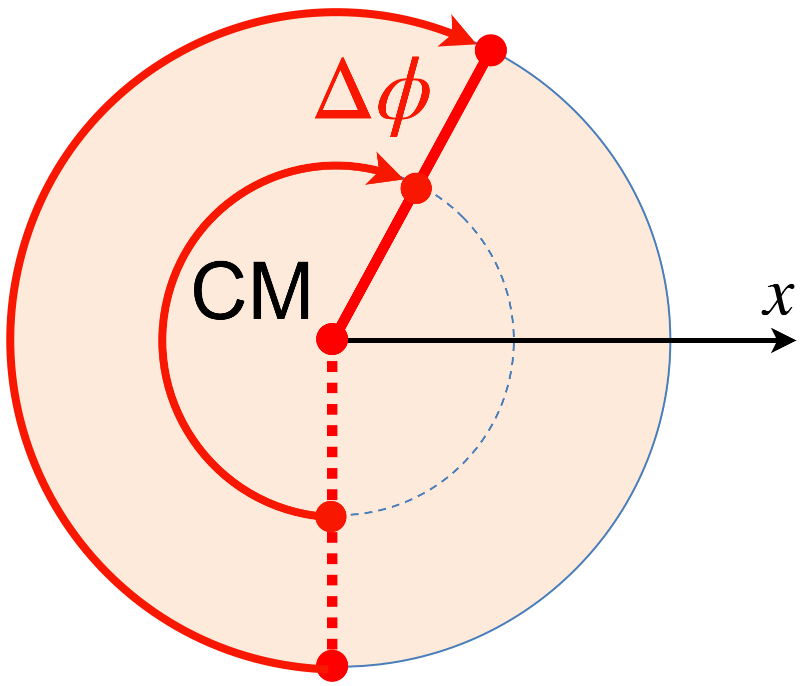
- In pure rotational motion, all points in the object move in circles around an **axis of rotation** through the CM



Pure rotation of a rigid body

- In pure rotational motion, all points in the object move in circles around an **axis of rotation** through the CM
- All points on a straight line drawn through the axis move through the same angle in the same time





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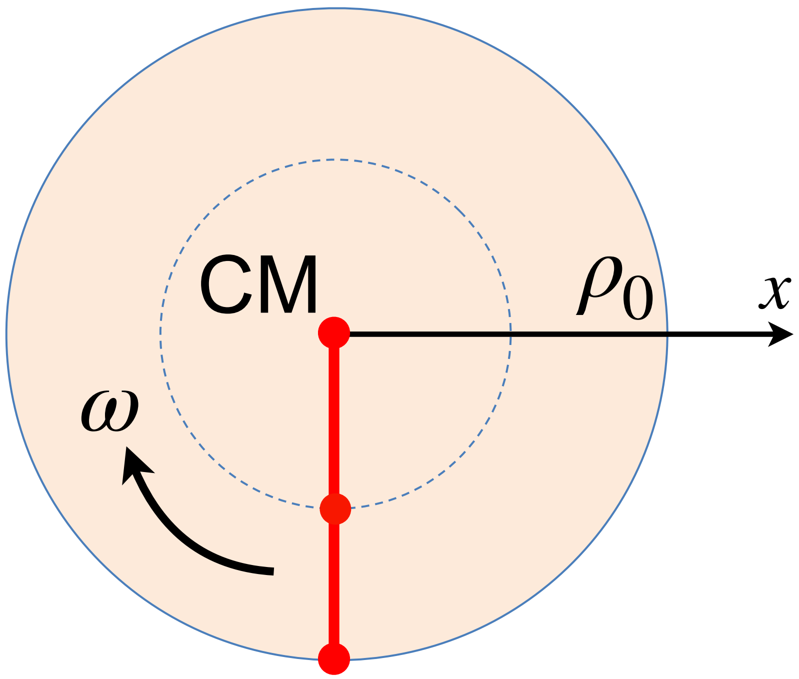
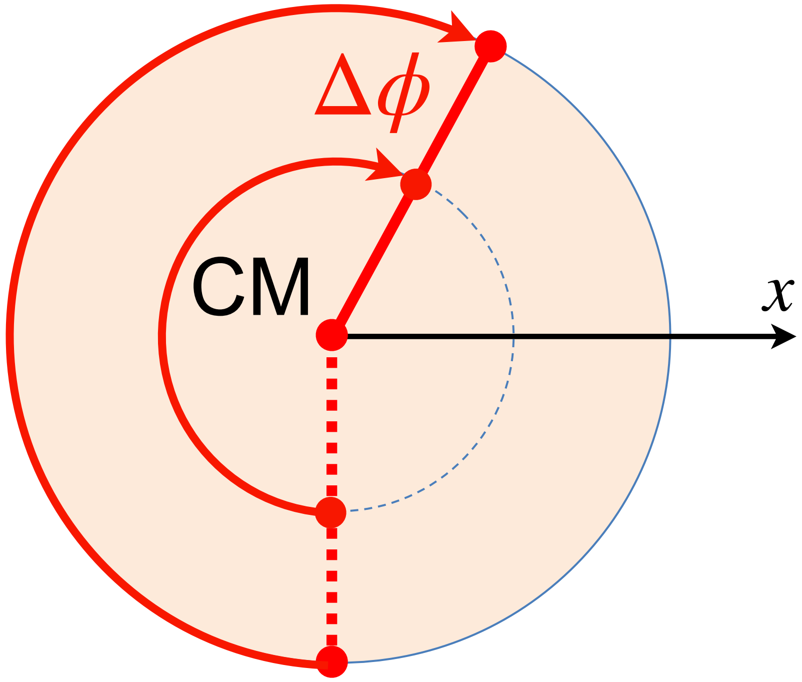
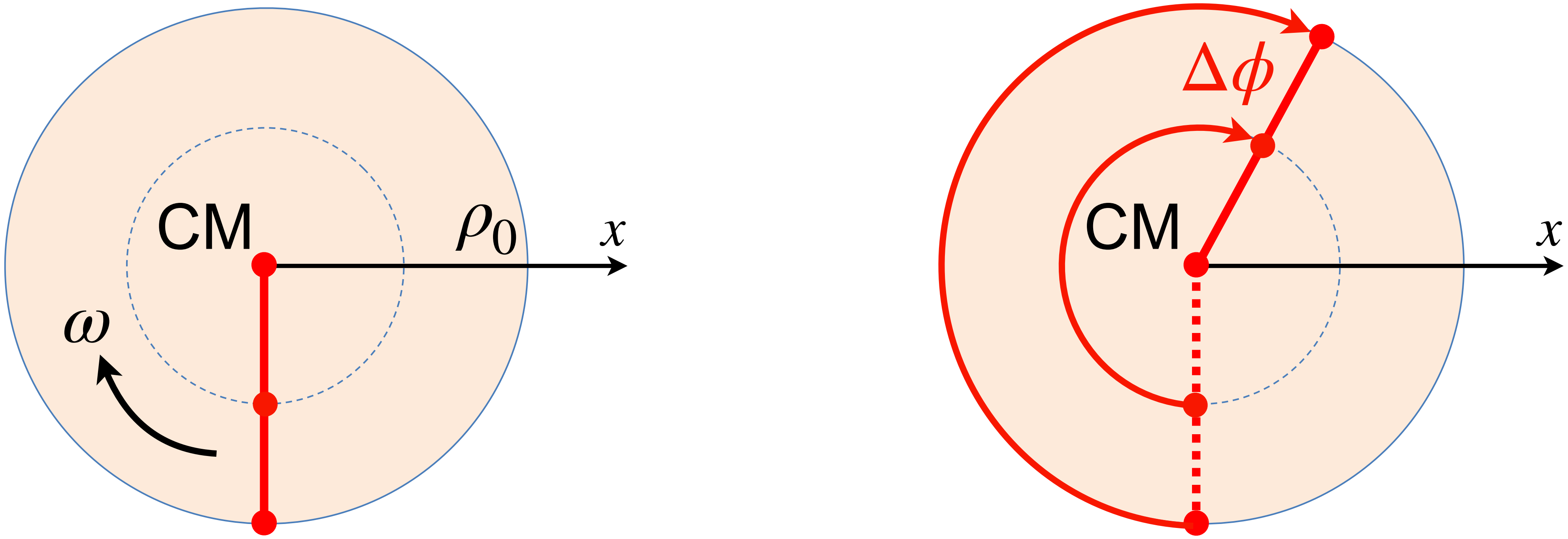
- Therefore, every point has the same value of $\omega = \dfrac{\Delta\phi}{\Delta t}$ (and α)





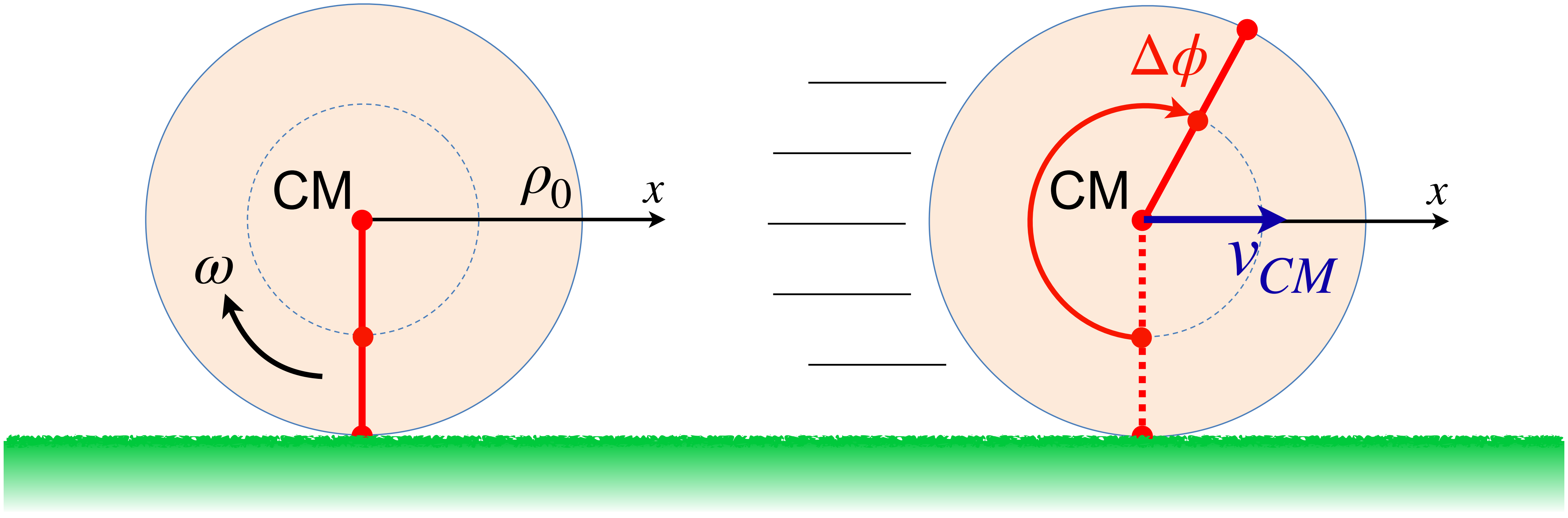
Pure rotation of a rigid body

- In pure rotational motion, all points in the object move in circles around an **axis of rotation** through the CM
- All points on a straight line drawn through the axis move through the same angle in the same time
- Therefore, every point has the same value of $\omega = \dfrac{\Delta\phi}{\Delta t}$ (and α)
- The distance they move is the arc length $\ell(\rho) = \rho\Delta\phi$



