Uebersicht Allgemeine Physik I

Kinematik	Dynamik & Gravitation (Newton)	Arbeit & Erhaltungssätze	Starrer Körper (Drehbewegungen)
Wie beschreibt man die Bewegung eines Objektes ?	Wodurch wird ein Objekt in Bewegung gesetzt ?	Welche Grössen sind erhalten während dieser Bewegungen ?	Wie charakterisiert man die Bewegung eines starren Körpers ?
Einführung geradliniege Kinematik; Vektorielle Kinematik; **Kinematik Drehbewegungen**	Lineare Dynamik; **Dynamique der Drehung**; Physique der Planeten	Arbeit & Ènergie; **Schwingungen**; Impuls und Drehimpuls	Massenschwerpunkt & Drehmoment; **Rollende Bewegungen**; **Drehimpuls**

Erstellen

1. Einer guten Arbeitsweise
2. Mathekompetenzen

Physik des Massenpunktes
(Alle Gesetze des Kurses eingeführt)

Verallgemeinerung des starren Körpers

Teil diese Kurses wurde schon im Grundlagenfach Physik der Schweizerischen Maturität eingeführt

Kurs mit besonders schwierigem Inhalt

1: Kinematik der Translation

I. Was ist Physik ?

Der Ansatz des Physikers

Analyse der Einheiten

Approximation/Abschätzung

II. Wie beschreibt man eine Translationsbewegung ?

Momentan-Geschwindigkeit und -Beschleunigung

Koordinatensysteme

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(3 Seiten)*:

- 1-7 Dimensions and dimensional analysis
- 2-1 Reference frames and displacement

Vorbereitende Übungen *(12)* **vor der Übungssession** zu erledigen**:**

- Das ch. 2.2 ex. P17; ch. 2.5 ex. 8
- Giancoli 1-2, 8, 36, 37;
- *MRUA (6):* Giancoli 2-1, 2-4, 2-21, 2-27, 2-30, 2-41

Giancoli Kapitel 1-3 bis 1-7 und 2-1 bis 2-5

*1–7 Dimensions and Dimensional Analysis

When we speak of the **dimensions** of a quantity, we are referring to the type of base units or base quantities that make it up. The dimensions of area, for example, are always length squared, abbreviated $[L^2]$, using square brackets; the units can be square meters, square feet, cm^2, and so on. Velocity, on the other hand, can be measured in units of km/h, m/s, or mi/h, but the dimensions are always a length $[L]$ divided by a time $[T]$: that is, $[L/T]$.

The formula for a quantity may be different in different cases, but the dimensions remain the same. For example, the area of a triangle of base b and height h is $A = \frac{1}{2}bh$, whereas the area of a circle of radius r is $A = \pi r^2$. The formulas are different in the two cases, but the dimensions of area are always $[L^2]$.

Dimensions can be used as a help in working out relationships, a procedure referred to as **dimensional analysis**. One useful technique is the use of dimensions to check if a relationship is *incorrect*. Note that we add or subtract quantities only if they have the same dimensions (we don't add centimeters and hours); and the quantities on each side of an equals sign must have the same dimensions. (In numerical calculations, the units must also be the same on both sides of an equation.)

For example, suppose you derived the equation $v = v_0 + \frac{1}{2}at^2$, where v is the speed of an object after a time t, v_0 is the object's initial speed, and the object undergoes an acceleration a. Let's do a dimensional check to see if this equation could be correct or is surely incorrect. Note that numerical factors, like the $\frac{1}{2}$ here, do not affect dimensional checks. We write a dimensional equation as follows, remembering that the dimensions of speed are $[L/T]$ and (as we shall see in Chapter 2) the dimensions of acceleration are $[L/T^2]$:

$$\left[\frac{L}{T}\right] \stackrel{?}{=} \left[\frac{L}{T}\right] + \left[\frac{L}{T^2}\right][T^2] = \left[\frac{L}{T}\right] + [L].$$

The dimensions are incorrect: on the right side, we have the sum of quantities whose dimensions are not the same. Thus we conclude that an error was made in the derivation of the original equation.

A dimensional check can only tell you when a relationship is wrong. It can't tell you if it is completely right. For example, a dimensionless numerical factor (such as $\frac{1}{2}$ or 2π) could be missing.

Dimensional analysis can also be used as a quick check on an equation you are not sure about. For example, suppose that you can't remember whether the equation for the period of a simple pendulum T (the time to make one back-and-forth swing) of length ℓ is $T = 2\pi\sqrt{\ell/g}$ or $T = 2\pi\sqrt{g/\ell}$, where g is the acceleration due to gravity and, like all accelerations, has dimensions $[L/T^2]$. (Do not worry about these formulas—the correct one will be derived in Chapter 14; what we are concerned about here is a person's recalling whether it contains ℓ/g or g/ℓ.)

A dimensional check shows that the former (ℓ/g) is correct:

$$[T] = \sqrt{\frac{[L]}{[L/T^2]}} = \sqrt{[T^2]} = [T],$$

whereas the latter (g/ℓ) is not:

$$[T] \neq \sqrt{\frac{[L/T^2]}{[L]}} = \sqrt{\frac{1}{[T^2]}} = \frac{1}{[T]}.$$

Note that the constant 2π has no dimensions and so can't be checked using dimensio

Further uses of dimensional analysis are found in Appendix C.

2–1 Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\text{km/h} + 5\,\text{km/h} = 85\,\text{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

FIGURE 2–1 The pinecone in (a) undergoes pure translation as it falls, whereas in (b) it is rotating as well as translating.

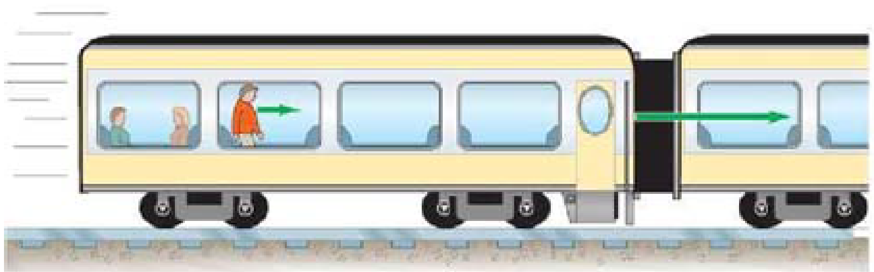
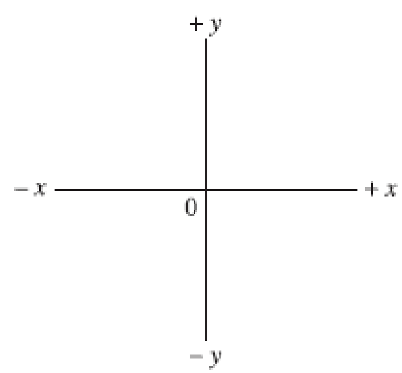


FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using the cardinal points, north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we usually choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

FIGURE 2–3 Standard set of xy coordinate axes.



CAUTION
The displacement may not equal the total distance traveled

We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point.* To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

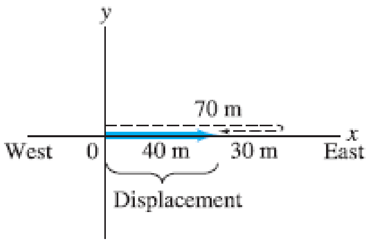


FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

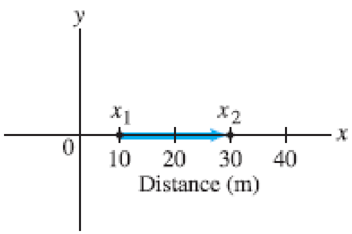
We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will have a positive sign, whereas vectors that point in the opposite direction will have a negative sign, along with their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 2–5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

$$\Delta x = x_2 - x_1,$$

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is the displacement. Note that the "change in" any quantity means the final value of that quantity, minus the initial value.

FIGURE 2–5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.



Suppose $x_1 = 10.0\,\mathrm{m}$ and $x_2 = 30.0\,\mathrm{m}$. Then

$$\Delta x = x_2 - x_1 = 30.0\,\mathrm{m} - 10.0\,\mathrm{m} = 20.0\,\mathrm{m},$$

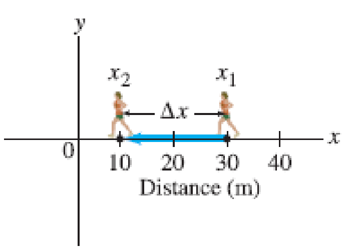
so the displacement is 20.0 m in the positive direction, Fig. 2–5.

Now consider an object moving to the left as shown in Fig. 2–6. Here the object, say, a person, starts at $x_1 = 30.0\,\mathrm{m}$ and walks to the left to the point $x_2 = 10.0\,\mathrm{m}$. In this case her displacement is

$$\Delta x = x_2 - x_1 = 10.0\,\mathrm{m} - 30.0\,\mathrm{m} = -20.0\,\mathrm{m},$$

and the blue arrow representing the vector displacement points to the left. For one-dimensional motion along the x axis, a vector pointing to the right has a positive sign, whereas a vector pointing to the left has a negative sign.

FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\,\mathrm{m} - 30.0\,\mathrm{m}$, the displacement vector points to the left.



EXERCISE A An ant starts at $x = 20\,\mathrm{cm}$ on a piece of graph paper and walks along the x axis to $x = -20\,\mathrm{cm}$. It then turns around and walks back to $x = -10\,\mathrm{cm}$. What is the ant's displacement and total distance traveled?

Vorbereitungsübungen (aus Giancoli)

= im Stil, die gute Formel finden und dann anwenden

2. (I) How many significant figures do each of the following numbers have: (*a*) 214, (*b*) 81.60, (*c*) 7.03, (*d*) 0.03, (*e*) 0.0086, (*f*) 3236, and (*g*) 8700?

8. (II) Multiply 2.079×10^{2} m by 0.082×10^{-1}, taking into account significant figures.

***36.** (II) The speed v of an object is given by the equation $v = At^3 - Bt$, where t refers to time. (*a*) What are the dimensions of A and B? (*b*) What are the SI units for the constants A and B?

***37.** (II) Three students derive the following equations in which x refers to distance traveled, v the speed, a the acceleration $(\mathrm{m/s^2})$, t the time, and the subscript zero $(_0)$ means a quantity at time $t = 0$: (*a*) $x = vt^2 + 2at$, (*b*) $x = v_0 t + \frac{1}{2}at^2$, and (*c*) $x = v_0 t + 2at^2$. Which of these could possibly be correct according to a dimensional check?

1. (I) If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

4. (I) A rolling ball moves from $x_1 = 3.4$ cm to $x_2 = -4.2$ cm during the time from $t_1 = 3.0$ s to $t_2 = 5.1$ s. What is its average velocity?

21. (I) At highway speeds, a particular automobile is capable of an acceleration of about $1.8\ \mathrm{m/s^2}$. At this rate, how long does it take to accelerate from 80 km/h to 110 km/h?

27. (II) A particle moves along the x axis. Its position as a function of time is given by $x = 6.8\,t + 8.5\,t^2$, where t is in seconds and x is in meters. What is the acceleration as a function of time?

30. (I) A car slows down from 25 m/s to rest in a distance of 85 m. What was its acceleration, assumed constant?

41. (II) Determine the stopping distances for an automobile with an initial speed of 95 km/h and human reaction time of 1.0 s: (*a*) for an acceleration $a = -5.0\ \mathrm{m/s^2}$; (*b*) for $a = -7.0\ \mathrm{m/s^2}$.

2: Kinematik im Raum

I. Wie kann man im allgemeinen die geradlinige Bewegung herleiten ?

Integration der Bewegungsgleichung

g – Erdbeschleunigung

II. Wovon hängt die allgemeine Beschreibung der Bewegung ab ?

Wurfparabel

Inertialsysteme

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(3 Seiten)*:

2-7 Freely falling objects

3-6 Vector Kinematics

Vorbereitende Übungen *(3)* **vor der Übungssession** zu erledigen **:**

Giancoli 3-28, 29, 57

Giancoli Kapitel 2-6 bis 3-9

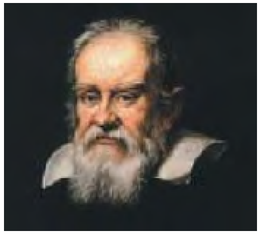


FIGURE 2–25 Galileo Galilei (1564–1642).

2–7 Freely Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed before the time of Galileo (Fig. 2–25), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.

Galileo made use of his new technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–26); that is, $d \propto t^2$. We can see this from Eq. 2–12b; but Galileo was the first to derive this mathematical relation.

CAUTION
A freely falling object increases in speed, but not in proportion to its mass or weight

To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

FIGURE 2–26 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.

Galileo claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 2–27a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 2–27b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–28). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall) but also for his approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

> **at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.**

We call this acceleration the **acceleration due to gravity** on the surface of the Earth, and we give it the symbol g. Its magnitude is approximately

Acceleration due to gravity

$$g = 9.80\ \mathrm{m/s^2}. \qquad \text{[at surface of Earth]}$$

In British units g is about 32 ft/s². Actually, g varies slightly according to latitude and elevation, but these variations are so small that we will ignore them for most purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector as is any acceleration, and its direction is downward, toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–12, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

FIGURE 2–27 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

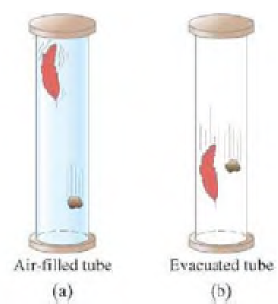


FIGURE 2–28 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.

3–6 Vector Kinematics

We can now extend our definitions of velocity and acceleration in a formal way to two- and three-dimensional motion. Suppose a particle follows a path in the xy plane as shown in Fig. 3–16. At time t_1, the particle is at point P_1, and at time t_2, it is at point P_2. The vector $\vec{\mathbf{r}}_1$ is the position vector of the particle at time t_1 (it represents the displacement of the particle from the origin of the coordinate system). And $\vec{\mathbf{r}}_2$ is the position vector at time t_2.

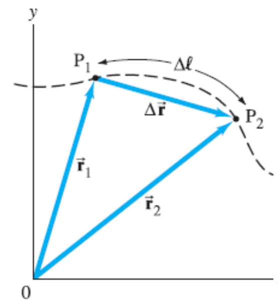
In one dimension, we defined displacement as the *change in position* of the particle. In the more general case of two or three dimensions, the **displacement vector** is defined as the vector representing change in position. We call it $\Delta\vec{\mathbf{r}}$,[†] where

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 .$$

This represents the displacement during the time interval $\Delta t = t_2 - t_1$.

[†]We used $\vec{\mathbf{D}}$ for the displacement vector earlier in the Chapter for illustrating vector addition. The new notation here, $\Delta\vec{\mathbf{r}}$, emphasizes that it is the difference between two position vectors.

