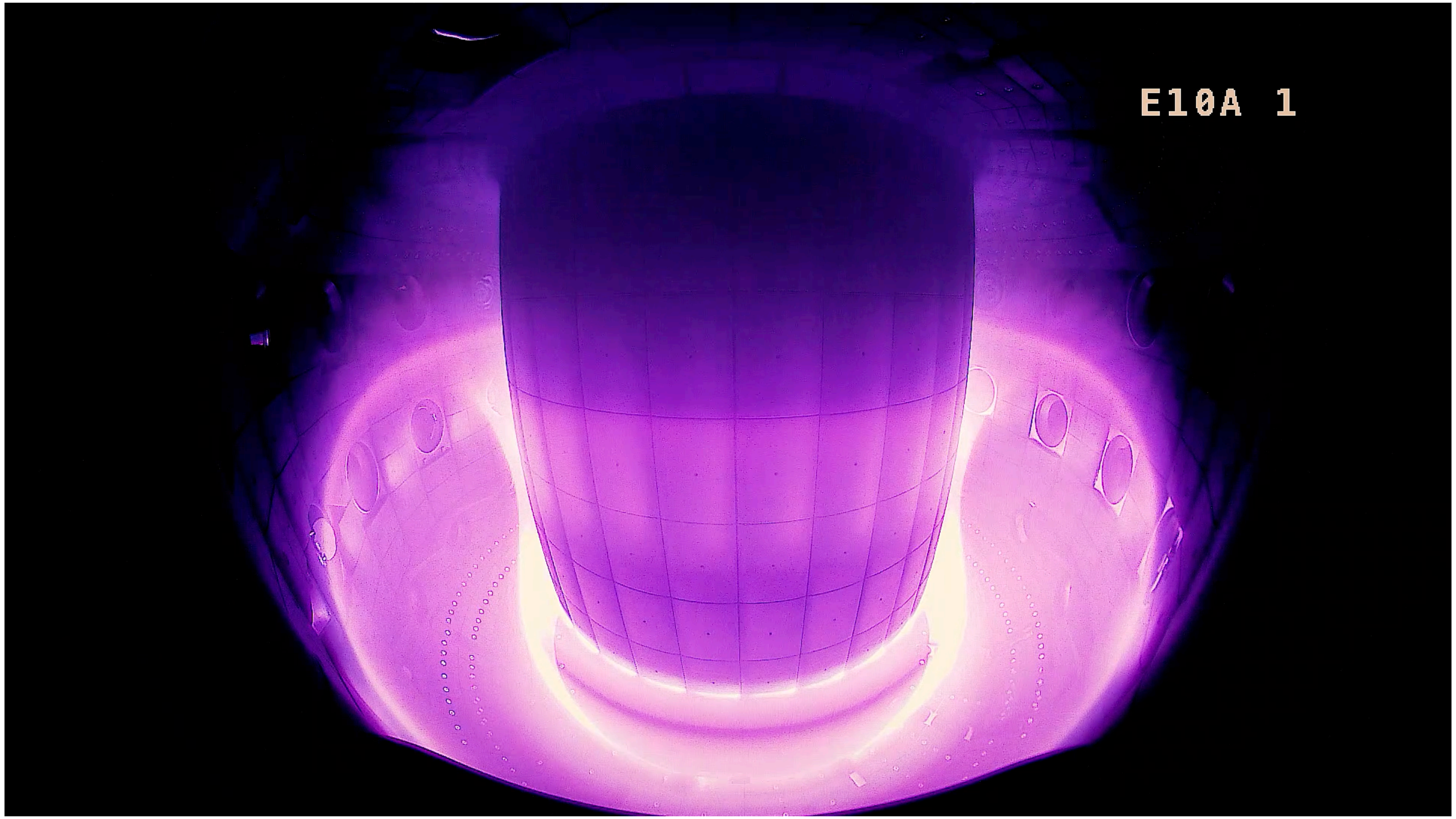
General Physics: Mechanics



PHYS-101(en)