Rolling without slipping

- If an object rolling on a surface does not slip, then the distance traveled along the ground must be equal to the distance traveled by the rim of the wheel



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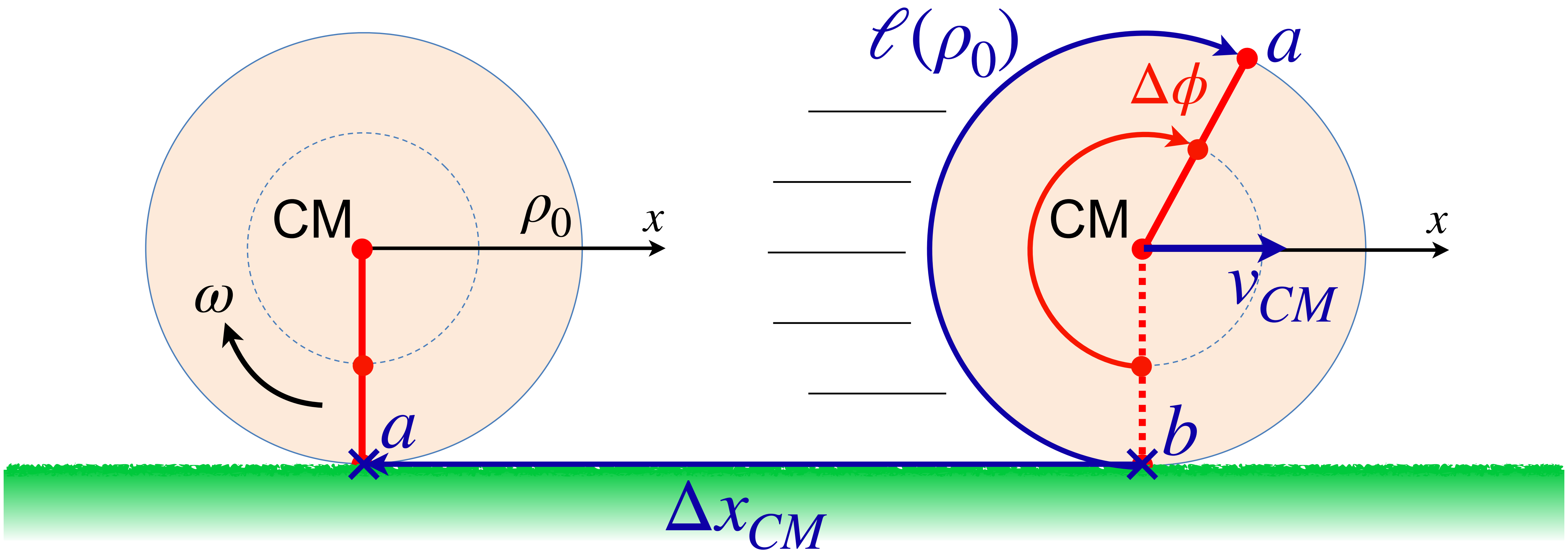

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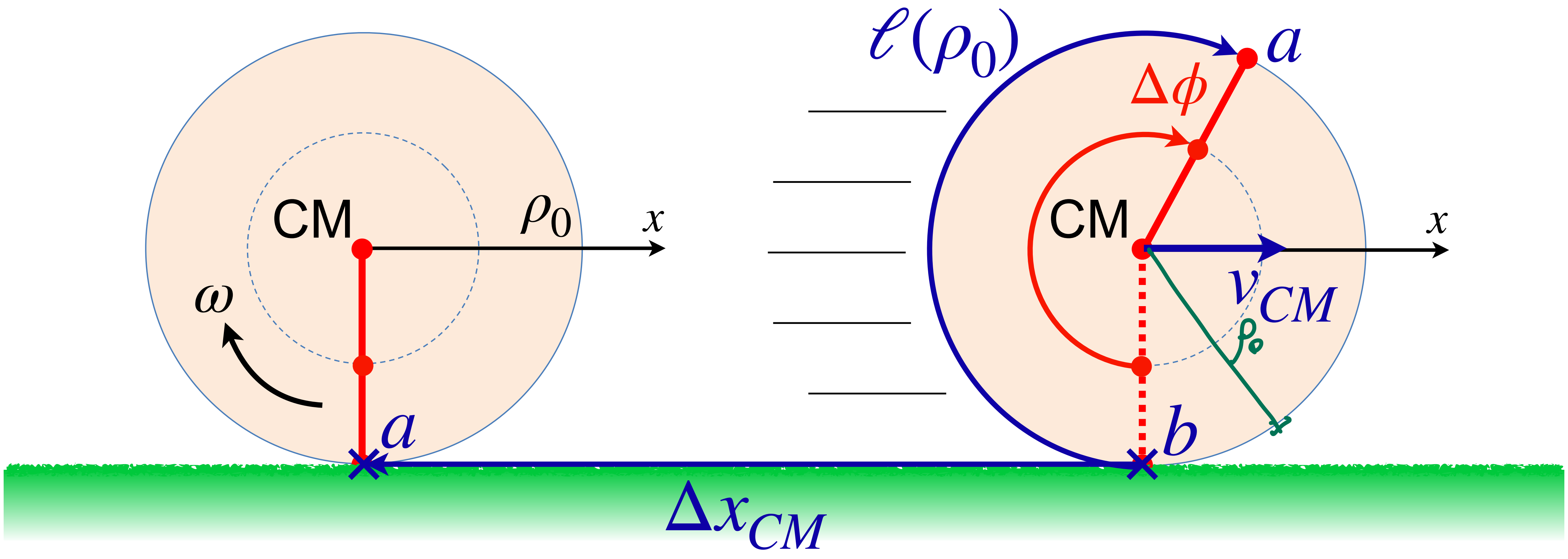
$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi$$



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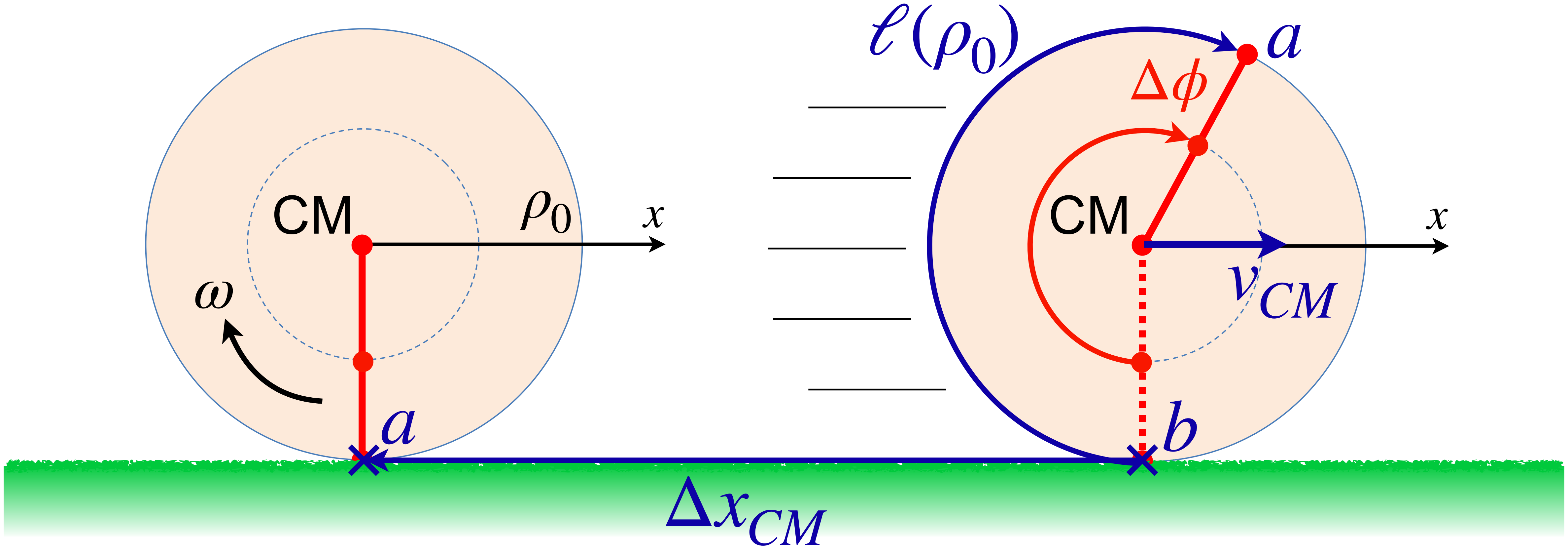
- If an object rolling on a surface does not slip, then the distance traveled along the ground must be equal to the distance traveled by the rim of the wheel