FIGURE 3–16 Path of a particle in the xy plane. At time t_1 the particle is at point P_1 given by the position vector $\vec{\mathbf{r}}_1$; at t_2 the particle is at point P_2 given by the position vector $\vec{\mathbf{r}}_2$. The displacement vector for the time interval $t_2 - t_1$ is $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$.



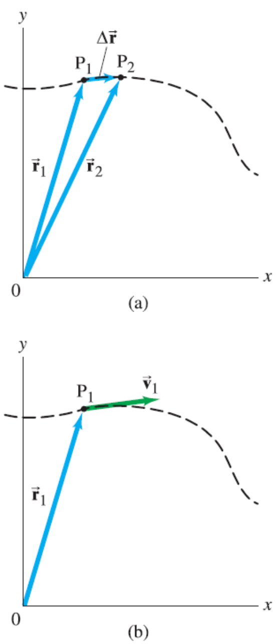


FIGURE 3–17 (a) As we take Δt and $\Delta\vec{\mathbf{r}}$ smaller and smaller [compare to Fig. 3–16] we see that the direction of $\Delta\mathbf{r}$ and of the instantaneous velocity ($\Delta\vec{\mathbf{r}}/\Delta t$, where $\Delta t \to 0$) is (b) tangent to the curve at P_1.

In unit vector notation, we can write

$$\vec{\mathbf{r}}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}, \tag{3–6a}$$

where x_1, y_1, and z_1 are the coordinates of point P_1. Similarly,

$$\vec{\mathbf{r}}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}.$$

Hence

$$\Delta\vec{\mathbf{r}} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}. \tag{3–6b}$$

If the motion is along the x axis only, then $y_2 - y_1 = 0$, $z_2 - z_1 = 0$, and the magnitude of the displacement is $\Delta r = x_2 - x_1$, which is consistent with our earlier one-dimensional equation (Section 2–1). Even in one dimension, displacement is a vector, as are velocity and acceleration.

The **average velocity vector** over the time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average velocity} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}. \tag{3–7}$$

Now let us consider shorter and shorter time intervals—that is, we let Δt approach zero so that the distance between points P_2 and P_1 also approaches zero, Fig. 3–17. We define the **instantaneous velocity vector** as the limit of the average velocity as Δt approaches zero:

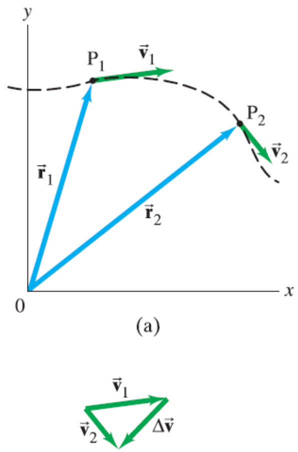
$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}. \tag{3–8}$$

The direction of $\vec{\mathbf{v}}$ at any moment is along the line tangent to the path at that moment (Fig. 3–17).

Note that the magnitude of the average velocity in Fig. 3–16 is not equal to the average speed, which is the actual distance traveled along the path, $\Delta\ell$, divided by Δt. In some special cases, the average speed and average velocity are equal (such as motion along a straight line in one direction), but in general they are not. However, in the limit $\Delta t \to 0$, Δr always approaches $\Delta\ell$, so the instantaneous speed *always* equals the magnitude of the instantaneous velocity at any time.

The instantaneous velocity (Eq. 3–8) is equal to the derivative of the position vector with respect to time. Equation 3–8 can be written in terms of components starting with Eq. 3–6a as:

FIGURE 3–18 (a) Velocity vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ at instants t_1 and t_2 for a particle at points P_1 and P_2, as in Fig. 3–16. (b) The direction of the average acceleration is in the direction of $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$.



$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}, \qquad \textbf{(3–9)}$$

where $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$ are the x, y, and z components of the velocity. Note that $d\hat{\mathbf{i}}/dt = d\hat{\mathbf{j}}/dt = d\hat{\mathbf{k}}/dt = 0$ since these unit vectors are constant in both magnitude and direction.

Acceleration in two or three dimensions is treated in a similar way. The **average acceleration vector**, over a time interval $\Delta t = t_2 - t_1$ is defined as

$$\text{average acceleration} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}, \qquad \textbf{(3–10)}$$

where $\Delta\vec{\mathbf{v}}$ is the change in the instantaneous velocity vector during that time interval: $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$. Note that $\vec{\mathbf{v}}_2$ in many cases, such as in Fig. 3–18a, may not be in the same direction as $\vec{\mathbf{v}}_1$. Hence the average acceleration vector may be in a different direction from either $\vec{\mathbf{v}}_1$ or $\vec{\mathbf{v}}_2$ (Fig. 3–18b). Furthermore, $\vec{\mathbf{v}}_2$ and $\vec{\mathbf{v}}_1$ may have the same magnitude but different directions, and the difference of two such vectors will not be zero. Hence acceleration can result from either a change in the magnitude of the velocity, or from a change in direction of the velocity, or from a change in both.

The **instantaneous acceleration vector** is defined as the limit of the average acceleration vector as the time interval Δt is allowed to approach zero:

$$\vec{\mathbf{a}} = \lim_{\Delta \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}, \qquad \textbf{(3–11)}$$

and is thus the derivative of $\vec{\mathbf{v}}$ with respect to t.

We can write $\vec{\mathbf{a}}$ using components:

$$\begin{aligned}\vec{\mathbf{a}} &= \frac{d\vec{\mathbf{v}}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}} \\ &= a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}},\end{aligned} \qquad \textbf{(3–12)}$$

where $a_x = dv_x/dt$, etc. Because $v_x = dx/dt$, then $a_x = dv_x/dt = d^2x/dt^2$, as we saw in Section 2–4. Thus we can also write the acceleration as

$$\vec{\mathbf{a}} = \frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}\hat{\mathbf{j}} + \frac{d^2z}{dt^2}\hat{\mathbf{k}}. \qquad \textbf{(3–12c)}$$

The instantaneous acceleration will be nonzero not only when the magnitude of the velocity changes but also if its direction changes. For example, a person riding in a car traveling at constant speed around a curve, or a child riding on a merry-go-round, will both experience an acceleration because of a change in the direction of the velocity, even though the speed may be constant. (More on this in Chapter 5.)

In general, we will use the terms "velocity" and "acceleration" to mean the instantaneous values. If we want to discuss average values, we will use the word "average."

28. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.2 m/s. How far from the base of the rock will she land?

29. (I) A diver running 2.3 m/s dives out horizontally from the edge of a vertical cliff and 3.0 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?

57. (I) A person going for a morning jog on the deck of a cruise ship is running toward the bow (front) of the ship at 2.0 m/s while the ship is moving ahead at 8.5 m/s. What is the velocity of the jogger relative to the water? Later, the jogger is moving toward the stern (rear) of the ship. What is the jogger's velocity relative to the water now?

3: Kinematik der Rotation

I. Wie beschreibt man eine kreisförmige (und oszillierende) Bewegung ?

II. Welche Bedingungen umschreiben ein Objekt auf einem Kreis ?

Winkelbeschleunigung

III. Wie kann man die kreisförmige Bewegung allgemein beschreiben ?

Präzession

III. Wie beschreibt man die Bewegung in zylindrischen Koordinaten ?

IV. Wie beschreibt man die Bewegung in einem linear beschleunigten Bezugssystem ?

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(3.5 Seiten)*:

5-2 Uniform circular motion - kinematics

10-1 Angular quantities

Vorbereitende Übungen *(6)* **vor der Übungssession** zu erledigen **:**

Giancoli 5-36, 38

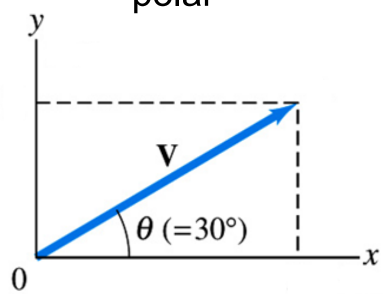
Giancoli 10-4, 5ab, 7abc, 12ab

Giancoli Kapitel 5-2 und 10-1 bis 10-3

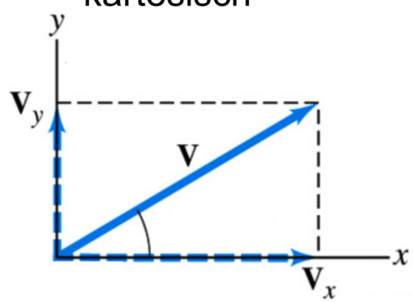
Anhang: Komponenten eines Vektors

kartesiche, polare & zylindrische Koordinaten (siche auch Mathe support, e.g. youtube)

polar



kartesisch



v= (v, θ) =v(cosθ **x**+$sin\theta$**y**)

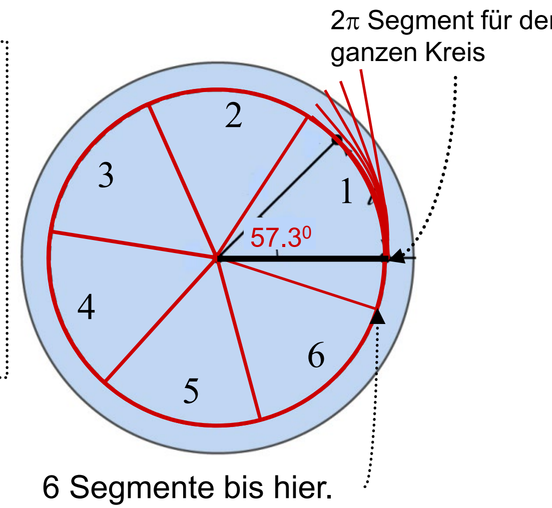
v= v_x **x** + v_y **y** ≡ (v_x,v_y)

v_x=vcosθ, v_y=vsinθ

Definition:

Winkelposition θ

- θ≡l/R
- Einheit: radian [rad]
  - 360^0=2π rad



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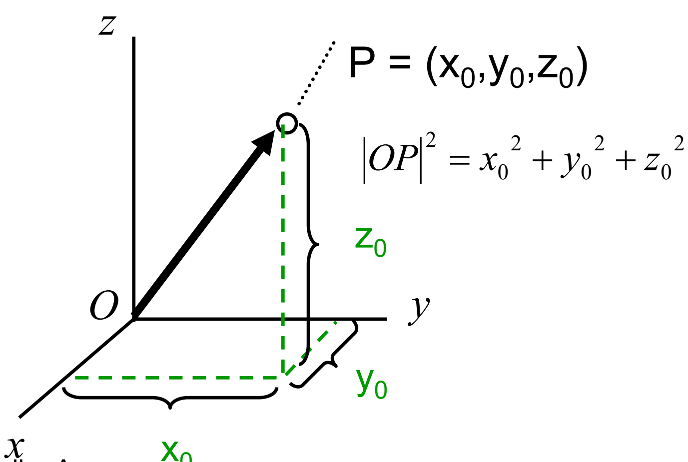
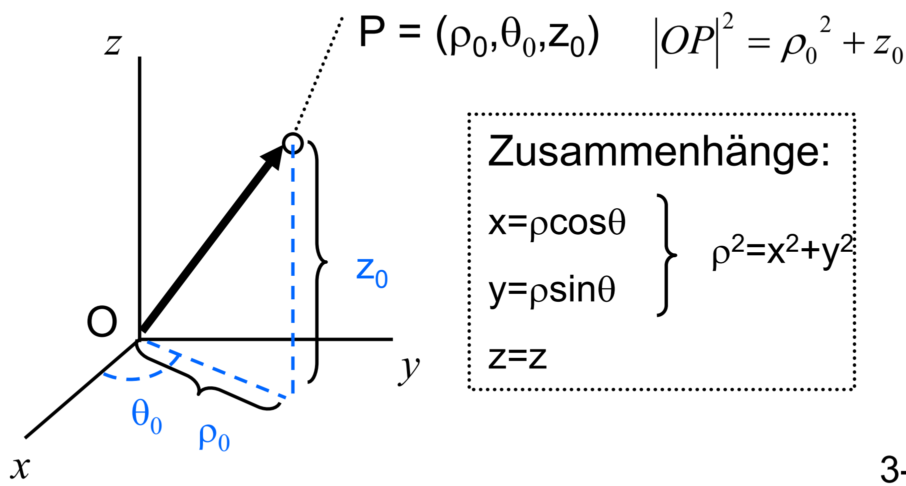


zylindrisch



Zusammenhänge:

x=ρcosθ
y=ρsinθ
z=z

$\rho^2=x^2+y^2$

5–2 Uniform Circular Motion—Kinematics

An object moves in a straight line if the net force on it acts in the direction of motion, or the net force is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string revolving around one's head, or the nearly circular motion of the Moon about the Earth.

An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity continuously changes as the object moves around the circle (Fig. 5–10). Because acceleration is defined as the rate of change of velocity, a change in direction of velocity constitutes an acceleration, just as a change in magnitude of velocity does. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant $(v_1 = v_2 = v)$. We now investigate this acceleration quantitatively.

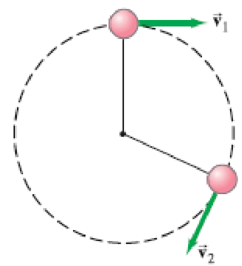


FIGURE 5–10 A small object moving in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.

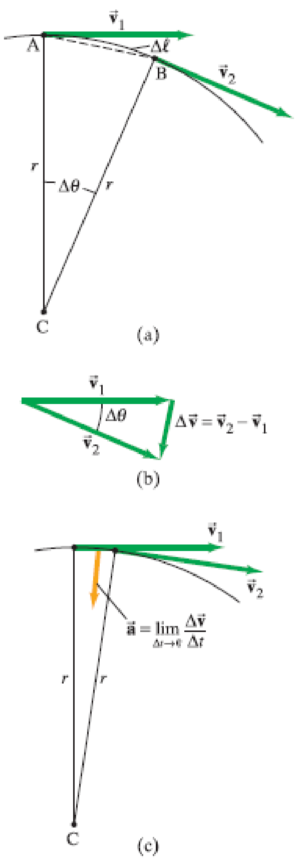


FIGURE 5–11 Determining the change in velocity, $\Delta\vec{\mathbf{v}}$, for a particle moving in a circle. The length $\Delta\ell$ is the distance along the arc, from A to B.

Acceleration is defined as

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt},$$

where $\Delta\vec{\mathbf{v}}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation in which Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing (Fig. 5–11), we consider a nonzero time interval. During the time interval Δt, the particle in Fig. 5–11a moves from point A to point B, covering a distance $\Delta\ell$ *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \Delta\vec{\mathbf{v}}$, and is shown in Fig. 5–11b.