Lecture 10b: Collisions

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November 19th, 2024

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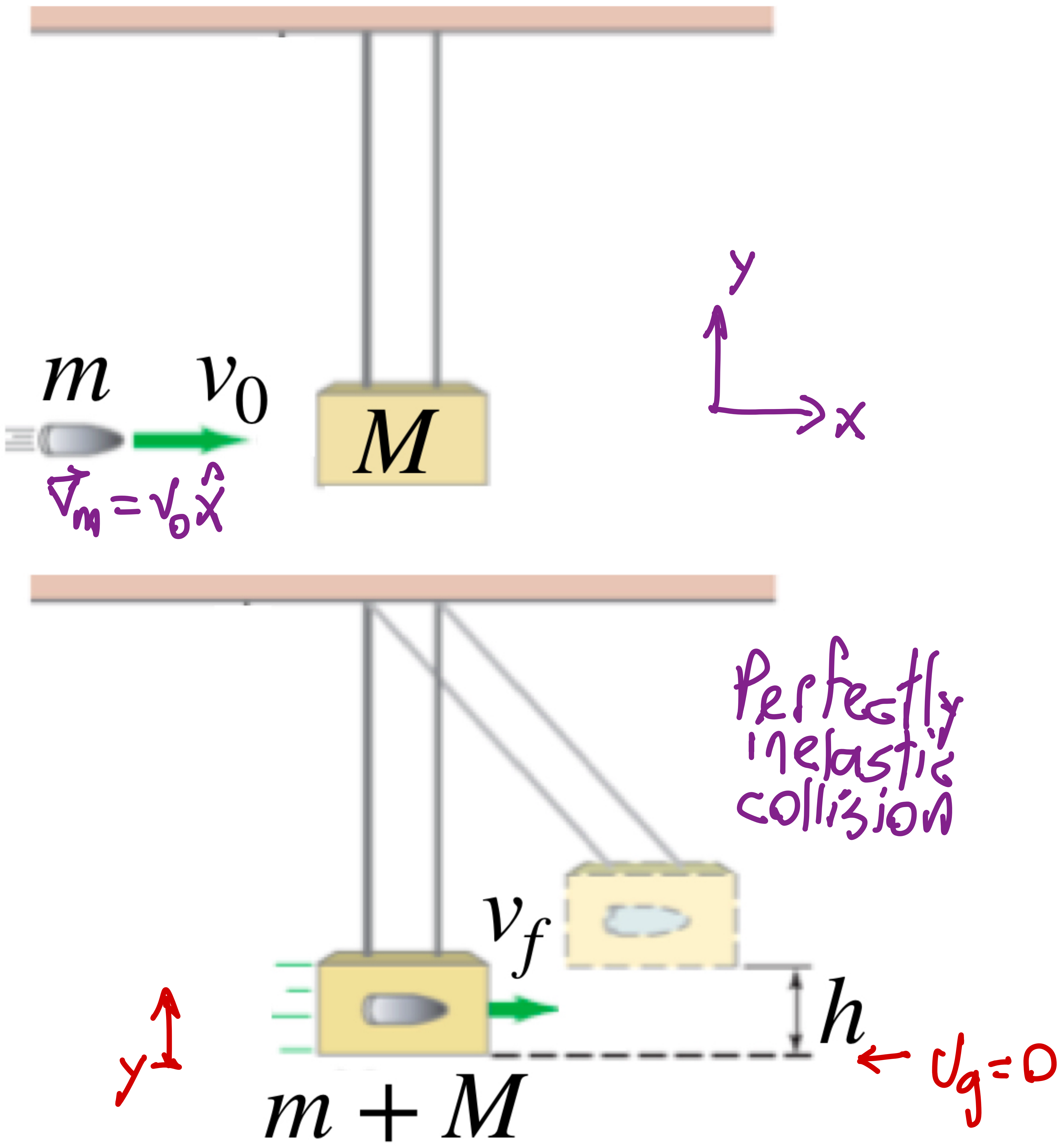
How to measure the speed of a bullet

Any projectile, of mass m, is fired into a large block of mass M, which is suspended like a pendulum. As a result of the collision, the pendulum and projectile swing up together to a maximum height h.