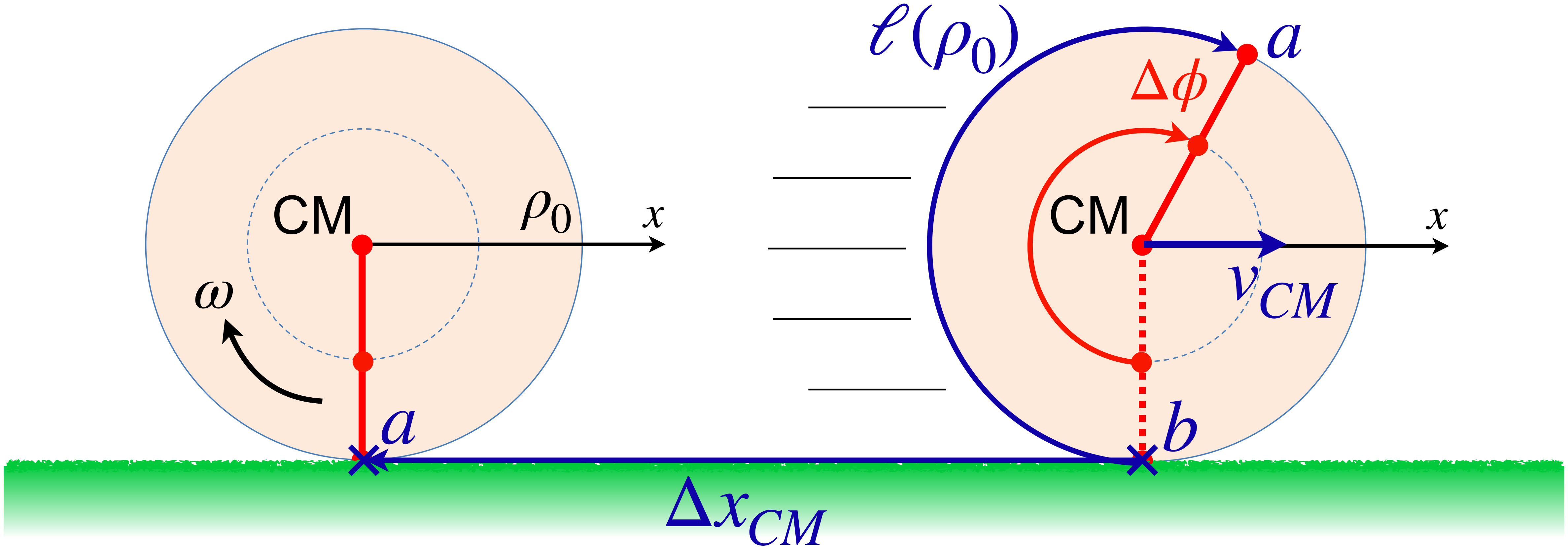
$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi \qquad \Rightarrow \lim_{\Delta t \to 0}\left[\frac{\Delta x_{CM}}{\Delta t}\right] = \lim_{\Delta t \to 0}\left[\rho_0 \frac{\Delta\phi}{\Delta t}\right]$$



Rolling without slipping

- If an object rolling on a surface does not slip, then the distance traveled along the ground must be equal to the distance traveled by the rim of the wheel

$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi \quad \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x_{CM}}{\Delta t} = \lim_{\Delta t \to 0} \rho_0 \frac{\Delta \phi}{\Delta t} \quad \Rightarrow \quad v_{CM} = \rho_0 \omega$$

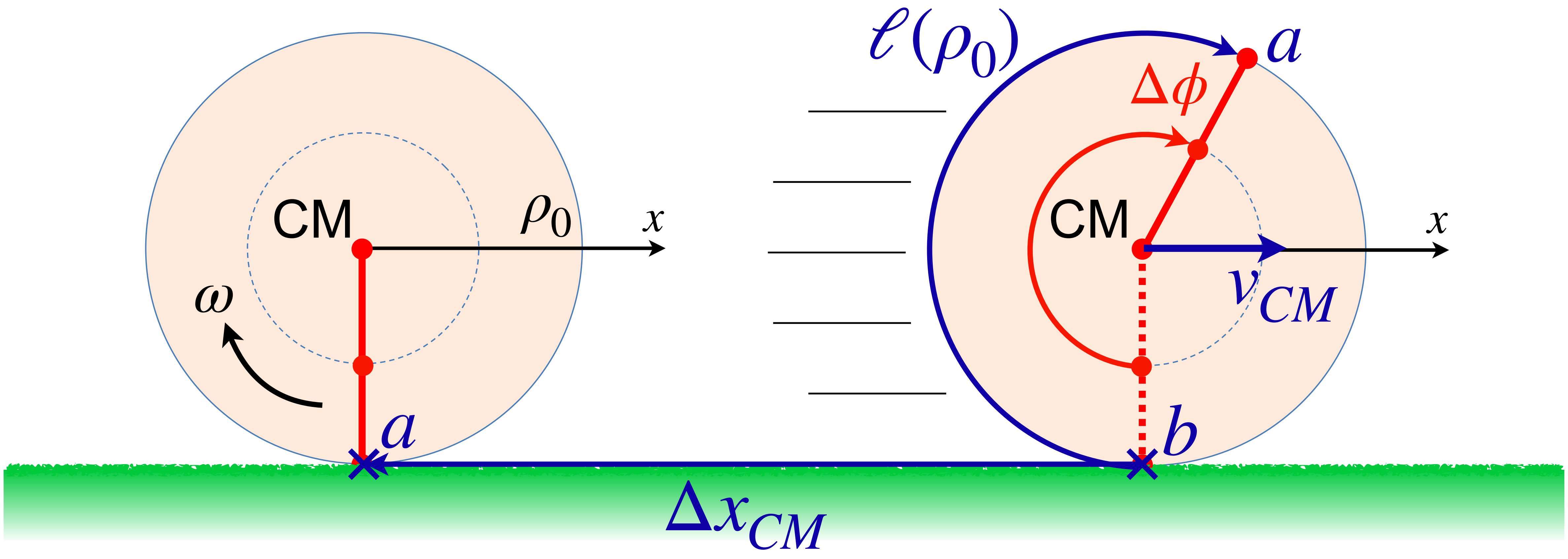


Rolling without slipping

- If an object rolling on a surface does not slip, then the distance traveled along the ground must be equal to the distance traveled by the rim of the wheel

$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi \quad \Rightarrow \quad \frac{\Delta x_{CM}}{\Delta t} = \rho_0 \frac{\Delta\phi}{\Delta t} \quad \Rightarrow \quad v_{CM} = \rho_0 \omega$$

- At point of contact b, $\vec{v}_{ground} = \vec{v}_{rim}$ (i.e. friction is static)



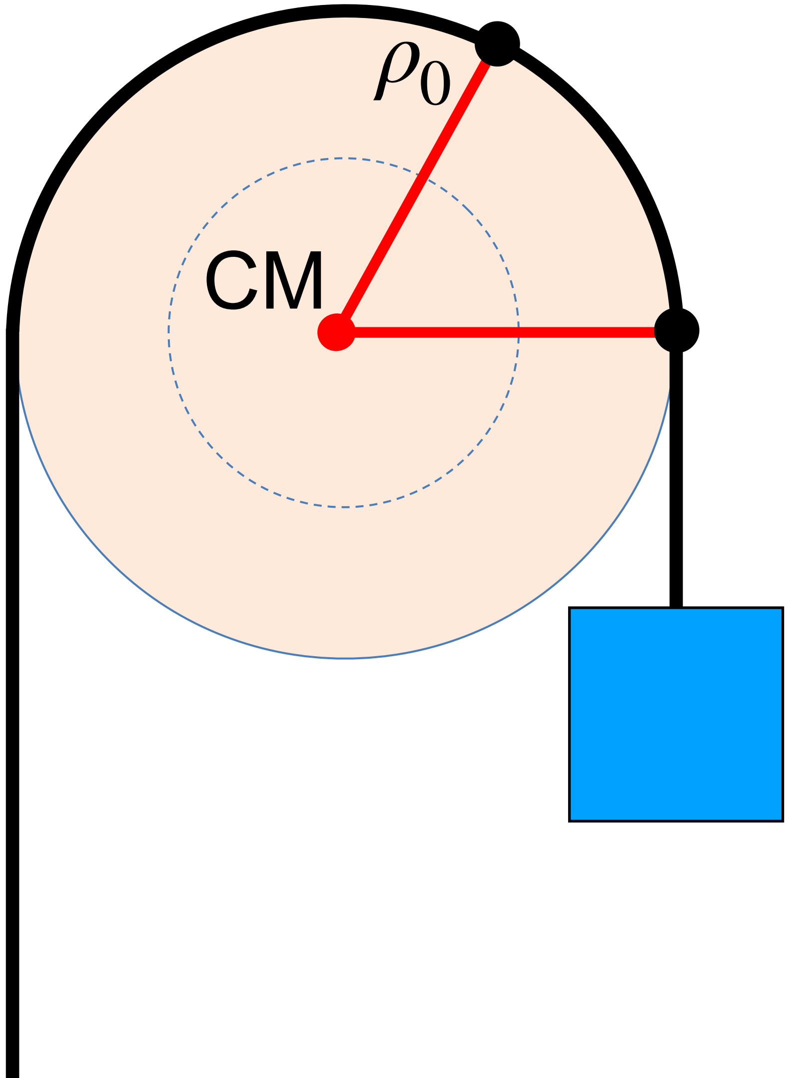
No-slip pulleys

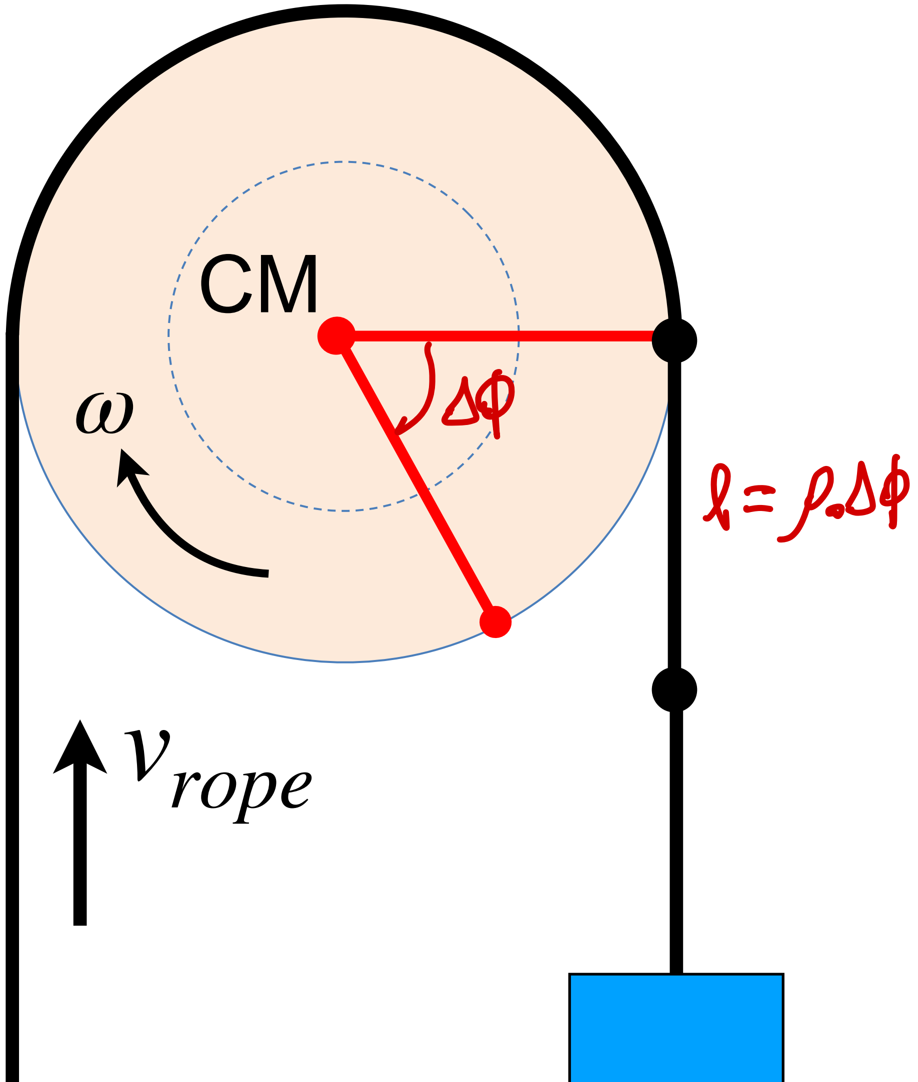
- If a rope rotates a pulley without slipping, then at the points of contact

$$v_{rope} = v_{rim} = \rho_0\omega$$

- Taking a derivative in time shows

$$a_{rope} = \rho_0\alpha$$





Rotational kinetic energy

- An object with no translational motion still has kinetic energy, if it is rotating