Now we let Δt be very small, approaching zero. Then $\Delta\ell$ and $\Delta\theta$ are also very small, and $\vec{\mathbf{v}}_2$ will be almost parallel to $\vec{\mathbf{v}}_1$ (Fig. 5–11c); $\Delta\vec{\mathbf{v}}$ will be essentially perpendicular to them. Thus $\Delta\vec{\mathbf{v}}$ points toward the center of the circle. Since $\vec{\mathbf{a}}$, by definition, is in the same direction as $\Delta\vec{\mathbf{v}}$, it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-pointing" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\vec{\mathbf{a}}_R$.

We next determine the magnitude of the radial (centripetal) acceleration, a_R. Because CA in Fig. 5–11a is perpendicular to $\vec{\mathbf{v}}_1$, and CB is perpendicular to $\vec{\mathbf{v}}_2$, it follows that the angle $\Delta\theta$, defined as the angle between CA and CB, is also the angle between $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$. Hence the vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, and $\Delta\vec{\mathbf{v}}$ in Fig. 5–11b form a triangle that is geometrically similar† to triangle CAB in Fig. 5–11a. If we take $\Delta\theta$ to be very small (letting Δt be very small) and setting $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change, we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta\ell}{r},$$

or

$$\Delta v \approx \frac{v}{r}\Delta\ell.$$

This is an exact equality when Δt approaches zero, for then the arc length $\Delta\ell$ equals the chord length AB. We want to find the instantaneous acceleration, a_R, so we use the expression above to write

$$a_R = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta\ell}{\Delta t}.$$

Then, because

$$\lim_{\Delta t \to 0} \frac{\Delta\ell}{\Delta t}$$

is just the linear speed, v, of the object, we have for the centripetal (radial) acceleration

$$a_R = \frac{v^2}{r}. \qquad \text{[centripetal (radial) acceleration]} \quad \textbf{(5–1)}$$

Equation 5–1 is valid even when v is not constant.

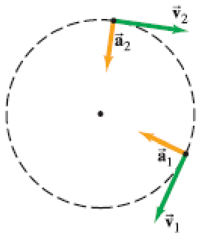
To summarize, *an object moving in a circle of radius r at constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is* $a_R = v^2/r$. It is not surprising that this acceleration depends on v and r. The greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

The acceleration vector points toward the center of the circle. But the velocity vector always points in the direction of motion, which is tangential to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion (Fig. 5–12). This is another example that illustrates the error in thinking that acceleration and velocity are always in the same direction. For an object falling vertically, $\vec{a}$ and $\vec{v}$ are indeed parallel. But in circular motion, $\vec{a}$ and $\vec{v}$ are perpendicular, not parallel (nor were they parallel in projectile motion, Section 3–7).

CAUTION

In uniform circular motion, the speed is constant, but the acceleration is not zero

FIGURE 5–12 For uniform circular motion, $\vec{a}$ is always perpendicular to $\vec{v}$.



EXERCISE C Can Equations 2–12, the kinematic equations for constant acceleration, be used for uniform circular motion? For example, could Eq. 2–12b be used to calculate the time for the revolving ball in Fig. 5–12 to make one revolution?

Circular motion is often described in terms of the **frequency** f, the number of revolutions per second. The **period** T of an object revolving in a circle is the time required for one complete revolution. Period and frequency are related by

$$T = \frac{1}{f}. \tag{5–2}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution takes $\frac{1}{3}$ s. For an object revolving in a circle (of circumference $2\pi r$) at constant speed v, we can write

$$v = \frac{2\pi r}{T},$$

since in one revolution the object travels one circumference.

36. (I) A jet plane traveling 1890 km/h (525 m/s) pulls out of a dive by moving in an arc of radius 4.80 km. What is the plane's acceleration in g's?

38. (II) How fast (in rpm) must a centrifuge rotate if a particle 8.00 cm from the axis of rotation is to experience an acceleration of 125,000 g's?

We will consider mainly the rotation of rigid objects. A **rigid object** is an object with a definite shape that doesn't change, so that the particles composing it stay in fixed positions relative to one another. Any real object is capable of vibrating or deforming when a force is exerted on it. But these effects are often very small, so the concept of an ideal rigid object is very useful as a good approximation.

Our development of rotational motion will parallel our discussion of translational motion: rotational position, angular velocity, angular acceleration, rotational inertia, and the rotational analog of force, "torque."

10–1 Angular Quantities

The motion of a rigid object can be analyzed as the translational motion of its center of mass plus rotational motion *about* its center of mass (Sections 9–8 and 9–9). We have already discussed translational motion in detail, so now we focus our attention on purely rotational motion. By *purely rotational motion* of an object about a fixed axis, we mean that all points in the object move in circles, such as the point P on the rotating wheel of Fig. 10–1, and that the centers of these circles all lie on a line called the **axis of rotation**. In Fig. 10–1 the axis of rotation is perpendicular to the page and passes through point O. We assume the axis is fixed in an inertial reference frame, but we will not always insist that the axis pass through the center of mass.

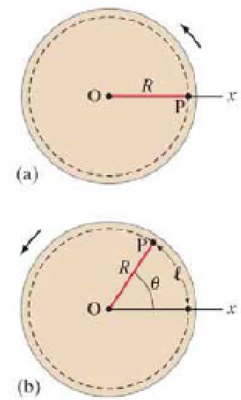
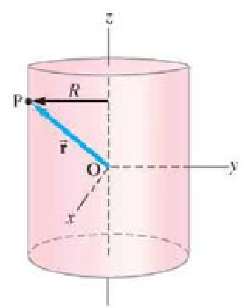


FIGURE 10–1 Looking at a wheel that is rotating counterclockwise about an axis through the wheel's center at O (axis perpendicular to the page). Each point, such as point P, moves in a circular path; ℓ is the distance P travels as the wheel rotates through the angle θ.

For a three-dimensional rigid object rotating about a fixed axis, we will use the symbol R to represent the perpendicular distance of a point or particle from the axis of rotation. We do this to distinguish R from r, which will continue to represent the position of a particle with reference to the origin (point) of some coordinate system. This distinction is illustrated in Fig. 10–2. This distinction may seem like a small point, but not being fully aware of it can cause huge errors when working with rotational motion. For a flat, very thin object, like a wheel, with the origin in the plane of the object (at the center of a wheel, for example), R and r will be nearly the same.

FIGURE 10–2 Showing the distinction between $\vec{\mathbf{r}}$ (the position vector) and R (the distance from the rotation axis) for a point P on the edge of a cylinder rotating about the z axis.



Every point in an object rotating about a fixed axis moves in a circle (shown dashed in Fig. 10–1 for point P) whose center is on the axis of rotation and whose radius is R, the distance of that point from the axis of rotation. A straight line drawn from the axis to any point in the object sweeps out the same angle θ in the same time interval.

To indicate the angular position of the object, or how far it has rotated, we specify the angle θ of some particular line in the object (red in Fig. 10–1) with respect to some reference line, such as the x axis in Fig. 10–1. A point in the object, such as P in Fig. 10–1b, moves through an angle θ when it travels the distance ℓ measured along the circumference of its circular path. Angles are commonly stated in degrees, but the mathematics of circular motion is much simpler if we use the *radian* for angular measure. One **radian** (abbreviated rad) is defined as the angle subtended by an arc whose length is equal to the radius. For example, in Fig. 10–1, point P is a distance R from the axis of rotation, and it has moved a distance ℓ along the arc of a circle. The arc length ℓ is said to "subtend" the angle θ. In general, any angle θ is given by

$$\theta = \frac{\ell}{R}, \qquad [\theta \text{ in radians}] \quad \textbf{(10–1a)}$$

where R is the radius of the circle and ℓ is the arc length subtended by the angle θ, which is specified in radians. If $\ell = R$, then $\theta = 1$ rad.

The radian, being the ratio of two lengths, is dimensionless. We thus do not have to mention it in calculations, although it is usually best to include it to remind us the angle is in radians and not degrees. We can rewrite Eq. 10–1a in terms of arc length ℓ:

$$\ell = R\theta. \qquad \textbf{(10–1b)}$$

Radians can be related to degrees in the following way. In a complete circle there are 360°, which must correspond to an arc length equal to the circumference of the circle, $\ell = 2\pi R$. Thus $\theta = \ell/R = 2\pi R/R = 2\pi$ rad in a complete circle, so

$$360^\circ = 2\pi \text{ rad}.$$

One radian is therefore $360^\circ/2\pi \approx 360^\circ/6.28 \approx 57.3^\circ$. An object that makes one complete revolution (rev) has rotated through 360°, or 2π radians:

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}.$$

Angular acceleration (denoted by α, the Greek lowercase letter alpha), in analogy to linear acceleration, is defined as the change in angular velocity divided by the time required to make this change. The **average angular acceleration** is defined as

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}, \qquad \textbf{(10–3a)}$$

where ω_1 is the angular velocity initially, and ω_2 is the angular velocity after a time interval Δt. **Instantaneous angular acceleration** is defined as the limit of this ratio as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}. \qquad \textbf{(10–3b)}$$

Since ω is the same for all points of a rotating object, Eq. 10–3 tells us that α also will be the same for all points. Thus, ω and α are properties of the rotating object as a whole. With ω measured in radians per second and t in seconds, α has units of radians per second squared $(\mathrm{rad/s^2})$.

Each point or particle of a rotating rigid object has, at any instant, a linear velocity v and a linear acceleration a. We can relate the linear quantities at each point, v and a, to the angular quantities, ω and α, of the rotating object. Consider a point P located a distance R from the axis of rotation, as in Fig. 10–5. If the object rotates with angular velocity ω, any point will have a linear velocity whose direction is tangent to its circular path. The magnitude of that point's linear velocity is $v = d\ell/dt$. From Eq. 10–1b, a change in rotation angle $d\theta$ (in radians) is related to the linear distance traveled by $d\ell = R\,d\theta$. Hence

$$v = \frac{d\ell}{dt} = R\frac{d\theta}{dt}$$

or

$$v = R\omega, \qquad \textbf{(10–4)}$$

where R is a fixed distance from the rotation axis and ω is given in rad/s. Thus, although ω is the same for every point in the rotating object at any instant, the linear velocity v is greater for points farther from the axis (Fig. 10–6). Note that Eq. 10–4 is valid both instantaneously and on the average.

If the angular velocity of a rotating object changes, the object as a whole—and each point in it—has an angular acceleration. Each point also has a linear acceleration whose direction is tangent to that point's circular path. We use Eq. 10–4 $(v = R\omega)$ to show that the angular acceleration α is related to the tangential linear acceleration $a_{\tan}$ of a point in the rotating object by

$$a_{\tan} = \frac{dv}{dt} = R\frac{d\omega}{dt}$$

or

$$a_{\tan} = R\alpha. \qquad \textbf{(10–5)}$$

In this equation, R is the radius of the circle in which the particle is moving, and the subscript "tan" in $a_{\tan}$ stands for "tangential."

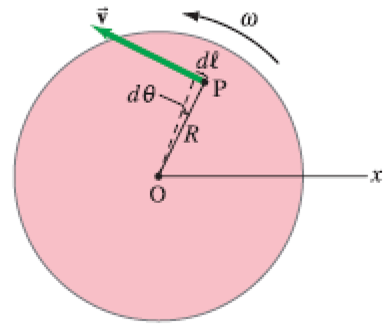
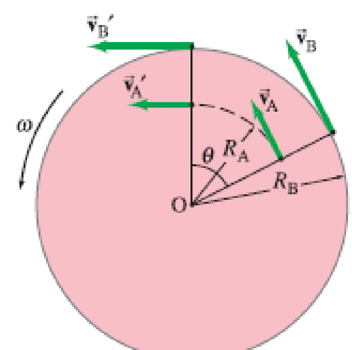


FIGURE 10–5 A point P on a rotating wheel has a linear velocity $\vec{\mathbf{v}}$ at any moment.

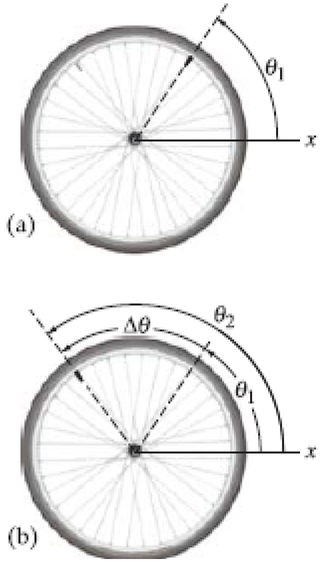
FIGURE 10–6 A wheel rotating uniformly counterclockwise. Two points on the wheel, at distances R_A and R_B from the center, have the same angular velocity ω because they travel through the same angle θ in the same time interval. But the two points have different linear velocities because they travel different distances in the same time interval. Since $R_B > R_A$, then $v_B > v_A$ (because $v = R\omega$).



G

is only 1%. For larger angles the error increases rapidly.

FIGURE 10–4 A wheel rotates from (a) initial position θ_1 to (b) final position θ_2. The angular displacement is $\Delta\theta = \theta_2 - \theta_1$.



To describe rotational motion, we make use of angular quantities, such as angular velocity and angular acceleration. These are defined in analogy to the corresponding quantities in linear motion, and are chosen to describe the rotating object as a whole, so they have the same value for each point in the rotating object. Each point in a rotating object also has translational velocity and acceleration, but they have different values for different points in the object.

When an object, such as the bicycle wheel in Fig. 10–4, rotates from some initial position, specified by θ_1, to some final position, θ_2, its **angular displacement** is

$$\Delta\theta = \theta_2 - \theta_1 .$$

The *angular velocity* (denoted by ω, the Greek lowercase letter omega) is defined in analogy with linear (translational) velocity that was discussed in Chapter 2. Instead of linear displacement, we use the angular displacement. Thus the **average angular velocity** of an object rotating about a fixed axis is defined as the time rate of change of angular position:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}, \tag{10–2a}$$

where $\Delta\theta$ is the angle through which the object has rotated in the time interval Δt. The **instantaneous angular velocity** is the limit of this ratio as Δt approaches zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \tag{10–2b}$$

Angular velocity has units of radians per second (rad/s). Note that *all points in a rigid object rotate with the same angular velocity*, since every position in the object moves through the same angle in the same time interval.

An object such as the wheel in Fig. 10–4 can rotate about a fixed axis either

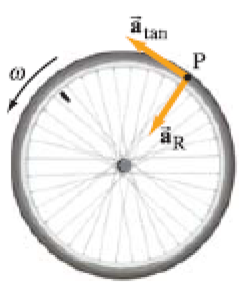


FIGURE 10–7 On a rotating wheel whose angular speed is increasing, a point P has both tangential and radial (centripetal) components of linear acceleration. (See also Chapter 5.)