Calculate the initial horizontal speed of the projectile v_0 from the maximum height h.

Ballistic pendulum



How to measure the speed of a bullet

Cons. of momentum (due to impulse approx.): $\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow m\vec{v}_m + \cancel{M\vec{v}_M}^{\,0} = (m+M)\vec{v}_f$

$\Rightarrow \vec{v}_f = \frac{m}{m+M}\vec{v}_m = \frac{m}{m+M} v_0 \hat{x}$

After collision we have $\Delta E_m = 0$: $K_i + \cancel{U_{gi}}^{\,0} = \cancel{K_f}^{\,0} + U_{gf} \Rightarrow$

$\Rightarrow K_i = \frac{1}{2}\cancel{(m+M)}v_f^2 = U_{gf} = \cancel{(m+M)}gh$

$\Rightarrow v_f^2 = 2gh \Rightarrow |v_f| = \sqrt{2gh}$

$\Rightarrow \frac{m}{m+M}|v_0| = \sqrt{2gh} \Rightarrow \boxed{|v_0| = \frac{m+M}{m}\sqrt{2gh}}$

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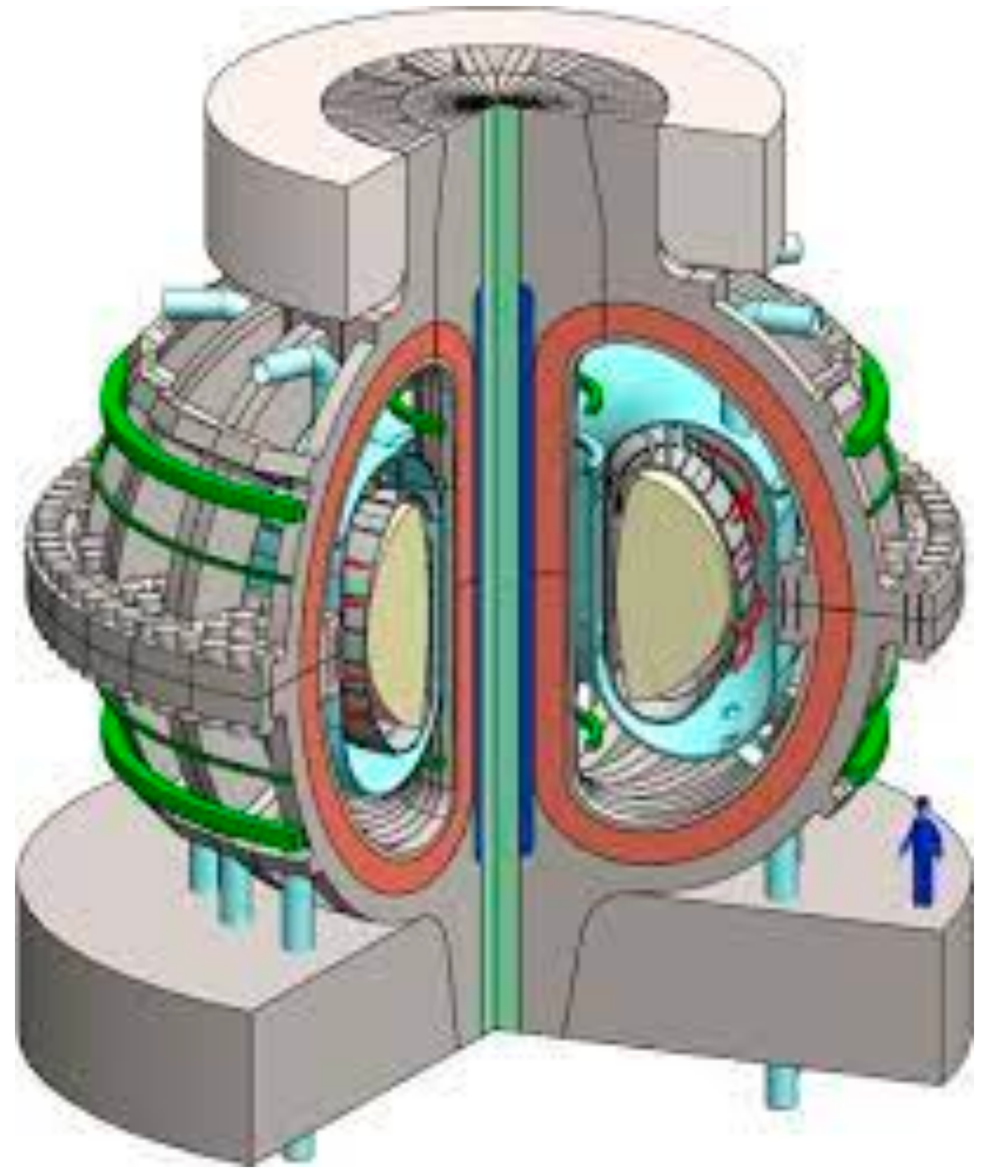
Example: Fusion reactions