Rotational kinetic energy

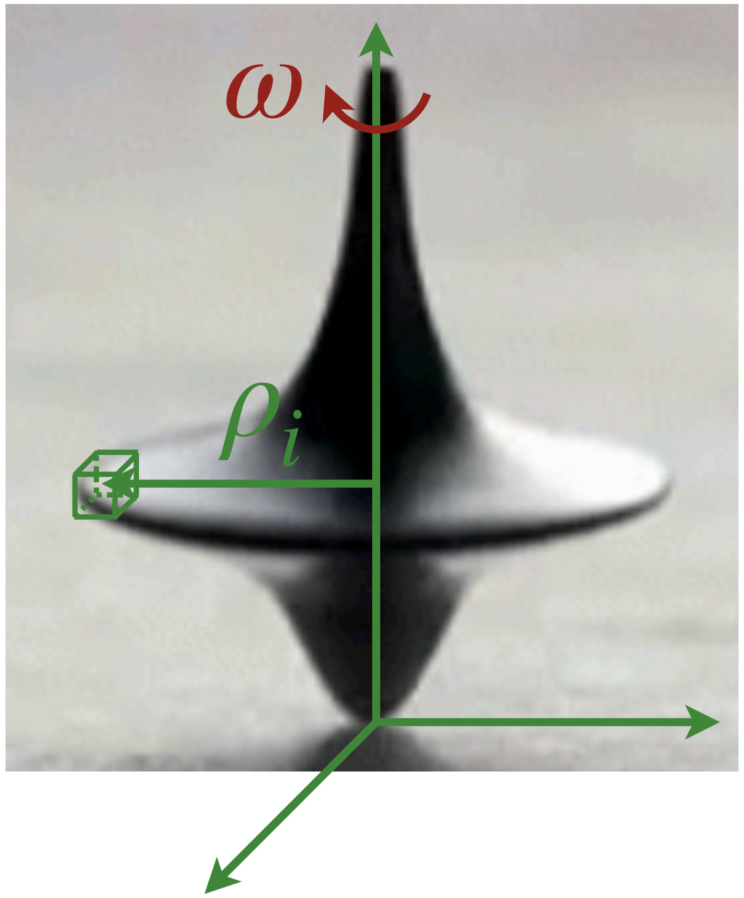
- An object with no translational motion still has kinetic energy, if it is rotating

- Rotational kinetic energy must be considered in conservation of energy

$$K = K^{trans} + K^{rot}$$

Rotational kinetic energy

- Again imagine an object is composed of many differential elements, labeled $i = 1, 2, 3, \ldots,$ at a distance ρ_i from the axis



Rotational kinetic energy

- Again imagine an object is composed of many differential elements, labeled $i = 1, 2, 3, \ldots,$ at a distance ρ_i from the axis

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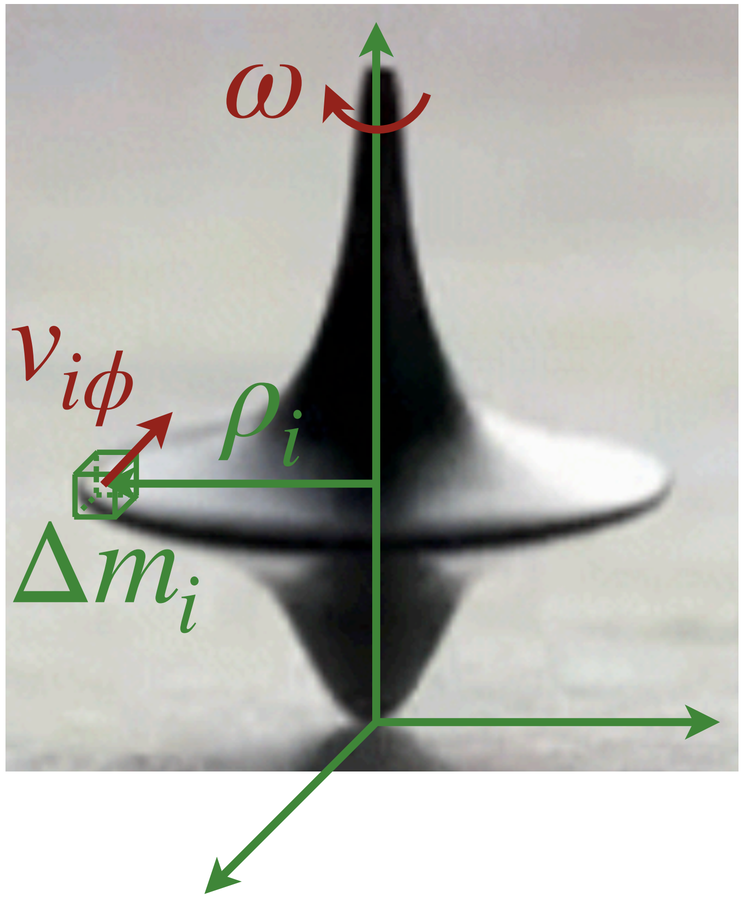

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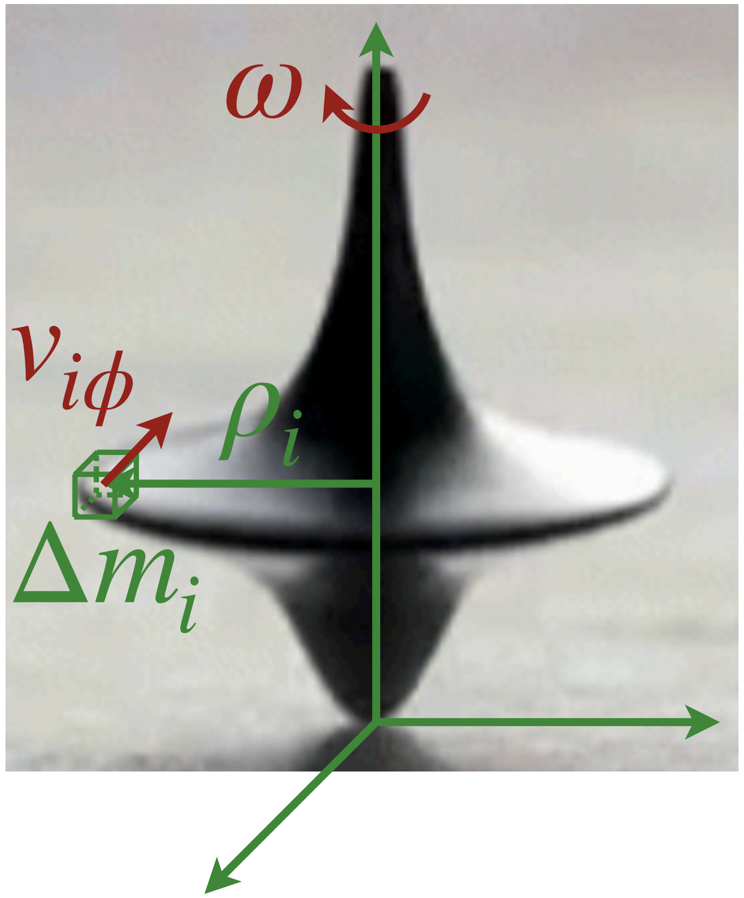

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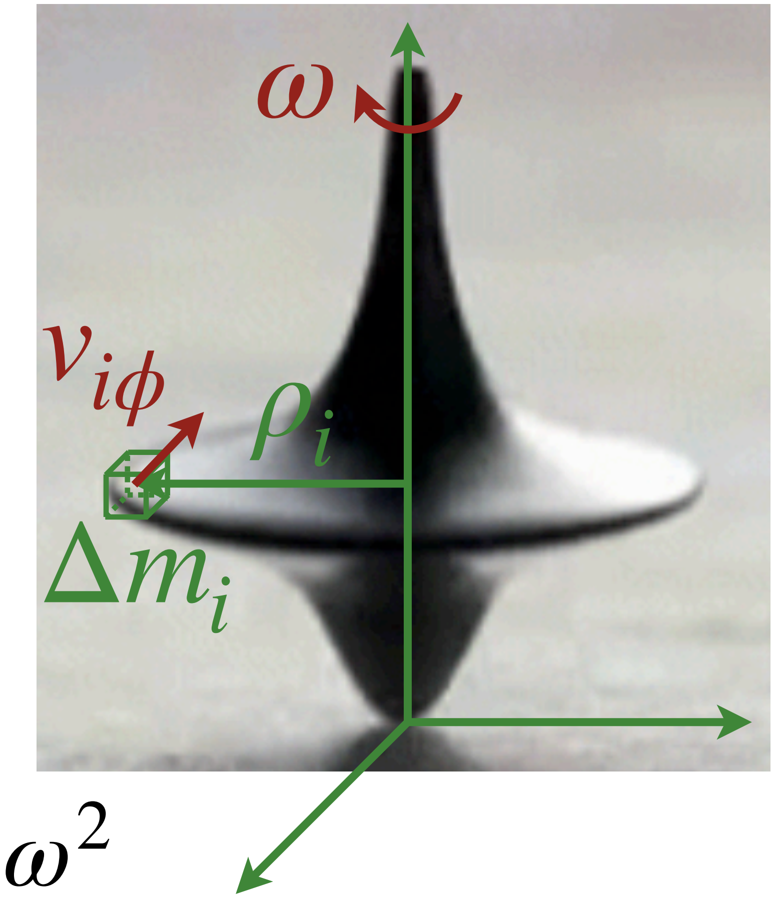