The total linear acceleration of a point at any instant is the vector sum of two components:

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\tan} + \vec{\mathbf{a}}_{\mathrm{R}} ,$$

where the radial component, $\vec{\mathbf{a}}_{\mathrm{R}}$, is the radial or "centripetal" acceleration and its direction is toward the center of the point's circular path; see Fig. 10–7. We saw in Chapter 5 (Eq. 5–1) that a particle moving in a circle of radius R with linear speed v has a radial acceleration $a_{\mathrm{R}} = v^2/R$; we can rewrite this in terms of ω using Eq. 10–4:

$$a_{\mathrm{R}} = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R. \tag{10–6}$$

Equation 10–6 applies to any particle of a rotating object. Thus the centripetal acceleration is greater the farther you are from the axis of rotation: the children farthest out on a carousel experience the greatest acceleration.

Table 10–1 summarizes the relationships between the angular quantities describing the rotation of an object to the linear quantities for each point of the object.

We can relate the angular velocity ω to the frequency of rotation, f. The **frequency** is the number of complete revolutions (rev) per second, as we saw in Chapter 5. One revolution (of a wheel, say) corresponds to an angle of 2π radians, and thus $1 \text{ rev/s} = 2\pi \text{ rad/s}$. Hence, in general, the frequency f is related to the angular velocity ω by

$$f = \frac{\omega}{2\pi}$$

or

$$\omega = 2\pi f. \qquad \textbf{(10–7)}$$

The unit for frequency, revolutions per second (rev/s), is given the special name the hertz (Hz). That is

$$1 \text{ Hz} = 1 \text{ rev/s}.$$

Note that "revolution" is not really a unit, so we can also write $1 \text{ Hz} = 1 \text{ s}^{-1}$.

The time required for one complete revolution is the **period** T, and it is related to the frequency by

$$T = \frac{1}{f}. \qquad \textbf{(10–8)}$$

If a particle rotates at a frequency of three revolutions per second, then the period of each revolution is $\frac{1}{3}$ s.

4. (I) The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 4.0 s. What is the angular acceleration as the blades slow down?

5. (II) (*a*) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in rad/s. (*b*) What are the linear speed and acceleration of a point on the edge of the grinding wheel?

7. (II) Calculate the angular velocity of (*a*) the second hand, (*b*) the minute hand, and (*c*) the hour hand, of a clock. State in rad/s. (*d*) What is the angular acceleration in each case?

12. (II) A 64-cm-diameter wheel accelerates uniformly about its center from 130 rpm to 280 rpm in 4.0 s. Determine (*a*) its angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.

4: Dynamik der geradlinigen Bewegung
Die drei Axiome von Newton

1. Welches ist der natürliche Zustand aller Dinge und wie kann man ihn ändern ?

    Die Masse widersetzt sich der Änderung der Geschwindigkeit

2. Welches ist das Verhältnis zweier wechselwirkender Objekte ?

    Federkraft

3. Wie beeinflussen Reibungskräfte die Bewegung eines Objektes ?

    Zwischen Oberflächen von Festkörpern

    Viskose Reibung

4. Wie geht man ein Problem der Dynamik an ?

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(1 Seite)*:

4-1 Force

4-3 Mass

Vorbereitende Übungen *(7)* **vor der Übungssession** zu erledigen **:**

Giancoli 4-2, 22, 37

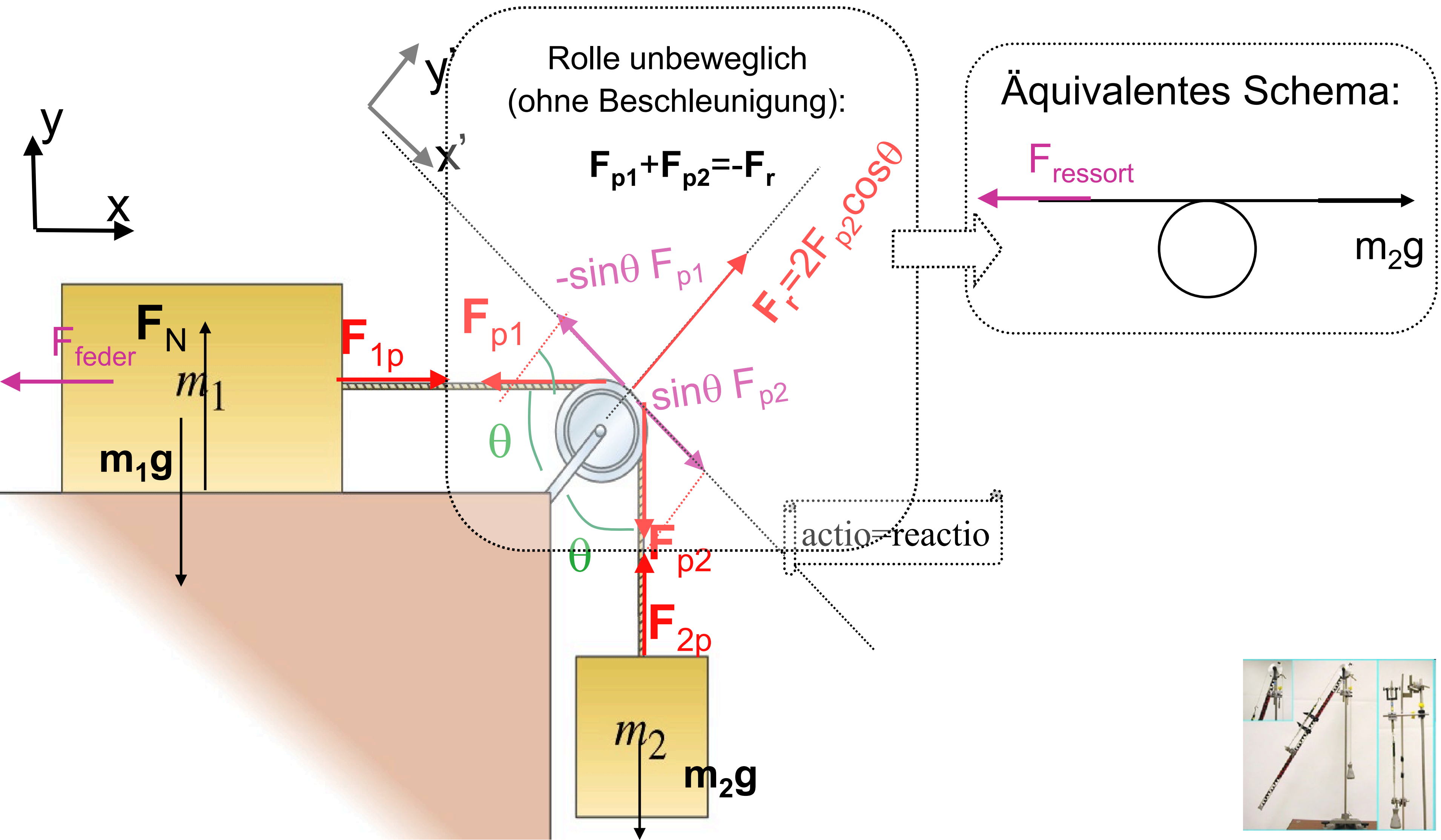
Giancoli 5-1, 2, 5, 66a

Inika McPherson, Moscow 2013

Giancoli Kapitel 4-1 bis 4-8 und 5-1, 5-6

Zugabe

Sind die Schnurkräfte, die auf eine Rolle einwirken, unabhängig von θ ?



We have discussed how motion is described in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes an object to accelerate or decelerate? What is involved when an object moves in a curved path? We can answer in each case that a force is required. In this Chapter[1], we will investigate the connection between force and motion, which is the subject called **dynamics**.

4–1 Force

FIGURE 4–1 A force exerted on a grocery cart—in this case exerted by a person.

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a stalled car or a grocery cart (Fig. 4–1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We often call these *contact forces* because the force is exerted when one object comes in contact with another object. On the other hand, we say that an object falls because of the *force of gravity*.

If an object is at rest, to start it moving requires force—that is, a force is needed to accelerate an object from zero velocity to a nonzero velocity. For an object already moving, if you want to change its velocity—either in direction or in magnitude—a force is required. In other words, to accelerate an object, a force is always required. In Section 4–4 we discuss the precise relation between acceleration and net force, which is Newton's second law.

One way to measure the magnitude (or strength) of a force is to use a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the object (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force exerted in a different direction has a different effect. Force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

FIGURE 4–2 A spring scale used to measure a force.

2. (I) A net force of 265 N accelerates a bike and rider at $2.30\ \mathrm{m/s^2}$. What is the mass of the bike and rider together?

37. (II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 4–40a and b (looking down) act on a 18.5-kg object on a frictionless tabletop. If $F_1 = 10.2$ N and $F_2 = 16.0$ N, find the net force on the object and its acceleration for (a) and (b).

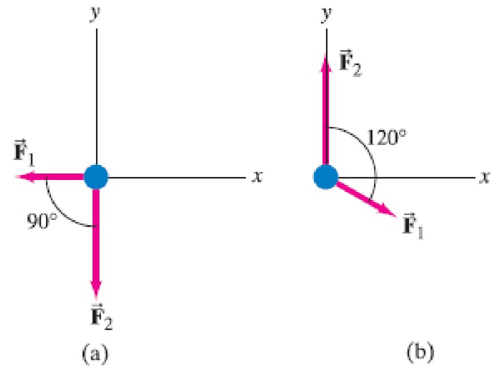


FIGURE 4–40 Problem 37.

22. (II) (*a*) What is the acceleration of two falling sky divers (mass = 132 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight? (*b*) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute? See Fig. 4–32.

FIGURE 4–32 Problem 22.

4–3 Mass

Newton's second law, which we come to in the next Section, makes use of the concept of mass. Newton used the term *mass* as a synonym for *quantity of matter*. This intuitive notion of the mass of an object is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia* of an object. The more mass an object has, the greater the force needed to give it a particular acceleration. It is harder to start it moving from rest, or to stop it when it is moving, or to change its velocity sideways out of a straight-line path. A truck has much more inertia than a baseball moving at the same speed, and a much greater force is needed to change the truck's velocity at the same rate as the ball's. The truck therefore has much more mass.

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg) as we discussed in Chapter 1, Section 1–4.

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of an object itself (a measure of an object's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the pull of gravity acting on an object. To see the difference, suppose we take an object to the Moon. The object will weigh only about one-sixth as much as it did on Earth, since the force of gravity is weaker. But its mass will be the same. It will have the same amount of matter as on Earth, and will have just as much inertia—for in the absence of friction, it will be just as hard to start it moving on the Moon as on Earth, or to stop it once it is moving. (More on weight in Section 4–6.)

1. (I) If the coefficient of kinetic friction between a 22-kg crate and the floor is 0.30, what horizontal force is required to move the crate at a steady speed across the floor? What horizontal force is required if μ_k is zero?

2. (I) A force of 35.0 N is required to start a 6.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 35.0-N force continues, the box accelerates at $0.60\,\mathrm{m/s^2}$. What is the coefficient of kinetic friction?

5. (I) What is the maximum acceleration a car can undergo if the coefficient of static friction between the tires and the ground is 0.90?

*66. (II) The terminal velocity of a 3×10^{-5} kg raindrop is about 9 m/s. Assuming a drag force $F_D = -bv$, determine (*a*) the value of the constant b and (*b*) the time required for such a drop, starting from rest, to reach 63% of terminal velocity.

5:Dynamik der Kreisbewegung (Rotation)

I. Welche Kräfte führen zu einer gleichmässigen Kreisbewegung ?

Die Bedingung der Zentripetalkraft

II. Welche Kräfte führen zu einer Schwingung (Oszillation) ?

III. Wie kann man die Winkelgeschwindigkeit ändern ?

Drehmoment
Inertial- oder Trägheitsmoment

III. Wie beschreibt man die Bewegung in Kugelkoordinaten ?

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(3 Seiten)*:

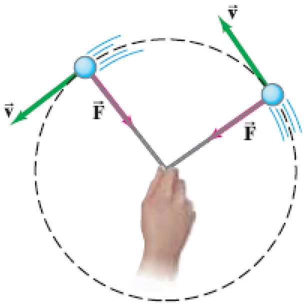
5-3 Dynamics of uniform circular motion
10-5 torque

Vorbereitende Übungen *(4)* **vor der Übungssession** zu erledigen **:**

Giancoli 5-34, 49
Giancoli 10-26, 27

Giancoli Kapitel 5-3 bis 5-5; 10-4(5); und 11-2

FIGURE 5–14 A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the circle's center.



⚠ CAUTION

Centripetal force is not a new kind of force (Every force must be exerted by an object)

5–3 Dynamics of Uniform Circular Motion

According to Newton's second law $(\Sigma\vec{F} = m\vec{a})$, an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in that circle. That is, a net force is necessary to give it centripetal acceleration. The magnitude of the required force can be calculated using Newton's second law for the radial component, $\Sigma F_R = ma_R$, where a_R is the centripetal acceleration, $a_R = v^2/r$, and ΣF_R is the total (or net) force in the radial direction:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}. \qquad \text{[circular motion]} \quad \textbf{(5–3)}$$

For uniform circular motion (v = constant), the acceleration is a_R, which is directed toward the center of the circle at any moment. Thus the *net force too must be directed toward the center of the circle*, Fig. 5–14. A net force is necessary because if no net force were exerted on the object, it would not move in a circle but in a straight line, as Newton's first law tells us. The direction of the net force is continually changing so that it is always directed toward the center of the circle. This force is sometimes called a centripetal ("pointing toward the center") force. But be aware that "centripetal force" does not indicate some new kind of force. The term merely describes the *direction* of the net force needed to provide a circular path: the net force is directed toward the circle's center. The force *must be applied by other objects*. For example, to swing a ball in a circle on the end of a string, you pull on the string and the string exerts the force on the ball. (Try it.)

⚠ CAUTION

There is no real "centrifugal force"

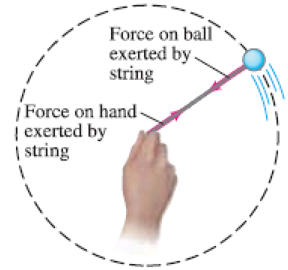
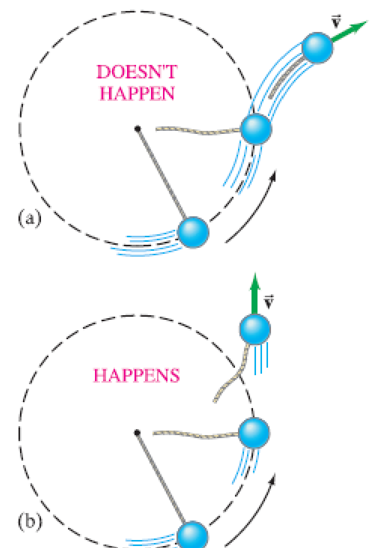


FIGURE 5–15 Swinging a ball on the end of a string.