In a fusion reactor, isotopes of hydrogen are confined as they are heated to temperatures exceeding those of the sun (i.e. 100 million Celsius = 0.01 MeV) in order to enable the fusion reaction

$${}^2\overset{D}{H} + {}^3\overset{T}{H} \rightarrow {}^4\overset{\alpha}{He} + {}^1\overset{n}{n} .$$ AMU

$m_D = 2AMU$ $m_T = 3$ $m_\alpha = 4$ $m_n = 1$



This reaction is inelastic as it releases $Q = 17.6$ MeV of energy, which is carried away in the form of the kinetic energy of the products. If the ${}^3H$ particle is at rest and the ${}^2H$ particle has 0.01 MeV of kinetic energy, how much energy does the neutron end up with?

Hints:

- You can still write down a version of conservation of energy, but you need to consider all the types of energy involved
- Look for an approximation that simplifies the problem significantly and barely affects the final answer

Example: Fusion reactions

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Energy budget: $\underbrace{K_D}_{\approx 0}^{\approx 0.01\,\mathrm{MeV}} + \cancel{K_T}^{0} + \overset{\approx 17.6\,\mathrm{MeV}}{Q} = K_\alpha + K_n \quad \Rightarrow v_D \approx 0$ (we approximate it due to $K_D \approx 0$)

$\Rightarrow Q \approx K_\alpha + K_n \quad (1)$

Cons. of momentum: $\overset{\approx 0}{\cancel{\vec{p}_D}} + \overset{0}{\cancel{\vec{p}_T}} = \vec{p}_\alpha + \vec{p}_n \Rightarrow 0 \approx \vec{p}_\alpha + \vec{p}_n \Rightarrow \vec{p}_\alpha = -\vec{p}_n \Rightarrow m_\alpha \vec{v}_\alpha \approx -m_n \vec{v}_n$

Squaring, $m_\alpha^2 v_\alpha^2 = m_n^2 v_n^2 \Rightarrow \frac{1}{2} m_\alpha v_\alpha^2 = \frac{m_n}{m_\alpha} \cdot \frac{1}{2} m_n v_n^2 \Rightarrow K_\alpha = \frac{m_n}{m_\alpha} K_n \quad (2)$

Now replace (2) in (1):

$$Q = K_\alpha + K_n = \frac{m_n}{m_\alpha} K_n + K_n = \left(\frac{m_n}{m_\alpha} + 1\right) K_n = \left(\frac{m_n + m_\alpha}{m_\alpha}\right) K_n \Rightarrow \boxed{K_n = \frac{m_\alpha}{m_n + m_\alpha} Q}$$

Replacing values $K_n = \frac{4\,\mathrm{AMU}}{1\,\mathrm{AMU} + 4\,\mathrm{AMU}} \cdot 17.6\,\mathrm{MeV} = \frac{4}{5} \cdot 17.6\,\mathrm{MeV} = 14.1\,\mathrm{MeV}$

$K_\alpha = \frac{1}{4} K_n \approx 3.5\,\mathrm{MeV}$