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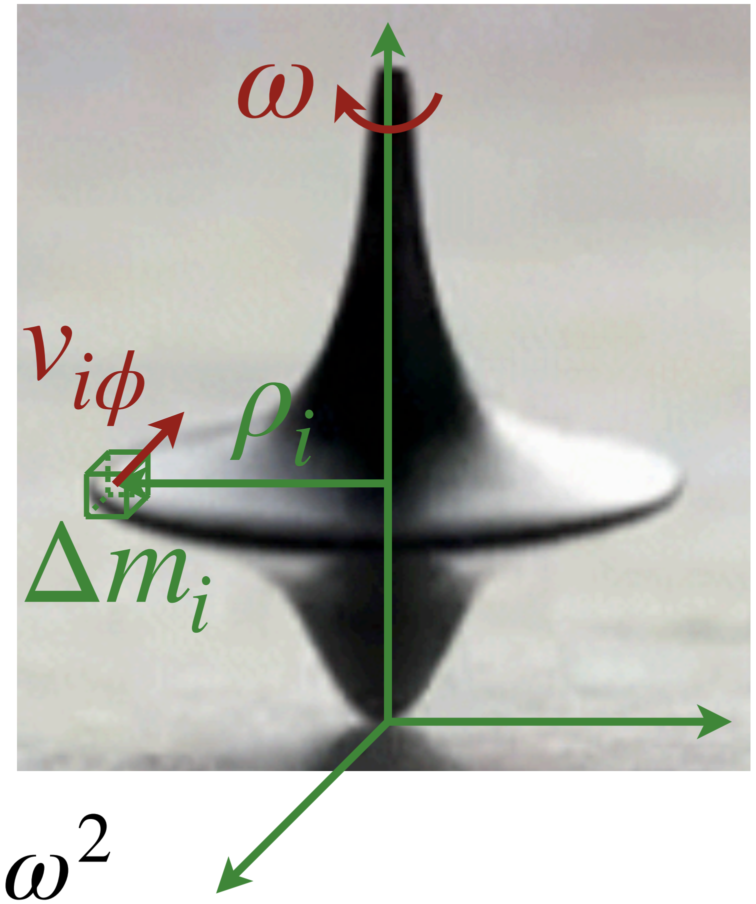

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- Thus, total kinetic energy is $K = \frac{m}{2}v^2 + \frac{I_{CM}}{2}\omega^2$

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- In the limit of infinitesimally small differential elements

$$I = \int_M \rho^2 dm$$

- Units of [kg·m^2]

Example: Uniform disk

$\rho_V = \frac{M}{V}$

What is the moment of inertia of a uniform disk with mass M, radius ρ_0, and height h, rotating about its axis of symmetry $\hat{z}$ at an angular velocity $\omega\hat{z}$?

$$I_{CM} = \int_M \rho^2 dm \qquad \frac{dm}{dV} = \rho_V = \frac{M}{V} = \frac{M}{\pi\rho_0^2 h} \Rightarrow dm = \frac{M}{\pi\rho_0^2 h} dV$$

$$I_{CM} = \int_M \rho^2 \frac{M}{\pi\rho_0^2 h} dV = \frac{M}{\pi\rho_0^2 h}\int_M \rho^2 dV$$

The volume of a cylinder of radius ρ and height h is $V = \pi\rho^2 h$

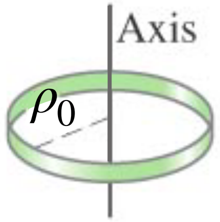
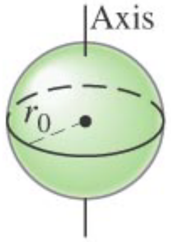
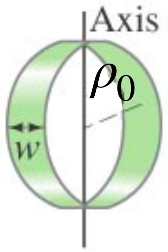
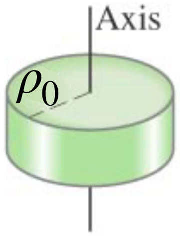
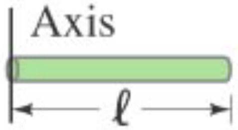
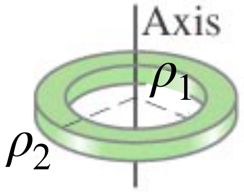
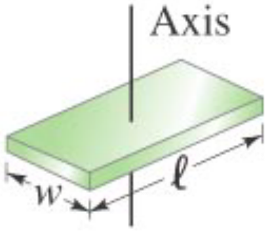
$$\frac{dV}{d\rho} = 2\pi\rho h \Rightarrow dV = 2\pi\rho h\, d\rho$$

$$I_{CM} = \frac{M}{\cancel{\pi}\rho_0^2 \cancel{h}}\int_0^{\rho_0}\rho^2(2\cancel{\pi}\rho\cancel{h})\, d\rho = \frac{2M}{\rho_0^2}\int_0^{\rho_0}\rho^3 d\rho = \frac{2M}{\rho_0^2}\left[\frac{\rho^4}{4}\right]_0^{\rho_0} = \frac{2M}{\rho_0^2}\frac{\rho_0^4}{4}$$

$$= \boxed{\frac{M}{2}\rho_0^2}$$

Moment of inertia for various uniform objects

- Moment of inertia depends on shape and mass distribution
- Also depends on the axis of rotation

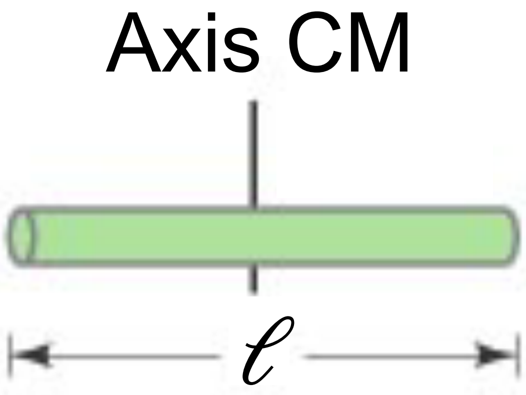
Object (rotation axis)	**Geometry**	**Moment of inertia**	**Object (rotation axis)**	**Geometry**	**Moment of inertia**
Thin hoop (about center)		$M\rho_0^2$	Uniform sphere (about center)		$\frac{2}{5}Mr_0^2$
Thin hoop (about diameter)		$\frac{M}{2}\rho_0^2 + \frac{M}{12}w^2$	Thin rod (about center)		$\frac{M}{12}\ell^2$
Solid cylinder (about center)		$\frac{M}{2}\rho_0^2$	Thin rod (about end)		$\frac{M}{3}\ell^2$
Hollow cylinder (about center)		$\frac{M}{2}\left(\rho_1^2 + \rho_2^2\right)$	Thin plate (about center)		$\frac{M}{12}\left(\ell^2 + w^2\right)$

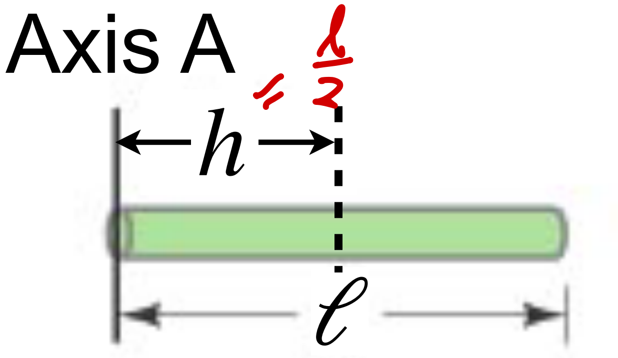
Parallel axis (or Steiner) theorem

- The moment of inertia about any axis parallel to an axis that goes through the center of mass is given by

$$I = I_{CM} + Mh^2$$

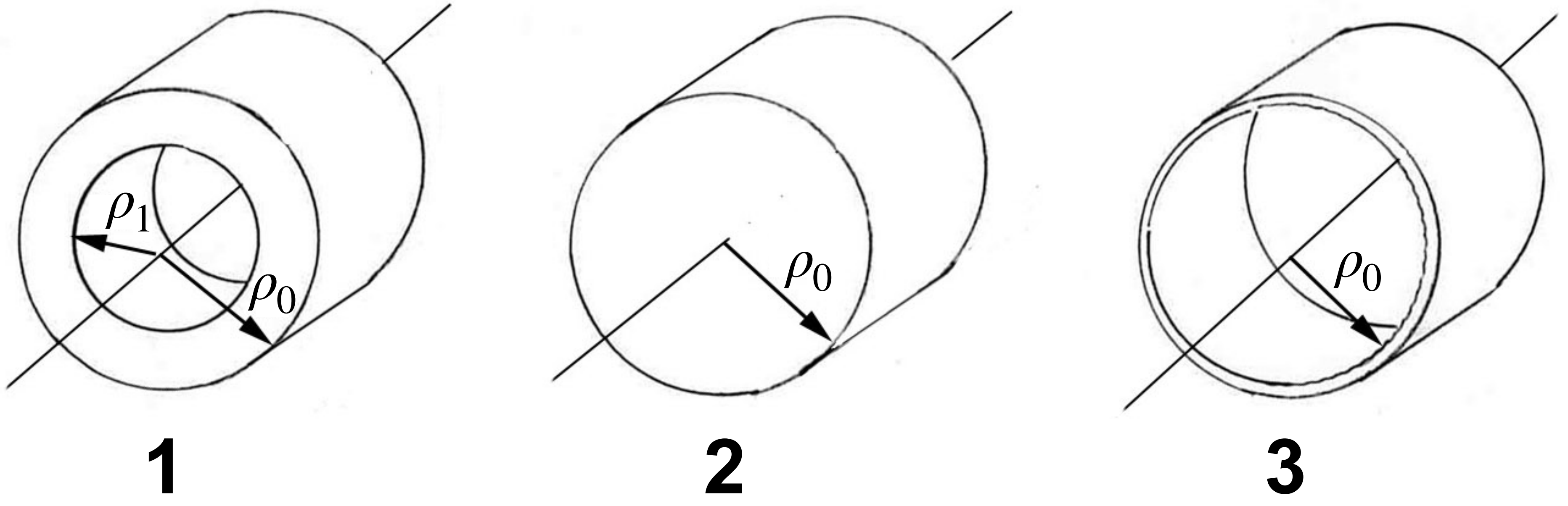
- For example:





$$I_A = I_{CM} + M\left(\frac{\ell}{2}\right)^2 = \frac{M}{12}\ell^2 + \frac{M}{4}\ell^2 = M\ell^2\left(\frac{1}{12}+\frac{1}{4}\right) = \frac{1}{3}M\ell^2$$

Conceptual question



All of the objects above have the same **mass**, the same **radius**, and are made of materials with different but **uniform density**. How are their moments of inertia about the axis related?