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: *there is no outward force* on the revolving object. Consider, for example, a person swinging a ball on the end of a string around her head (Fig. 5–15). If you have ever done this yourself, you know that you feel a force pulling outward on your hand. The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to your hand. This is not what is happening at all. To keep the ball moving in a circle, you pull *inwardly* on the string, and the string exerts this force on the ball. The ball exerts an equal and opposite force on the string (Newton's third law), and *this* is the outward force your hand feels (see Fig. 5–15).

The force *on the ball* is the one exerted *inwardly* on it by you, via the string. To see even more convincing evidence that a "centrifugal force" does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly outward, as shown in Fig. 5–16a. But it doesn't; the ball flies off tangentially (Fig. 5–16b), in the direction of the velocity it had at the moment it was released, because the inward force no longer acts. Try it and see!

FIGURE 5–16 If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b). For example, in (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.



EXERCISE E Return to the Chapter-Opening Question, page 112, and answer it again now. Try to explain why you may have answered differently the first time.

34. (I) What is the maximum speed with which a 1200-kg car can round a turn of radius 80.0 m on a flat road if the coefficient of friction between tires and road is 0.65? Is this result independent of the mass of the car?

49. (II) On an ice rink two skaters of equal mass grab hands and spin in a mutual circle once every 2.5 s. If we assume their arms are each 0.80 m long and their individual masses are 60.0 kg, how hard are they pulling on one another?

10–5 Rotational Dynamics; Torque and Rotational Inertia

We discussed in Section 10–4 that the angular acceleration α of a rotating object is proportional to the net torque τ applied to it:

$$\alpha \propto \Sigma\tau,$$

where we write $\Sigma\tau$ to remind us that it is the *net* torque (sum of all torques acting on the object) that is proportional to α. This corresponds to Newton's second law for translational motion, $a \propto \Sigma F$, but here torque has taken the place of force, and, correspondingly, the angular acceleration α takes the place of the linear acceleration a. In the linear case, the acceleration is not only proportional to the net force, but it is also inversely proportional to the inertia of the object, which we call its mass, m. Thus we could write $a = \Sigma F/m$. But what plays the role of mass for the rotational case? That is what we now set out to determine. At the same time, we will see that the relation $\alpha \propto \Sigma\tau$ follows directly from Newton's second law, $\Sigma F = ma$.

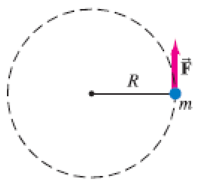


FIGURE 10–17 A mass m rotating in a circle of radius R about a fixed point.

NEWTON'S SECOND LAW FOR ROTATION

We first consider a very simple case: a particle of mass m rotating in a circle of radius R at the end of a string or rod whose mass we can ignore compared to m (Fig. 10–17), and we assume that a single force F acts on m tangent to the circle as shown. The torque that gives rise to the angular acceleration is $\tau = RF$. If we use Newton's second law for linear quantities, $\Sigma F = ma$, and Eq. 10–5 relating the angular acceleration to the tangential linear acceleration, $a_{\tan} = R\alpha$, then we have

$$F = ma$$
$$= mR\alpha,$$

where α is given in rad/s^2. When we multiply both sides of this equation by R, we find that the torque $\tau = RF = R(mR\alpha)$, or

$$\tau = mR^2\alpha. \qquad \text{[single particle]} \qquad \textbf{(10–11)}$$

26. (II) A person exerts a horizontal force of 32 N on the end of a door 96 cm wide. What is the magnitude of the torque if the force is exerted (*a*) perpendicular to the door and (*b*) at a 60.0° angle to the face of the door?

27. (II) Two blocks, each of mass m, are attached to the ends of a massless rod which pivots as shown in Fig. 10–48. Initially the rod is held in the horizontal position and then released. Calculate the magnitude and direction of the net torque on this system when it is first released.

FIGURE 10–48 Problem 27.

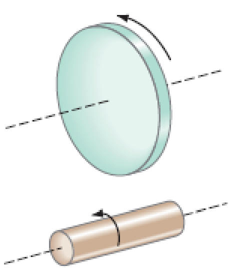


FIGURE 10–18 A large-diameter cylinder has greater rotational inertia than one of equal mass but smaller diameter.

CAUTION

Mass can not be considered concentrated at CM for rotational motion

Here at last we have a direct relation between the angular acceleration and the applied torque τ. The quantity mR^2 represents the *rotational inertia* of the particle and is called its *moment of inertia*.

Now let us consider a rotating rigid object, such as a wheel rotating about a fixed axis through its center, such as an axle. We can think of the wheel as consisting of many particles located at various distances from the axis of rotation. We can apply Eq. 10–11 to each particle of the object; that is, we write $\tau_i = m_i R_i^2 \alpha$ for the i^{th} particle of the object. Then we sum over all the particles. The sum of the various torques is just the total torque, $\Sigma\tau$, so we obtain:

$$\Sigma\tau_i = (\Sigma m_i R_i^2)\alpha \qquad \text{[axis fixed]} \qquad \textbf{(10–12)}$$

where we factored out the α since it is the same for all the particles of a rigid object. The resultant torque, $\Sigma\tau$, represents the sum of all internal torques that each particle exerts on another, plus all external torques applied from the outside: $\Sigma\tau = \Sigma\tau_{\text{ext}} + \Sigma\tau_{\text{int}}$. The sum of the internal torques is zero from Newton's third law. Hence $\Sigma\tau$ represents the resultant *external* torque.

The sum $\Sigma m_i R_i^2$ in Eq. 10–12 represents the sum of the masses of each particle in the object multiplied by the square of the distance of that particle from the axis of rotation. If we give each particle a number (1, 2, 3, …), then

$$\Sigma m_i R_i^2 = m_1 R_1^2 + m_2 R_2^2 + m_3 R_3^2 + \cdots.$$

This summation is called the **moment of inertia** (or *rotational inertia*) I of the object:

$$I = \Sigma m_i R_i^2 = m_1 R_1^2 + m_2 R_2^2 + \cdots. \qquad \textbf{(10–13)}$$

Combining Eqs. 10–12 and 10–13, we can write

$$\Sigma\tau = I\alpha. \qquad \begin{bmatrix}\text{axis fixed in}\\ \text{inertial reference frame}\end{bmatrix} \qquad \textbf{(10–14)}$$

This is the rotational equivalent of Newton's second law. It is valid for the rotation of a rigid object about a fixed axis.† It can be shown (see Chapter 11) that Eq. 10–14 is valid even when the object is translating with acceleration, as long as I and α are calculated about the center of mass of the object, and the rotation axis through the CM doesn't change direction. (A ball rolling down a ramp is an example.) Then

$$(\Sigma\tau)_{\text{CM}} = I_{\text{CM}}\alpha_{\text{CM}}, \qquad \begin{bmatrix}\text{axis fixed in direction,}\\ \text{but may accelerate}\end{bmatrix} \qquad \textbf{(10–15)}$$

where the subscript CM means "calculated about the center of mass."

We see that the moment of inertia, I, which is a measure of the rotational inertia of an object, plays the same role for rotational motion that mass does for translational motion. As can be seen from Eq. 10–13, the rotational inertia of an object depends not only on its mass, but also on how that mass is distributed with respect to the axis. For example, a large-diameter cylinder will have greater rotational inertia than one of equal mass but smaller diameter (and therefore greater length), Fig. 10–18. The former will be harder to start rotating, and harder to stop. When the mass is concentrated farther from the axis of rotation, the rotational inertia is greater. For rotational motion, the mass of an object *cannot* be considered as concentrated at its center of mass.

6: Physik der Planeten
Gravitation

I. Wie kann man die Bewegung in einem rotierenden Bezugssystem beschreiben ?

Zentrifugalkraft und Corioliseffekt

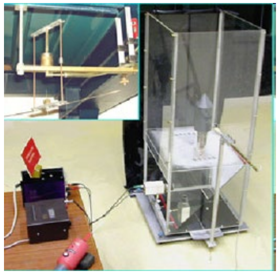
II. Wodurch wird die Umlaufbahn eines Planeten bestimmt ?

Die 3 Keplerschen Gesetze

Ursprung der Gezeiten

III. Warum ist der Raum gekrümmt ?

Äquivalenz-prinzip



Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(2 Seiten)*:

11-8 Rotating frames of reference

6-1 Newton's law of universal gravitation

Vorbereitende Übungen *(3)* **vor der Übungssession** zu erledigen **:**

Giancoli 6-1, 3, 37

Giancoli Kapitel 6-1 bis 6-5; 6-8 sowie 11-8 bis 11-9

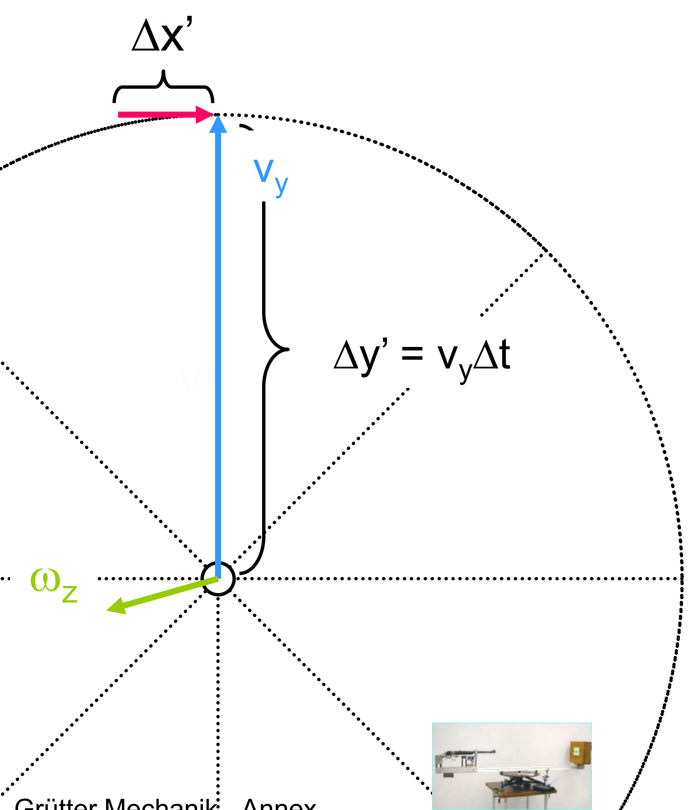
Anhang A. Herleitung der Norm des Corioliseffektes.

(Siehe auch Lektion 3)

Situation: Ein Objekt passiert das Zentrum des Karussel.

Im rotierenden Bezugssystem ist dieses Objekt einer zu seiner Geschwindigkeit senkrechten Beschleunigung ausgesetzt.

(nur eine kurze Zeit Δt berücksichtigt)



$$\Delta x' = \Delta y' \sin\Delta\theta = v_y \Delta t \sin\Delta\theta$$

$$= v_y \Delta t \Delta\theta \quad \sin\Delta\theta \cong \Delta\theta$$

$$= v_y \Delta t \Delta t \omega$$

$\Delta\theta = \omega\Delta t$

Kinematik:

$$\left.\Delta x' = a^c \Delta t^2/2 \right\} \quad a_x^c = 2 v_y \omega_z$$

In vektorieller Form ?

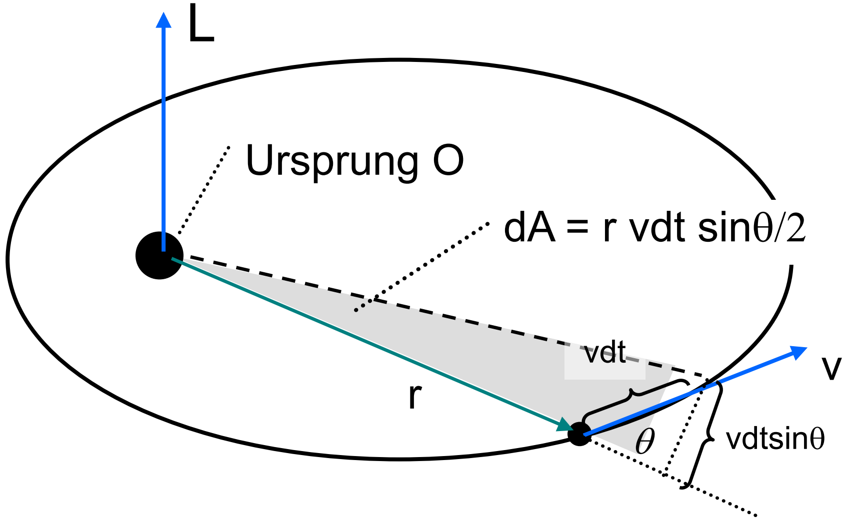
$$\Rightarrow \quad \boxed{\vec{a}_{Co} = -2\vec{\omega} \times \vec{v}}$$

NB. Bewegungen parallel zu ω erfahren keine solche Beschleunigung und werden nicht berücksichtigt. Warum ? Siehe …

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

Anhang B: Direkte Herleitung des 2. Keplerschen Gesetzes aus dem Gravitationsgesetz

« Der Ortsvektor des Planeten überstreicht in Δt gleich große Flächen »



Definition : $\vec{L} \equiv \vec{r} \times m\vec{v} = m(\vec{r} \times \vec{v})$

$$\underbrace{\frac{dA}{dt} = \frac{rv\sin\theta}{2} \qquad L = mrv\sin\theta}$$

$$dA = \frac{L}{2m}dt$$

$$\int_{t_1}^{t_2} dA = \Delta A = \frac{L}{2m}\Delta t \qquad \Delta t = \int_{t_1}^{t_2} dt$$

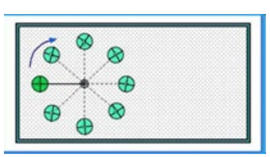
M und Δt sind konstant, aber L ?

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt} = (\vec{r} \times \vec{F}) = 0$$

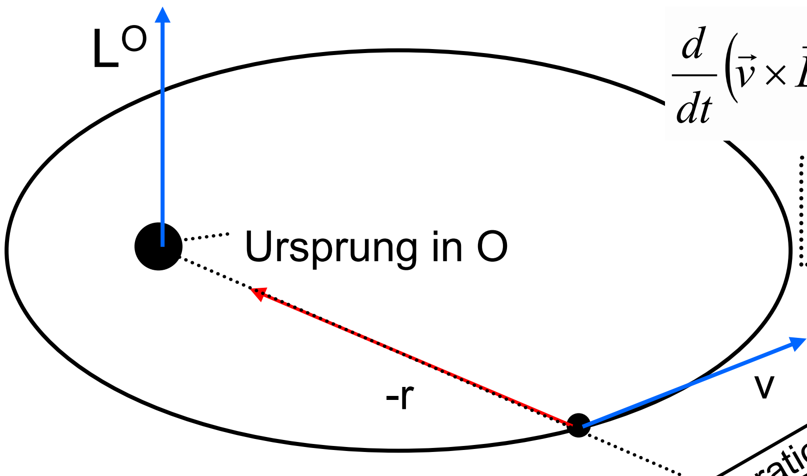
$$(fg)' = f'g + fg'$$

Zentralkraft (Gravitation) :
τ^O =**rxF**= 0

ΔA = konst.
(Konsequenz der Erhaltung von L, siehe Lektion 9)



Anhang C: Beweis dass das 1. Keplersche Gesetz eine direkte Konsequenz des Gravitationsgesetzes ist.



$$\frac{d}{dt}\left(\vec{v}\times\vec{L}\right)=\frac{\vec{F}}{m}\times\vec{L}=-\frac{GmM}{r^2}\left[\frac{\vec{r}}{r}\times\left(\vec{r}\times\vec{v}\right)\right]$$

Zentralfraft : τ^O = 0 = dL/dt

$$=-\frac{GmM}{r^2}\left[\frac{\vec{r}\cdot\vec{v}}{r}\vec{r}-\frac{\vec{r}\cdot\vec{r}}{r}\vec{v}\right]=-GmM\overbrace{\left[\left(\frac{\vec{r}}{r}\cdot\frac{\vec{v}}{r}\right)\frac{\vec{r}}{r}-\frac{\vec{v}}{r}\right]}^{-\frac{d\left(\vec{r}/r\right)}{dt}}$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{L}\right)=\frac{d}{dt}\left(GMm\frac{\vec{r}}{r}\right)$$

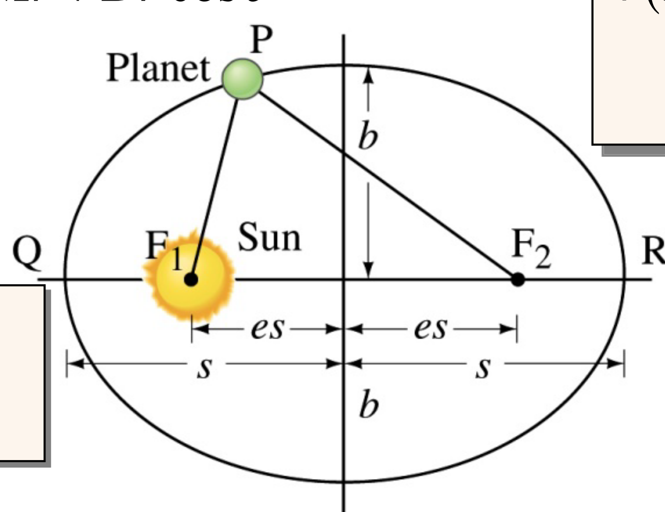
Integration

$$\left(\vec{v}\times\vec{L}\right)(t)=GmM\frac{\vec{r}}{r}(t)+\vec{D}$$

$$\vec{r}\cdot\left(\vec{v}\times\vec{L}\right)=\underbrace{\left(\vec{r}\times\vec{v}\right)\cdot\vec{L}}_{\frac{\vec{L}\cdot\vec{L}}{m}}=GmMr+Dr\cos\theta$$

$$r(\theta)=\frac{L^2/m}{GM\left(1+\frac{D}{GmM}\cos\theta\right)}$$

D/GmM : Exzentrizität



1. Gesetz Die Planeten bewegen sich auf elliptischen Bahnen, in deren einem Brennpunkt sich die Sonne befindet.

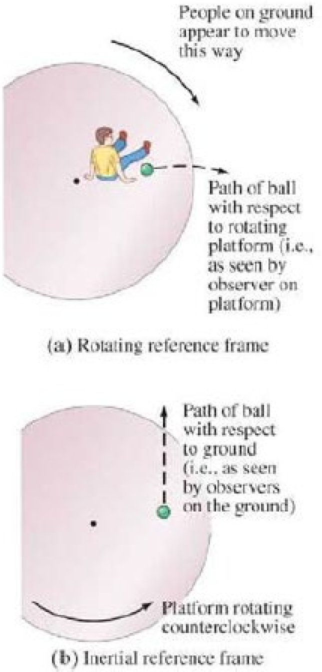
*11–8 Rotating Frames of Reference; Inertial Forces

Inertial and Noninertial Reference Frames

Up to now, we have examined the motion of objects, including circular and rotational motion, from the outside, as observers fixed on the Earth. Sometimes it is convenient to place ourselves (in theory, if not physically) into a reference frame that is rotating. Let us examine the motion of objects from the point of view, or frame of reference, of persons seated on a rotating platform such as a merry-go-round. It looks to them as if the rest of the world is going around *them*. But let us focus attention on what they observe when they place a tennis ball on the floor of the rotating platform, which we assume is frictionless. If they put the ball down gently, without giving it any push, they will observe that it accelerates from rest and moves outward as shown in Fig. 11–25a. According to Newton's first law, an object initially at rest should stay at rest if no net force acts on it. But, according to the observers on the rotating platform, the ball starts moving even though there is no net force acting on it. To observers on the ground this is all very clear: the ball has an initial velocity when it is released (because the platform is moving), and it simply continues moving in a straight-line path as shown in Fig. 11–25b, in accordance with Newton's first law.

But what shall we do about the frame of reference of the observers on the rotating platform? Since the ball moves without any net force on it, Newton's first law, the law of inertia, does not hold in this rotating frame of reference. For this reason, such a frame is called a **noninertial reference frame**. An **inertial reference frame** (as we discussed in Chapter 4) is one in which the law of inertia—Newton's first law—does hold, and so do Newton's second and third laws. In a noninertial reference frame, such as our rotating platform, Newton's second law also does not hold. For instance in the situation described above, there is no net force on the ball; yet, with respect to the rotating platform, the ball accelerates.

FIGURE 11–25 Path of a ball released on a rotating merry-go-round (a) in the reference frame of the merry-go-round, and (b) in a reference frame fixed on the ground.



Fictitious (Inertial) Forces

Because Newton's laws do not hold when observations are made with respect to a rotating frame of reference, calculation of motion can be complicated. However, we can still make use of Newton's laws in such a reference frame if we make use of a trick. The ball on the rotating platform of Fig. 11–25a flies outward when released (even though no force is actually acting on it). So the trick we use is to write down the equation $\Sigma F = ma$ as if a force equal to mv^2/r (or $m\omega^2 r$) were acting radially outward on the object in addition to any other forces that may be acting. This extra force, which might be designated as "centrifugal force" since it *seems* to act outward, is called a **fictitious force** or **pseudoforce**. It is a pseudoforce ("pseudo" means "false") because there is no object that exerts this force. Furthermore, when viewed from an inertial reference frame, the effect doesn't exist at all. We have made up this pseudoforce so that we can make calculations in a noninertial frame using Newton's second law, $\Sigma F = ma$. Thus the observer in the noninertial frame of Fig. 11–25a uses Newton's second law for the ball's outward motion by assuming that a force equal to mv^2/r acts on it. Such pseudoforces are also called **inertial forces** since they arise only because the reference frame is not an inertial one.

The Earth itself is rotating on its axis. Thus, strictly speaking, Newton's laws are not valid on the Earth. However, the effect of the Earth's rotation is usually so small that it can be ignored, although it does influence the movement of large air masses and ocean currents. Because of the Earth's rotation, the material of the Earth is concentrated slightly more at the equator. The Earth is thus not a perfect sphere but is slightly fatter at the equator than at the poles.

6–1 Newton's Law of Universal Gravitation

Among his many great accomplishments, Sir Isaac Newton examined the motion of the heavenly bodies—the planets and the Moon. In particular, he wondered about the nature of the force that must act to keep the Moon in its nearly circular orbit around the Earth.

Newton was also thinking about the problem of gravity. Since falling objects accelerate, Newton had concluded that they must have a force exerted on them, a force we call the force of gravity. Whenever an object has a force exerted *on* it, that force is exerted *by* some other object. But what exerts the force of gravity? Every object on the surface of the Earth feels the force of gravity, and no matter where the object is, the force is directed toward the center of the Earth (Fig. 6–1). Newton concluded that it must be the Earth itself that exerts the gravitational force on objects at its surface.

According to legend, Newton noticed an apple drop from a tree. He is said to have been struck with a sudden inspiration: If gravity acts at the tops of trees, and even at the tops of mountains, then perhaps it acts all the way to the Moon! With this idea that it is Earth's gravity that holds the Moon in its orbit, Newton developed his great theory of gravitation. But there was controversy at the time. Many thinkers had trouble accepting the idea of a force "acting at a distance." Typical forces act through contact—your hand pushes a cart and pulls a wagon, a bat hits a ball, and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the Moon, even though there is no contact, and the two objects may even be very far apart.

Newton set about determining the magnitude of the gravitational force that the Earth exerts on the Moon as compared to the gravitational force on objects at the Earth's surface. At the surface of the Earth, the force of gravity accelerates objects at $9.80\,\mathrm{m/s^2}$. The centripetal acceleration of the Moon is calculated from $a_R = v^2/r$ (see Example 5–9) and gives $a_R = 0.00272\,\mathrm{m/s^2}$. In terms of the acceleration of gravity at the Earth's surface, g, this is equivalent to

$$a_R = \frac{0.00272\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}}\,g \approx \frac{1}{3600}\,g.$$

That is, the acceleration of the Moon toward the Earth is about $\frac{1}{3600}$ as great as the acceleration of objects at the Earth's surface. The Moon is 384,000 km from the Earth, which is about 60 times the Earth's radius of 6380 km. That is, the Moon is 60 times farther from the Earth's center than are objects at the Earth's surface. But $60 \times 60 = 60^2 = 3600$. Again that number 3600! Newton concluded that the gravitational force F exerted by the Earth on any object decreases with the square of its distance, r, from the Earth's center:

$$F \propto \frac{1}{r^2}.$$

The Moon is 60 Earth radii away, so it feels a gravitational force only $\frac{1}{60^2} = \frac{1}{3600}$ times as strong as it would if it were a point at the Earth's surface.

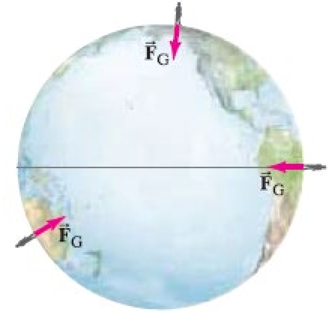


FIGURE 6–1 Anywhere on Earth, whether in Alaska, Australia, or Peru, the force of gravity acts downward toward the center of the Earth.

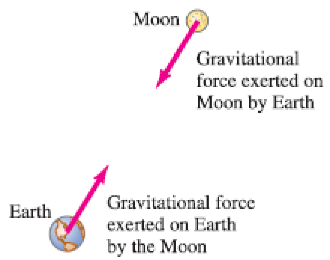


FIGURE 6–2 The gravitational force one object exerts on a second object is directed toward the first object, and is equal and opposite to the force exerted by the second object on the first.

Newton realized that the force of gravity on an object depends not only on distance but also on the object's mass. In fact, it is directly proportional to its mass, as we have seen. According to Newton's third law, when the Earth exerts its gravitational force on any object, such as the Moon, that object exerts an equal and opposite force on the Earth (Fig. 6–2). Because of this symmetry, Newton reasoned, the magnitude of the force of gravity must be proportional to *both* the masses. Thus

$$F \propto \frac{m_E m_B}{r^2},$$

where m_E is the mass of the Earth, m_B the mass of the other object, and r the distance from the Earth's center to the center of the other object.

Newton went a step further in his analysis of gravity. In his examination of the orbits of the planets, he concluded that the force required to hold the different planets in their orbits around the Sun seems to diminish as the inverse square of their distance from the Sun. This led him to believe that it is also the gravitational force that acts between the Sun and each of the planets to keep them in their orbits. And if gravity acts between these objects, why not between all objects? Thus he proposed his **law of universal gravitation**, which we can state as follows:

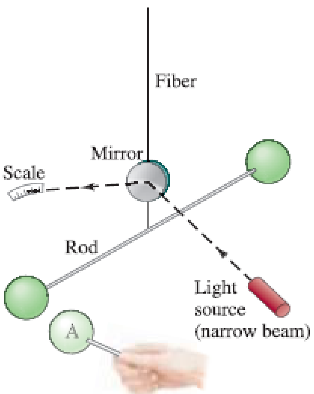
Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

The magnitude of the gravitational force can be written as

$$\boxed{F = G\frac{m_1 m_2}{r^2},} \qquad \textbf{(6–1)}$$

where m_1 and m_2 are the masses of the two particles, r is the distance between them, and G is a universal constant which must be measured experimentally and has the same numerical value for all objects.

FIGURE 6–3 Schematic diagram of Cavendish's apparatus. Two spheres are attached to a light horizontal rod, which is suspended at its center by a thin fiber. When a third sphere (labeled A) is brought close to one of the suspended spheres, the gravitational force causes the latter to move, and this twists the fiber slightly. The tiny movement is magnified by the use of a narrow light beam directed at a mirror mounted on the fiber. The beam reflects onto a scale. Previous determination of how large a force will twist the fiber a given amount then allows the experimenter to determine the magnitude of the gravitational force between two objects.



The value of G must be very small, since we are not aware of any force of attraction between ordinary-sized objects, such as between two baseballs. The force between two ordinary objects was first measured by Henry Cavendish in 1798, over 100 years after Newton published his law. To detect and measure the incredibly small force between ordinary objects, he used an apparatus like that shown in Fig. 6–3. Cavendish confirmed Newton's hypothesis that two objects attract one another and that Eq. 6–1 accurately describes this force. In addition, because Cavendish could measure F, m_1, m_2, and r accurately, he was able to determine the value of the constant G as well. The accepted value today is

$$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}.$$

(See Table inside front cover for values of all constants to highest known precision.)

Strictly speaking, Eq. 6–1 gives the magnitude of the gravitational force that one particle exerts on a second particle that is a distance r away. For an extended object (that is, not a point), we must consider how to measure the distance r. You might think that r would be the distance between the centers of the objects. This is true for two spheres, and is often a good approximation for other objects. A correct calculation treats each extended body as a collection of particles, and the total force is the sum of the forces due to all the particles. The sum over all these particles is often best done using integral calculus, which Newton himself invented. When extended bodies are small compared to the distance between them (as for the Earth–Sun system), little inaccuracy results from considering them as point particles.

1. (I) Calculate the force of Earth's gravity on a spacecraft 2.00 Earth radii above the Earth's surface if its mass is 1480 kg.

3. (I) A hypothetical planet has a radius 2.3 times that of Earth, but has the same mass. What is the acceleration due to gravity near its surface?