A. $I_3 > I_2 > I_1$

B. $I_1 > I_2 > I_3$

[✓ C.] $I_3 > I_1 > I_2$

D. $I_2 > I_1 > I_3$

DEMO (60): Racing cylinders

A cylinder with moment of inertia I, radius ρ_0, and mass m is initially at rest on an inclined plane. It rolls without slipping, descending a vertical distance h. What is its *translational* speed at the bottom?

$$\Delta E_m = 0$$

$$E_{mi} = \cancelto{0}{K_i^t} + \cancelto{0}{K_i^r} + U_{gi} = E_{mf} = K_f^t + K_f^r + U_{gf}$$

$$\Rightarrow K_f^t + K_f^r = U_{gi} - U_{gf} = mgh \quad ①$$

$$K_f^t + K_f^r = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \left(\frac{1}{\rho_0} v_{CM}\right)^2$$

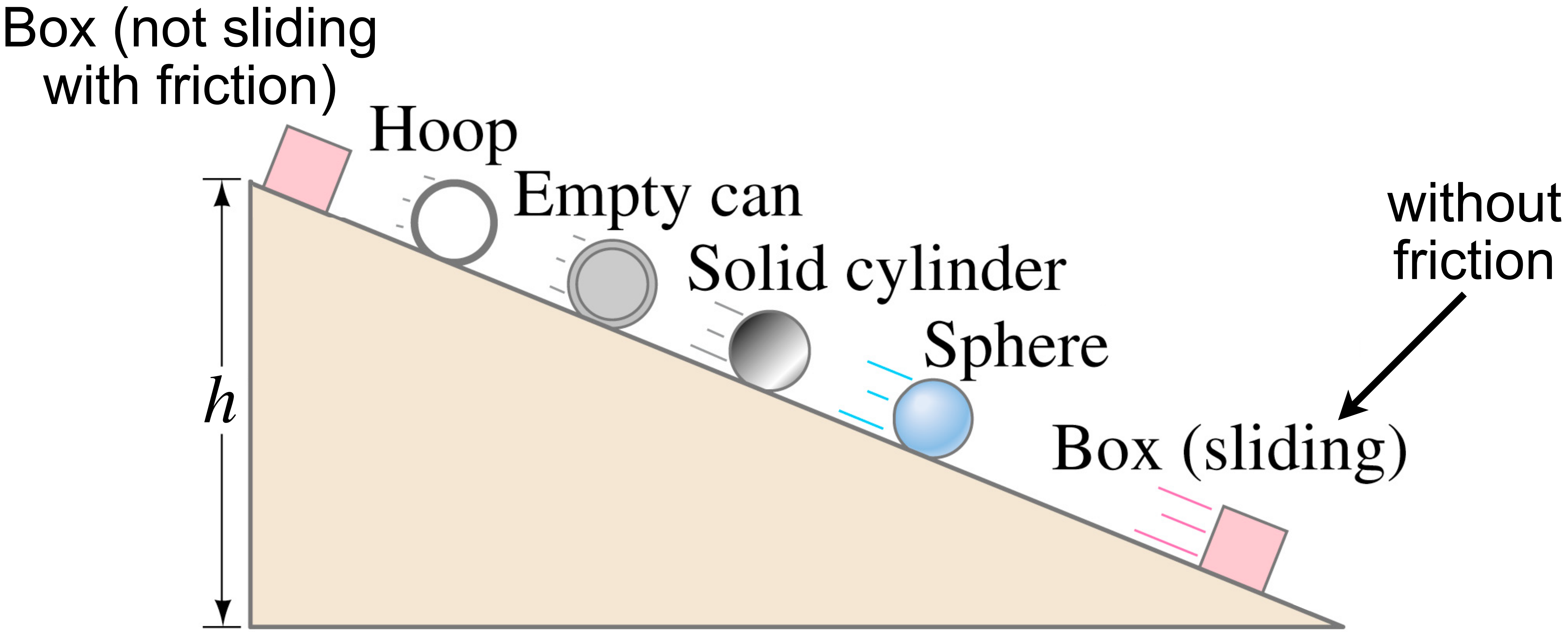
$$= \frac{1}{2} m v_{CM}^2 \left[1 + \frac{I}{m\rho_0^2}\right] \quad ②$$

$$v_{CM} = \rho_0 \omega \Rightarrow \omega = \frac{1}{\rho_0} v_{CM}$$

Substituting ② in ① we get: $\frac{1}{2} \cancel{m} v_{CM}^2 \left(1 + \frac{I}{m\rho_0^2}\right) = \cancel{m} g h$

$$v_{CM} = \sqrt{2gh\left(1 + \frac{I}{m\rho_0^2}\right)^{-1}}$$

Rotation steals energy from translation



- More rotational inertia means rotation takes more energy
- But rolling without slipping enables an object to avoid friction

Torque

- Defined to be the *moment* of a force about a pivot point

$$\vec{\tau} = \vec{r} \times \overrightarrow{F}$$

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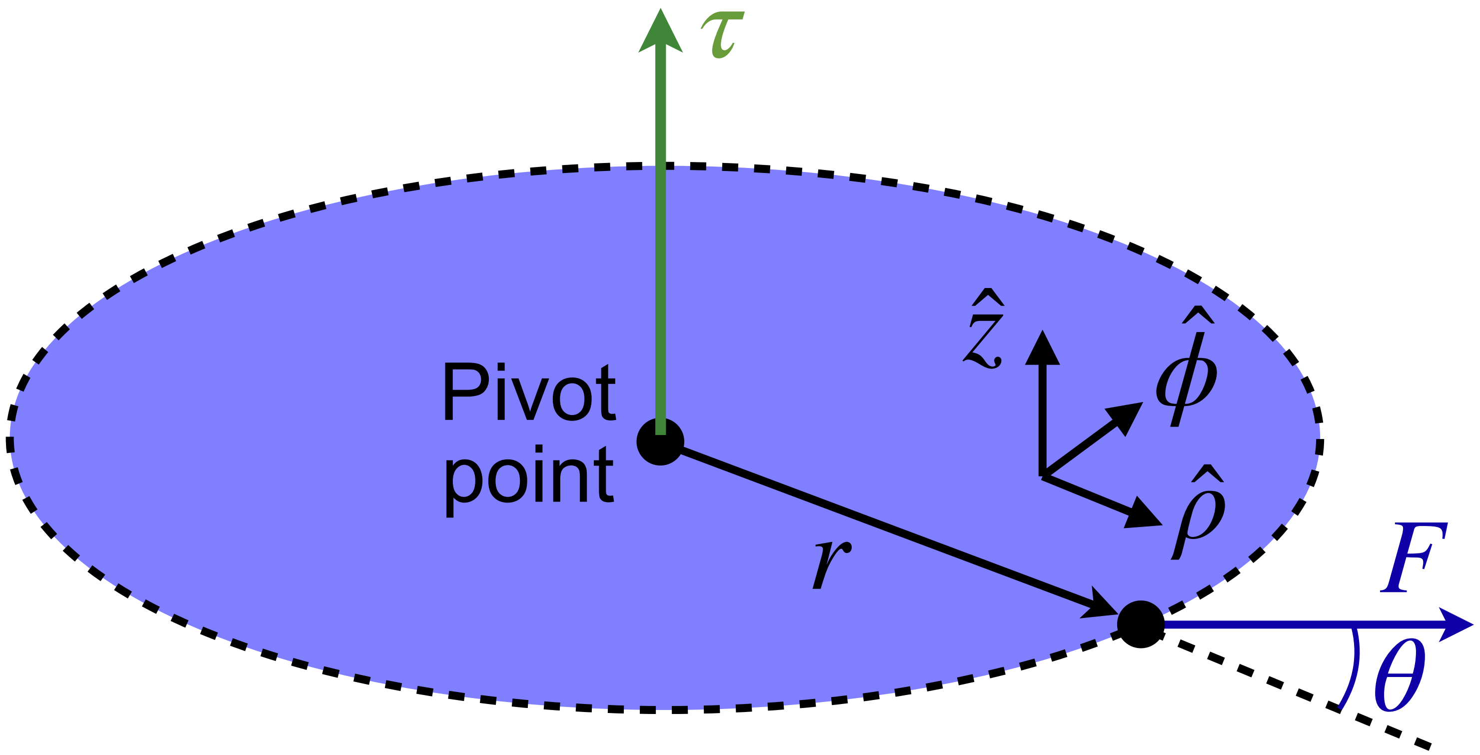

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$$\vec{\tau} = \vec{r} \times \overrightarrow{F} = rF\sin\theta\hat{z}$$

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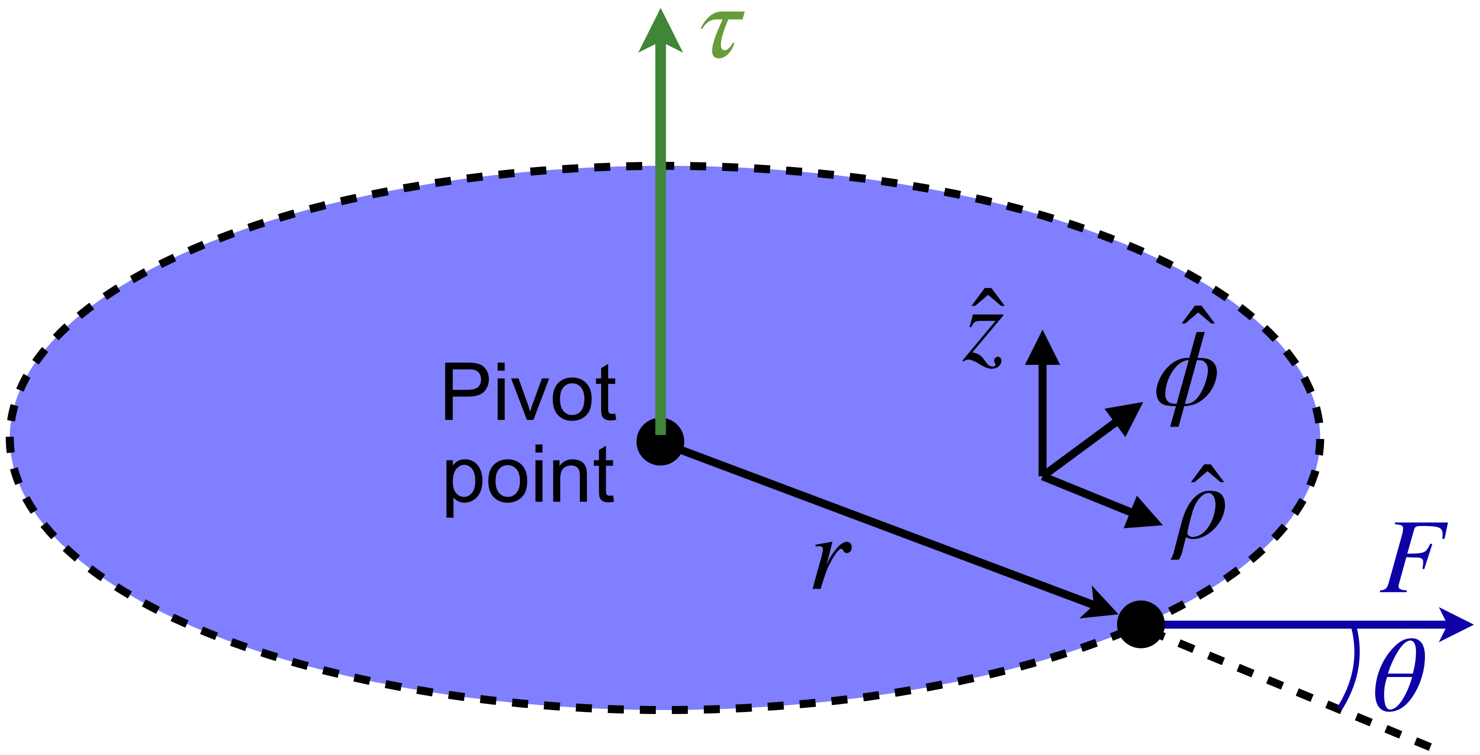