37. (I) Use Kepler's laws and the period of the Moon (27.4 d) to determine the period of an artificial satellite orbiting very near the Earth's surface.

7: Arbeit und Energie

I. Wie kann man die Geschwindigkeit eines Objektes aufgrund seiner Bahnkurve bestimmen (ohne die benötigte Zeit zu berechnen) ?

Das Arbeit-Energie Prinzip

Konservative Kräfte

II. Welche Energieformen hängen von der Position des Objektes ab ?

Mechanische Energie

III. Wie berechnet man die Leistung eines sich in Bewegung befindenden Objektes ?

IV. Unter welchen Umständen ist die mechanische Energie nicht erhalten ?

Energieerhaltungssatz

Nicht-konservative Kräfte

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(4 Seiten)*:

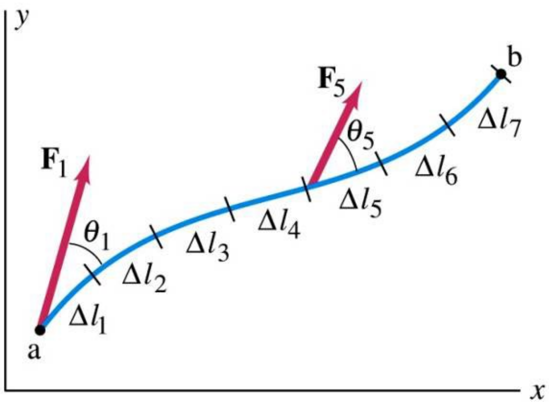
7-1 Work done by a constant force
7-4 Kinetic energy and the work-energy principle
8-5 Law of conservation of energy

Vorbereitende Übungen *(9)* **vor der Übungssession** zu erledigen **:**

Giancoli 7-5, 50, 53, (59), 63abc
8-12, 62, 70, 39

Giancoli Kapitel 7-1 bis 7-4; 8-1 bis 8-8

Anhang: Die Arbeit einer Kraft im Allgemeinen



$$\Delta W_i = F_i \Delta l_i \cos\theta_i = \mathbf{F_i} \cdot \mathbf{\Delta l_i}$$

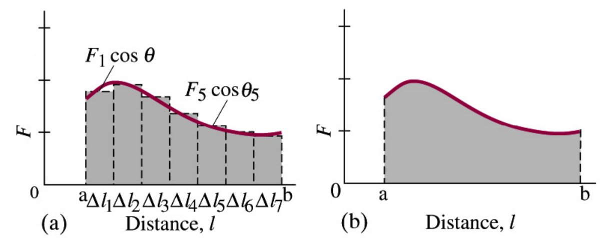
$$\text{Limes } \Delta l \to 0:$$

$$dW = F\cos\theta dl = \vec{F} \cdot d\vec{l}$$

$$W_{a-b} = \int_a^b \vec{F} \cdot d\vec{l}$$

$$= \int_{x_a}^{x_b} F_x dx + \int_{y_a}^{y_b} F_y dy + \int_{z_a}^{z_b} F_z dz$$

Diese Integrale (=Fläche der Funktion **F**(**x**)) berechnet man in dem man der Bahnkurve folgt



$$dW = \vec{F} \cdot d\vec{l} = m\vec{a} \cdot d\vec{l}$$

$$= m\frac{d\vec{v}}{dt} \cdot d\vec{l} = m d\vec{v} \cdot \frac{d\vec{l}}{dt}$$

$$= m\vec{v} \cdot d\vec{v} \begin{cases} m\vec{v} \cdot d\vec{v} = m\left(v_x dv_x + v_y dv_y + v_z dv_z\right) \\ = m\dfrac{(d(v_x^2) + d(v_y^2) + d(v_y^2))}{2} \\ = m\dfrac{d(\vec{v}\cdot\vec{v})}{2} \underset{m=const}{=} d(\dfrac{mv^2}{2}) \equiv dK \end{cases}$$

Kinetische Energie

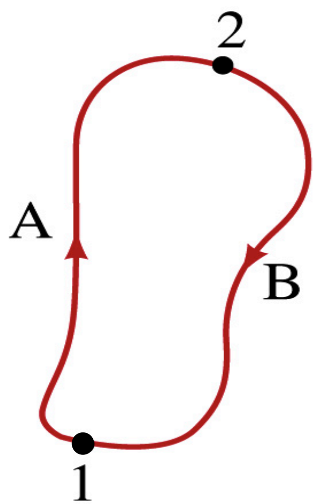
$$K \equiv \frac{mv^2}{2}$$

QED

Anhang: Konservative Kräft auf einer geschlossenen Bahnkurve

Äquivalente Definition:

Die Arbeit einer konservativen Kraft ist gleich null auf einer geschlossenen Bahnkurve.



Beweis: $W_{1-1}^{A} = W_{1-2}^{A} + W_{2-1}^{B}$

per Definition $W_{2-1}^{B} = W_{2-1}^{A}$ (konservative Kraft)

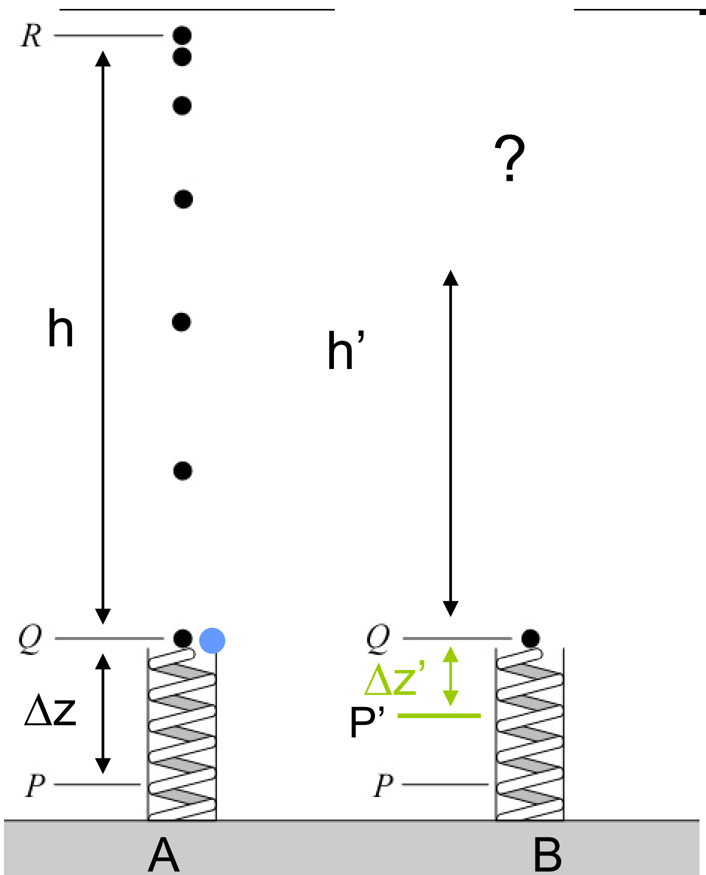
$$W_{2-1}^{A} = \int_2^1 \vec{F}\cdot d\vec{l} = \int_1^2 \vec{F}\cdot(-d\vec{l}\,) = -W_{1-2}^{A}$$

und es folgt, dass

$$\rightarrow W_{1-1}^{A} = W_{1-2}^{A} - W_{1-2}^{A} = 0$$

QED

Beispiel und Bonus-Quiz



Annahme $\Delta z \ll h$

Situation: Das Ende einer Feder im Ruhezustand befindet sich im Punkt Q. Die Feder wird um eine Länge Δz komprimiert, bevor sie losgelassen wird. Beim Loslassen wird eine Kugel bis in eine Höhe h katapultiert. (Fig. A)

Frage 1: Würde man die Feder um die Hälfte komprimieren ($\Delta z'=0.5\ \Delta z$), wie in B illustriert, bis in welche Höhe würde die Kugel fliegen ?

a) h'=2h
b) h'=h
c) h'=h/2
d) h'=h/4

Frage 2: Verdoppelt man die Masse der Kugel ($m'=2m$ mit $\Delta z'=\Delta z$) in A, welche Höhe würde sie nun erreichen?

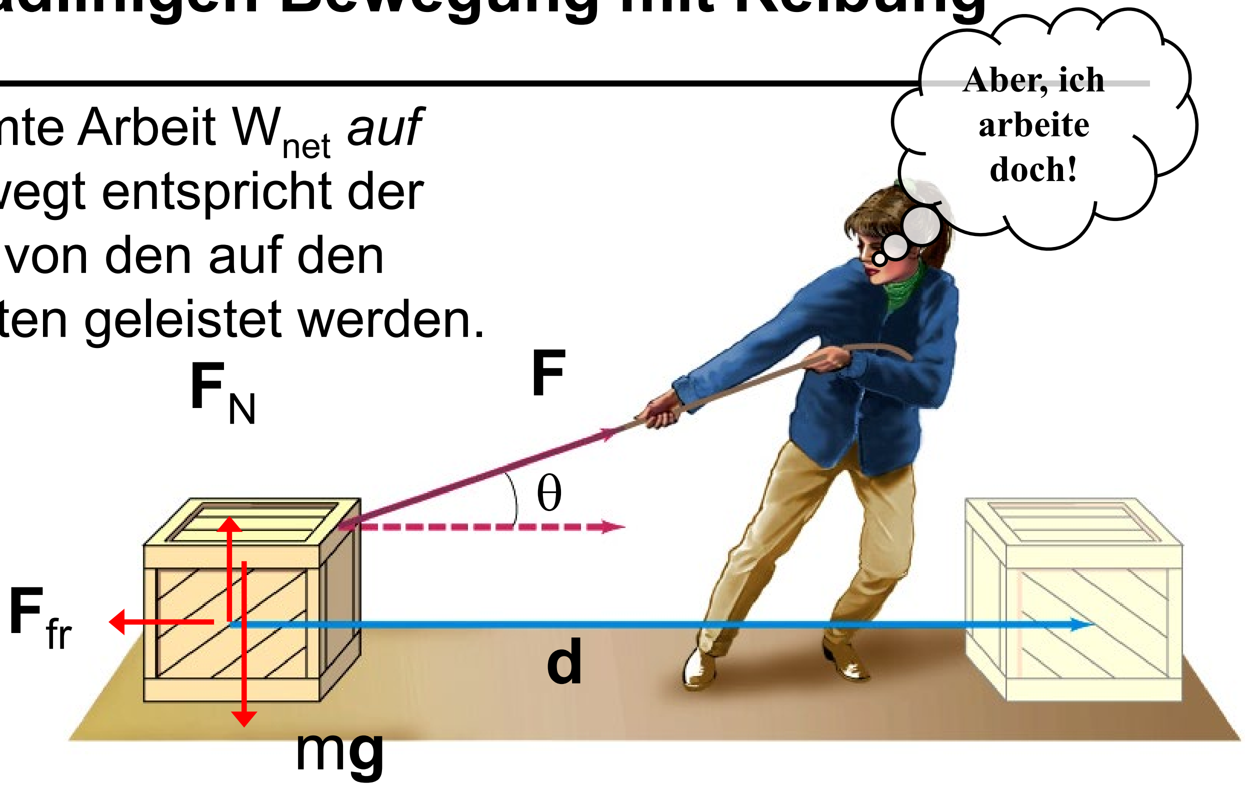
a) h'=2h
b) h'=h
c) h'=h/2
d) h'=h/4

zB. Arbeit einer geradlinigen Bewegung mit Reibung

z. Erinnerung: Die gesamte Arbeit W_{net} *auf einen Körper* der sich bewegt entspricht der Summe aller Arbeiten die von den auf den Körper einwirkenden Kräften geleistet werden.

$$W_{net} = \sum_k W_k$$

$$\vec{F}_{net} = \sum_k \vec{F}_k$$



Beispiel: $W_{net} = \underbrace{F_N d\cos 90}_{0} + \underbrace{mgd\cos 90}_{0} + \underbrace{Fd\cos\theta + F_{fr}d\cos 180}_{(F\cos\theta - F_{fr})d} = 0 \cdot d = 0$

(v=const$\rightarrow F\cos\theta = F_{fr}$)

Die Kräfte normal zur Bahnkurve (m**g**, $\mathbf{F}_N$ sont ⊥ à **d**) verrichten keine Arbeit. Die Stosskraft **F** und die Reibungskraft $\mathbf{F}_{fr}$ verrichten Arbeit

7–1 Work Done by a Constant Force

The word *work* has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished when a force acts on an object, and the object moves through a distance. We consider only translational motion for now and, unless otherwise explained, objects are assumed to be rigid with no complicating internal motion, and can be treated like particles. Then the **work** done on an object by a constant force (constant in both magnitude and direction) is defined to be *the product of the magnitude of the displacement times the component of the force parallel to the displacement*. In equation form, we can write

$$W = F_{\parallel} d,$$

where $F_{\parallel}$ is the component of the constant force $\vec{\mathbf{F}}$ parallel to the displacement $\vec{\mathbf{d}}$. We can also write

$$W = Fd\cos\theta, \qquad \textbf{(7–1)}$$

where F is the magnitude of the constant force, d is the magnitude of the displacement of the object, and θ is the angle between the directions of the force and the displacement (Fig. 7–1). The $\cos\theta$ factor appears in Eq. 7–1 because $F\cos\theta\,(= F_{\parallel})$ is the component of $\vec{\mathbf{F}}$ that is parallel to $\vec{\mathbf{d}}$. Work is a scalar quantity—it has only magnitude, which can be positive or negative.

Let us consider the case in which the motion and the force are in the same direction, so $\theta = 0$ and $\cos\theta = 1$; in this case, $W = Fd$. For example, if you push a loaded grocery cart a distance of 50 m by exerting a horizontal force of 30 N on the cart, you do $30\text{ N} \times 50\text{ m} = 1500\text{ N}\cdot\text{m}$ of work on the cart.

As this example shows, in SI units work is measured in newton-meters (N·m). A special name is given to this unit, the **joule** (J): $1\text{ J} = 1\text{ N}\cdot\text{m}$.

[In the cgs system, the unit of work is called the *erg* and is defined as $1\text{ erg} = 1\text{ dyne}\cdot\text{cm}$. In British units, work is measured in foot-pounds. It is easy to show that $1\text{ J} = 10^7\text{ erg} = 0.7376\text{ ft}\cdot\text{lb}$.]

A force can be exerted on an object and yet do no work. If you hold a heavy bag of groceries in your hands at rest, you do no work on it. You do exert a force on the bag, but the displacement of the bag is zero, so the work done by you on the bag is $W = 0$. You need both a force and a displacement to do work. You also do no work on the bag of groceries if you carry it as you walk horizontally across the floor at constant velocity, as shown in Fig. 7–2. No horizontal force is required to move the bag at a constant velocity. The person shown in Fig. 7–2 does exert an upward force $\vec{\mathbf{F}}_P$ on the bag equal to its weight. But this upward force is perpendicular to the horizontal displacement of the bag and thus is doing no work. This conclusion comes from our definition of work, Eq. 7–1: $W = 0$, because $\theta = 90°$ and $\cos 90° = 0$.