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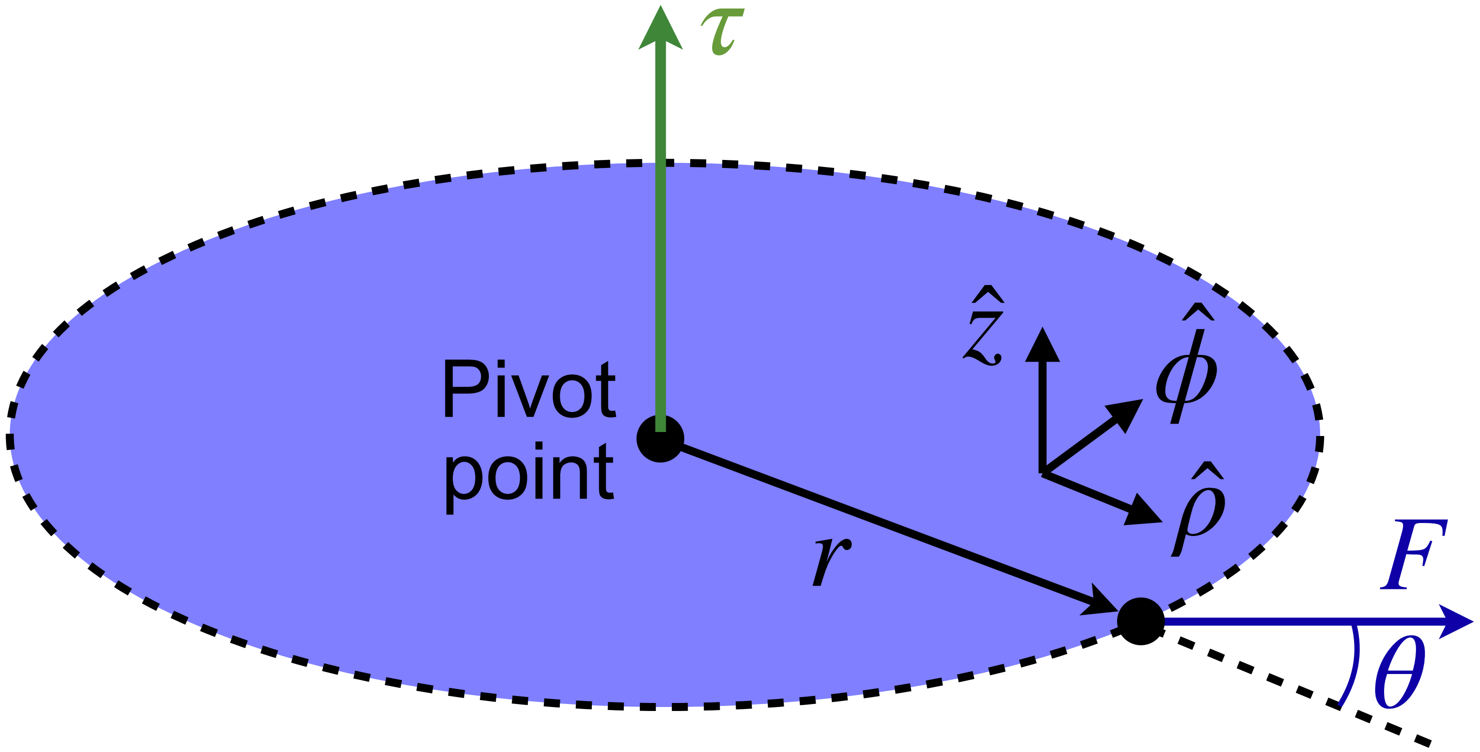
- It has units of [N·m], which is similar to [J], but torques are never expressed in Joules

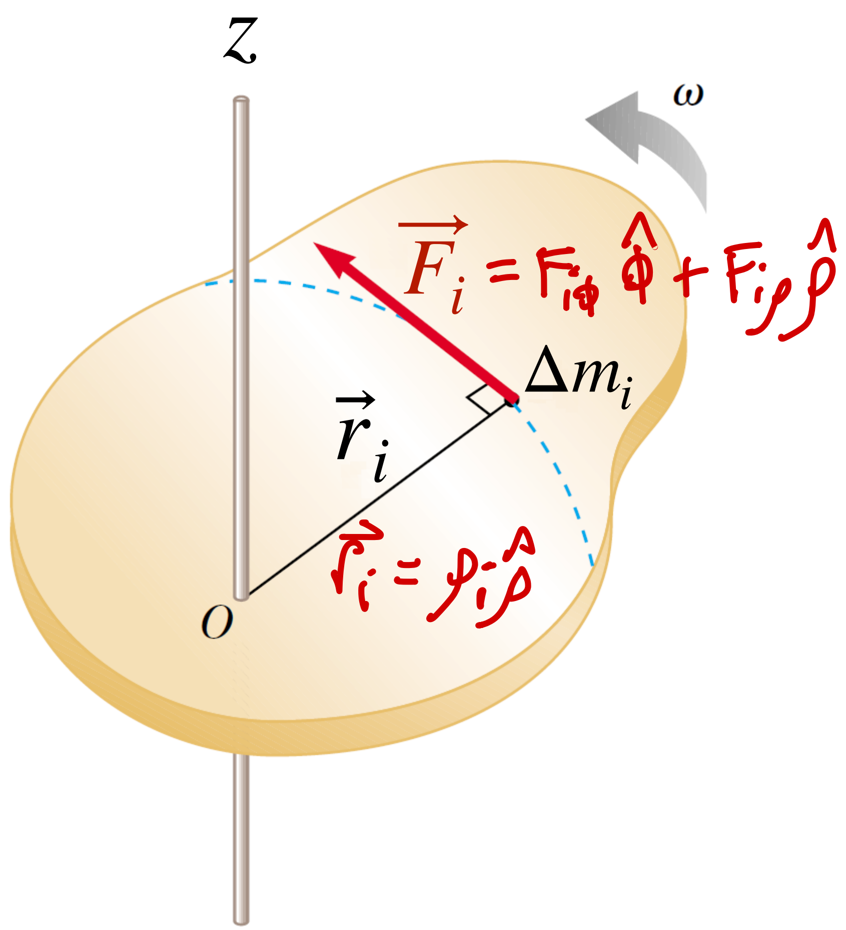
- It is the analogue of a force for rotation



Newton's laws for rotation about a fixed axis

- Consider rigid body rotation about a fixed axis

$$\hat{\rho} \times \hat{\rho} = 0$$
$$\hat{\rho} \times \hat{\phi} = \hat{z}$$



$$\vec{\tau}_i = \vec{r}_i \times \vec{F}_i = \rho_i \hat{\rho} \times (F_{i\phi}\hat{\phi} + F_{i\rho}\hat{\rho})$$
$$= \rho_i F_{i\phi} \hat{\rho} \times \hat{\phi} = \rho_i F_{i\phi} \hat{z}$$

Now $F_{i\phi} = \Delta m_i a_{i\phi} = \Delta m_i \cdot \rho_i \alpha$ $\quad | \; a_{i\phi} = \rho_i \alpha$

$$\Rightarrow \vec{\tau}_i = \rho_i (\Delta m_i \cdot \rho_i \alpha) \hat{z} = \rho_i^2 \Delta m_i \alpha \hat{z}$$

Total torque: $\vec{\tau}_{net} = \sum_i \vec{\tau}_i$

$$\Rightarrow \vec{\tau}_{net} = \sum_i \rho_i^2 \Delta m_i \alpha \hat{z}$$
$$= \alpha \left[\sum_i \rho_i^2 \Delta m_i \right] \hat{z}$$
$$= \alpha I_z \hat{z} = \boxed{I_z \vec{\alpha}}$$

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- Fictitious forces act at the center of mass

DEMO (22)

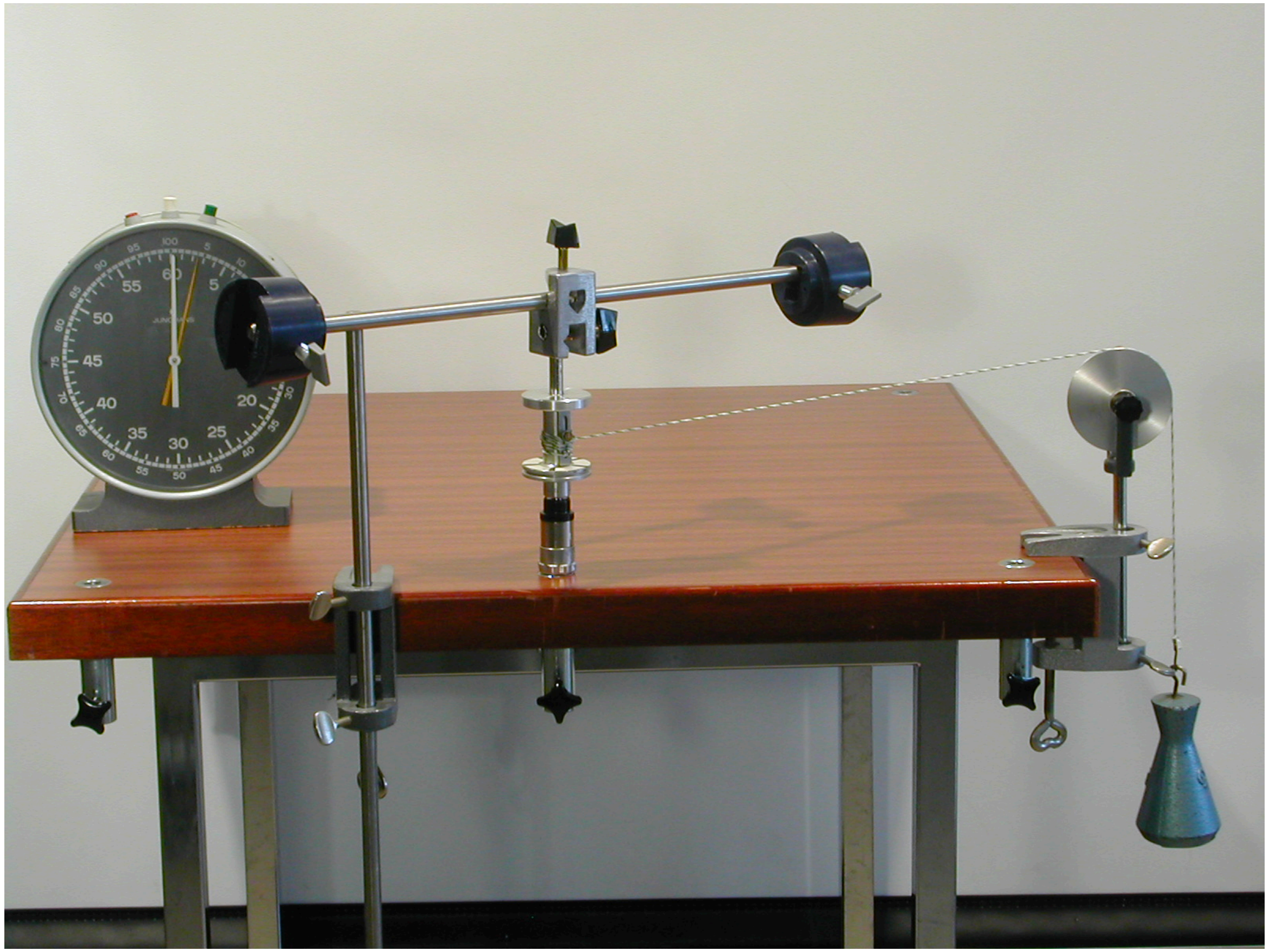
Torque and static equilibrium

DEMO (30): Conceptual question

responseware.eu
Session ID: epflphys101en

A fixed torque is applied to rotate the shaft of a beam. If the two weights on the beam are slid out, the angular acceleration of the wheel will…

A. increase.
[✓ B.] decrease.
C. remain the same.
D. Not enough information.



Summary of rotation and translation

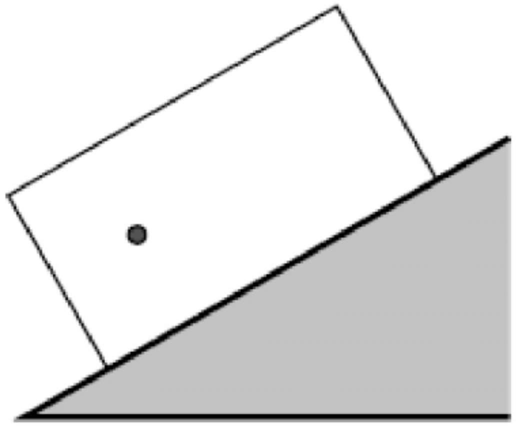
Rotational motion (about a fixed axis)		**Translational motion (in one dimension)**	
Angular position	ϕ	Position	x
Angular speed	$\omega = d\phi/dt$	Speed	$v = dx/dt$
Angular acceleration	$\alpha = d\omega/dt$	Acceleration	$a = dv/dt$
Moment of inertia	$I = \int \rho^2 dm$	Mass	m
Net torque	$\Sigma\tau_{ext} = I\alpha$	Net force	$\Sigma F_{ext} = ma$
Rotational kinetic energy	$K^{rot} = I\omega^2/2$	Translational kinetic energy	$K^{trans} = mv^2/2$
Work	$W = \int_{\phi_a}^{\phi_b} \tau d\phi$	Work	$W = \int_{x_a}^{x_b} F dx$
Power	$P = \tau\omega$	Power	$P = Fv$
Angular momentum	$L = I\omega$	Momentum	$p = mv$
Net torque	$\Sigma\tau_{ext} = dL/dt$	Net torque	$\Sigma F_{ext} = dp/dt$

Conceptual question

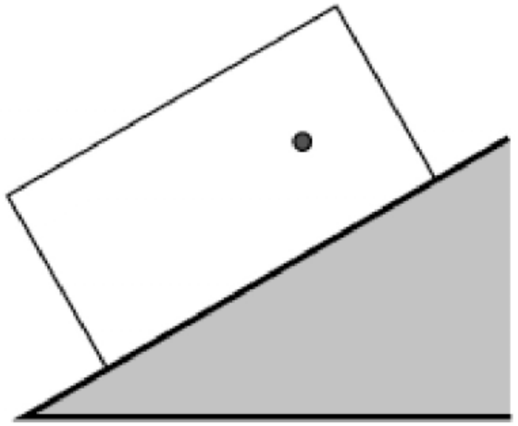
responseware.eu
Session ID: epflphys101en

A box, with its center of mass indicated by the dot, is placed on an inclined plane. In which of the four orientations shown, if any, does the box tip over?

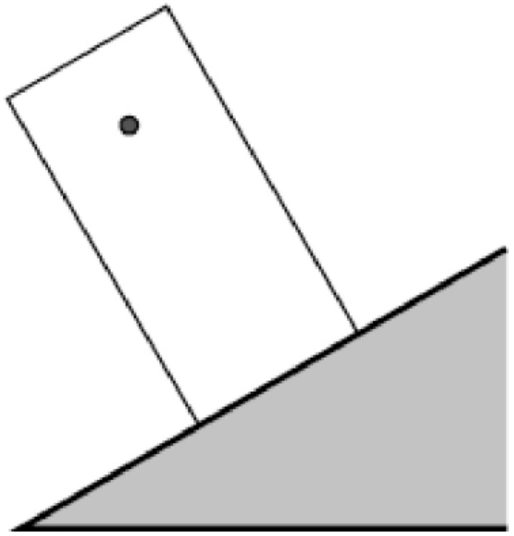
A. 1
B. 2
[✓ C. 3]
D. 4
E. None of them.



1



2



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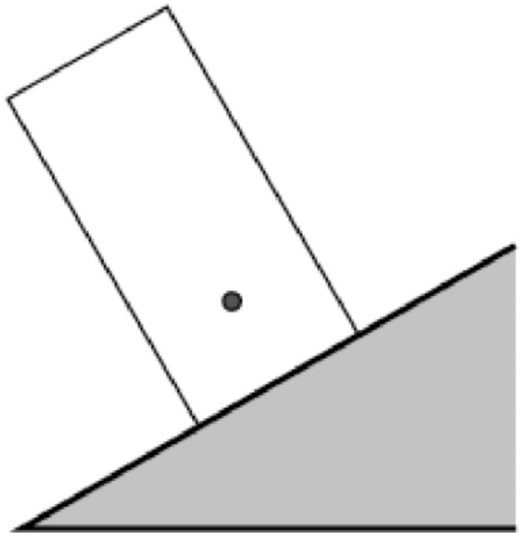


4

Conceptual question

An object is in static equilibrium when the net force and the net torque on it are zero. Which of the following statements are correct for an object in an inertial frame of reference?

A. Any object in equilibrium is at rest
[✓ B.] An object in equilibrium need not be at rest
C. An object at rest must be in equilibrium

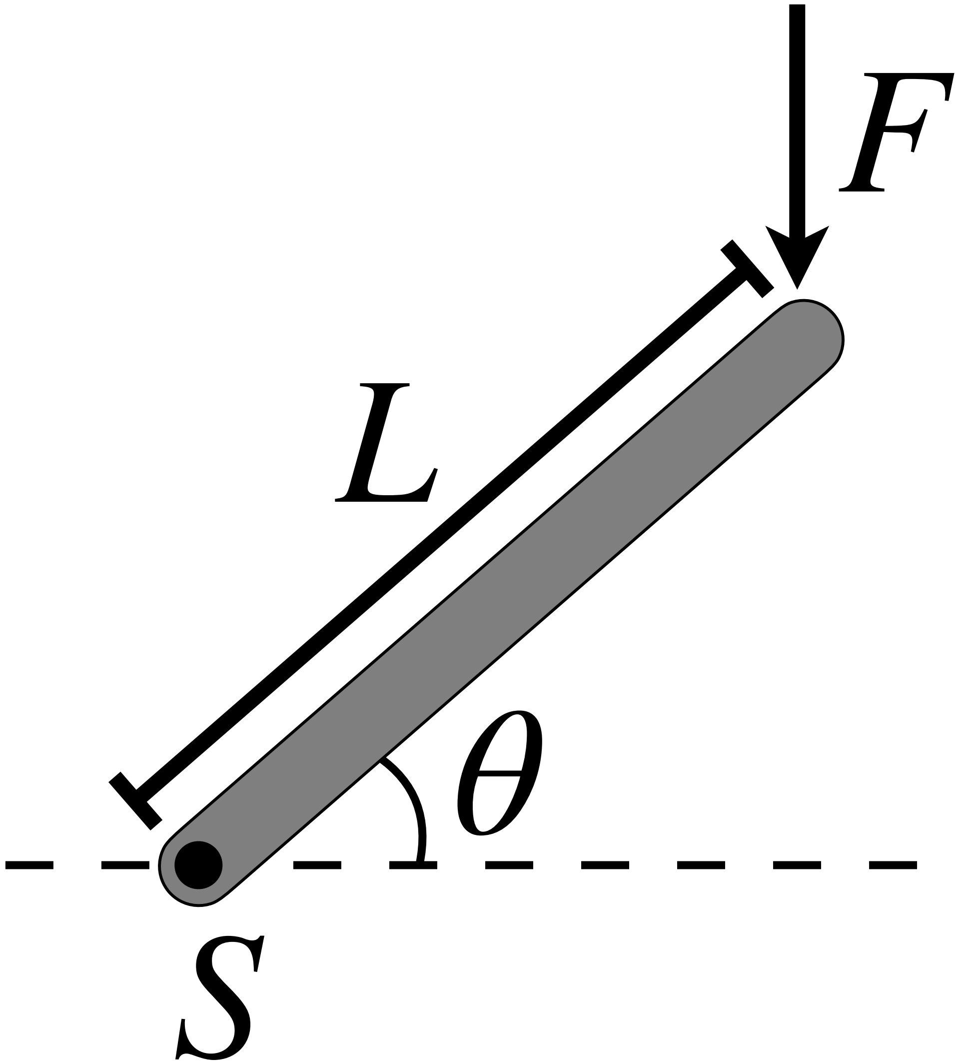
Conceptual question

Point A sits at the outer edge (rim) of a merry-go-round, and point B sits halfway between the rim and the axis of rotation. The merry-go-round makes a complete revolution once every thirty seconds. The magnitude of the average angular velocity of point B is…

A. half the angular speed of point A.
[✓ B.] the same as the angular speed of point A.
C. twice the angular speed of point A.
D. Not enough information is given to decide.

Conceptual question

In the figure, a force of magnitude F is applied to one end of a lever of length L. What is the magnitude of the torque about the point S?



A. $FL\sin\theta$
[✓ B.] $FL\cos\theta$
C. $FL\tan\theta$
D. None of the above.