5. (II) Estimate the work you do to mow a lawn 10 m by 20 m with a 50-cm wide mower. Assume you push with a force of about 15 N.

50. (I) At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a kinetic energy of about 6.21×10^{-21} J. How fast is it moving?

53. (I) How much work must be done to stop a 1300-kg car traveling at 95 km/h?

Thus, when a particular force is perpendicular to the displacement, no work is done by that force. When you start or stop walking, there is a horizontal acceleration and you do briefly exert a horizontal force, and thus do work on the bag.

When we deal with work, as with force, it is necessary to specify whether you are talking about work done *by* a specific object or done *on* a specific object. It is also important to specify whether the work done is due to one particular force (and which one), or the total (net) work done by the *net force* on the object.

CAUTION
State that work is done on or by an object

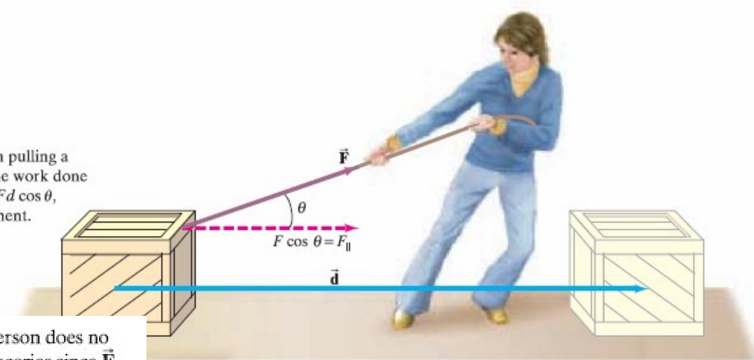


FIGURE 7–1 A person pulling a crate along the floor. The work done by the force $\vec{\mathbf{F}}$ is $W = Fd\cos\theta$, where $\vec{\mathbf{d}}$ is the displacement.

FIGURE 7–2 The person does no work on the bag of groceries since $\vec{\mathbf{F}}_P$ is perpendicular to the displacement $\vec{\mathbf{d}}$.

59. (II) A 1200-kg car rolling on a horizontal surface has speed $v = 66$ km/h when it strikes a horizontal coiled spring and is brought to rest in a distance of 2.2 m. What is the spring constant of the spring?

63. (II) (*a*) How much work is done by the horizontal force $F_P = 150$ N on the 18-kg block of Fig. 7–29 when the force pushes the block 5.0 m up along the 32° frictionless incline? (*b*) How much work is done by the gravitational force on the block during this displacement? (*c*) How much work is done by the normal force? (*d*) What is the speed of the block (assume that it is zero initially) after this displacement? [*Hint*: Work-energy involves *net* work done.]

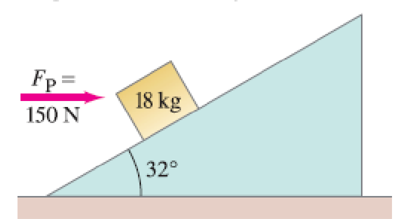


FIGURE 7–29
Problems 63 and 64.

7–4 Kinetic Energy and the Work-Energy Principle

Energy is one of the most important concepts in science. Yet we cannot give a simple general definition of energy in only a few words. Nonetheless, each specific type of energy can be defined fairly simply. In this Chapter we define translational kinetic energy; in the next Chapter, we take up potential energy. In later Chapters we will examine other types of energy, such as that related to heat (Chapters 19 and 20). The crucial aspect of all the types of energy is that the sum of all types, the *total energy*, is the same after any process as it was before: that is, energy is a conserved quantity.

For the purposes of this Chapter, we can define energy in the traditional way as "the ability to do work." This simple definition is not very precise, nor is it really valid for all types of energy.† It works, however, for mechanical energy which we discuss in this Chapter and the next. We now define and discuss one of the basic types of energy, kinetic energy.

A moving object can do work on another object it strikes. A flying cannonball does work on a brick wall it knocks down; a moving hammer does work on a nail it drives into wood. In either case, a moving object exerts a force on a second object which undergoes a displacement. An object in motion has the ability to do work and thus can be said to have energy. The energy of motion is called **kinetic energy**, from the Greek word *kinetikos*, meaning "motion."

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass m (treated as a particle) that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a displacement d, Fig. 7–14. Then the net work done on the object is $W_{\text{net}} = F_{\text{net}}d$. We apply Newton's second law, $F_{\text{net}} = ma$, and use Eq. 2–12c $(v_2^2 = v_1^2 + 2ad)$, which we rewrite as

$$a = \frac{v_2^2 - v_1^2}{2d},$$

where v_1 is the initial speed and v_2 the final speed. Substituting this into $F_{\text{net}} = ma$, we determine the work done:

$$W_{\text{net}} = F_{\text{net}}d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{\text{net}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(7–9)}$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy**, K, of the object:

$$K = \tfrac{1}{2}mv^2. \qquad \textbf{(7–10)}$$

(We call this "translational" kinetic energy to distinguish it from rotational kinetic energy, which we discuss in Chapter 10.) Equation 7–9, derived here for one-dimensional motion with a constant force, is valid in general for translational motion of an object in three dimensions and even if the force varies, as we will show at the end of this Section.

†Energy associated with heat is often not available to do work, as we will discuss in Chapter 20.

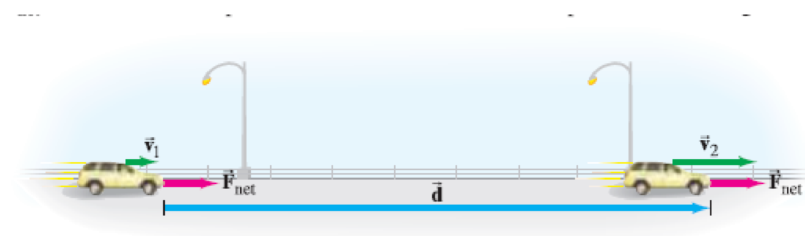


FIGURE 7–14 A constant net force F_{net} accelerates a car from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{\text{net}} = F_{\text{net}}d$.

We can rewrite Eq. 7–9 as:

$$W_{\text{net}} = K_2 - K_1$$

or

$$W_{\text{net}} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(7–11)}$$

WORK-ENERGY PRINCIPLE

Equation 7–11 (or Eq. 7–9) is a useful result known as the **work-energy principle**. It can be stated in words:

The net work done on an object is equal to the change in the object's kinetic energy.

WORK-ENERGY PRINCIPLE

Notice that we made use of Newton's second law, $F_{\text{net}} = ma$, where F_{net} is the *net* force—the sum of all forces acting on the object. Thus, the work-energy principle is valid only if W is the *net work* done on the object—that is, the work done by all forces acting on the object.

CAUTION
Work-energy valid only for net work

The work-energy principle is a very useful reformulation of Newton's laws. It tells us that if (positive) net work W is done on an object, the object's kinetic energy increases by an amount W. The principle also holds true for the reverse situation: if the net work W done on an object is negative, the object's kinetic energy decreases by an amount W. That is, a net force exerted on an object opposite to the object's direction of motion decreases its speed and its kinetic energy. An example is a moving hammer (Fig. 7–15) striking a nail. The net force on the hammer ($-\vec{\mathbf{F}}$ in Fig. 7–15, where $\vec{\mathbf{F}}$ is assumed constant for simplicity) acts toward the left, whereas the displacement $\vec{\mathbf{d}}$ of the hammer is toward the right. So the net work done on the hammer, $W_h = (F)(d)(\cos 180°) = -Fd$, is negative and the hammer's kinetic energy decreases (usually to zero).

Figure 7–15 also illustrates how energy can be considered the ability to do work. The hammer, as it slows down, does positive work on the nail: $W_n = (+F)(+d)(\cos 0°) = Fd$ and is positive. The decrease in kinetic energy of the hammer ($= Fd$ by Eq. 7–11) is equal to the work the hammer can do on another object, the nail in this case.

The translational kinetic energy $(= \frac{1}{2}mv^2)$ is directly proportional to the mass of the object, and it is also proportional to the *square* of the speed. Thus, if the mass is doubled, the kinetic energy is doubled. But if the speed is doubled, the object has four times as much kinetic energy and is therefore capable of doing four times as much work.

Because of the direct connection between work and kinetic energy, energy is measured in the same units as work: joules in SI units. [The energy unit is ergs in the cgs, and foot-pounds in the British system.] Like work, kinetic energy is a scalar quantity. The kinetic energy of a group of objects is the sum of the kinetic energies of the individual objects.

The work-energy principle can be applied to a particle, and also to an object that can be approximated as a particle, such as an object that is rigid or whose internal motions are insignificant. It is very useful in simple situations, as we will see in the Examples below. The work-energy principle is not as powerful and encompassing as the law of conservation of energy which we treat in the next Chapter, and should not itself be considered a statement of energy conservation.

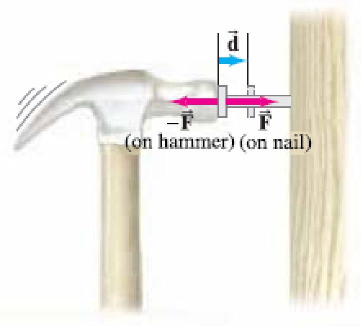


FIGURE 7–15 A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force $-F$ on the hammer (Newton's third law). The work done on the nail by the hammer is positive $(W_n = Fd > 0)$. The work done on the hammer by the nail is negative $(W_h = -Fd)$.

12. (I) Jane, looking for Tarzan, is running at top speed (5.0 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine affect your answer?

62. (I) How long will it take a 1750-W motor to lift a 335-kg piano to a sixth-story window 16.0 m above?

70. (II) A pump lifts 21.0 kg of water per minute through a height of 3.50 m. What minimum output rating (watts) must the pump motor have?

39. (II) You drop a ball from a height of 2.0 m, and it bounces back to a height of 1.5 m. (*a*) What fraction of its initial energy is lost during the bounce? (*b*) What is the ball's speed just before and just after the bounce? (*c*) Where did the energy go?

8–5 The Law of Conservation of Energy

We now take into account nonconservative forces such as friction, since they are important in real situations. For example, consider again the roller-coaster car in Fig. 8–8, but this time let us include friction. The car will not in this case reach the same height on the second hill as it had on the first hill because of friction.

In this, and in other natural processes, the mechanical energy (sum of the kinetic and potential energies) does not remain constant but decreases. Because frictional forces reduce the mechanical energy (but *not* the total energy), they are called **dissipative forces**. Historically, the presence of dissipative forces hindered the formulation of a comprehensive conservation of energy law until well into the nineteenth century. It was not until then that heat, which is always produced when there is friction (try rubbing your hands together), was interpreted in terms of energy. Quantitative studies in the nineteenth-century (Chapter 19) demonstrated that if heat is considered as a transfer of energy (sometimes called **thermal energy**), then the total energy is conserved in any process. For example, if the roller-coaster car in Fig. 8–8 is subject to frictional forces, then the initial total energy of the car will be equal to the car's kinetic energy plus the potential energy at any subsequent point along its path plus the amount of thermal energy produced in the process. A block sliding freely across a table, for example, comes to rest because of friction. Its initial kinetic energy is all transformed into thermal energy. The block and table are a little warmer as a result of this process: both have absorbed some thermal energy. Another example of the transformation of kinetic energy into thermal energy can be observed by vigorously striking a nail several times with a hammer and then gently touching the nail with your finger.

According to the atomic theory, thermal energy represents kinetic energy of rapidly moving molecules. We shall see in Chapter 18 that a rise in temperature corresponds to an increase in the average kinetic energy of the molecules. Because thermal energy represents the energy of atoms and molecules that make up an object, it is often called **internal energy**. Internal energy, from the atomic point of view, can include not only kinetic energy of molecules but also potential energy (usually electrical in nature) because of the relative positions of atoms within molecules. On a macroscopic level, thermal or internal energy corresponds to nonconservative forces such as friction. But at the atomic level, the energy is partly kinetic, partly potential corresponding to forces that are conservative. For example, the energy stored in food or in a fuel such as gasoline can be regarded as potential energy stored by virtue of the relative positions of the atoms within a molecule due to electric forces between atoms (referred to as chemical bonds). For this energy to be used to do work, it must be released, usually through chemical reactions (Fig. 8–15). This is analogous to a compressed spring which, when released, can do work.

To establish the more general law of conservation of energy, it required nineteenth-century physicists to recognize electrical, chemical, and other forms of energy in addition to heat and to explore if in fact they could fit into a conservation law. For each type of force, conservative or nonconservative, it has always been found possible to define a type of energy that corresponds to the work done by such a force. And it has been found experimentally that the total energy E always remains constant. That is, the change in the total energy, kinetic plus potential plus all other forms of energy, equals zero:

$$\Delta K + \Delta U + [\text{change in all other forms of energy}] = 0. \qquad \textbf{(8–14)}$$

This is one of the most important principles in physics. It is called the **law of conservation of energy** and can be stated as follows:

The total energy is neither increased nor decreased in any process. Energy can

For conservative mechanical systems, this law can be derived from Newton's laws (Section 8–3) and thus is equivalent to them. But in its full generality, the validity of the law of conservation of energy rests on experimental observation.

Even though Newton's laws have been found to fail in the submicroscopic world of the atom, the law of conservation of energy has been found to hold there and in every experimental situation so far tested.

FIGURE 8–15 The burning of fuel (a chemical reaction) releases energy to boil water in this steam engine. The steam produced expands against a piston to do work in turning the wheels.

8: Schwingungen

I. Wie analysiert man die Kräfte eines Potentials?

Energie-diagramm

(In-)stabile Gleichgewichte

II. Wie beschreibt man die Bewegung am stabilen Gleichgewicht ?

Harmonische Schwingungen

Gedämpfte harmonische Schwingungen

III. Was beschreibt die erzwungene harmonische Schwingung ?

Resonanz

Vorbereitung für den Kurs und die Uebungen

Kapitel im Giancoli **vor dem Kurs** zu lesen *(4 p)*:

8-9 Potential energy diagrams
14-1 Oscillations of a spring; 14-2 Simple harmonic motion
14-7 Damped harmonic oscillator
14-8 Forced oscillations

Vorbereitungsübungen *(4)* **vor den Übungen** zu lösen:

Giancoli 14-3, 9, 59, 65

Giancoli Kapitel 8-9;14-1 bis 5; 14-7 und 14-8

Anhang A. Wie kann man die mathematische Lösung des gedämpften Pendels bestimmen ?

Mit dem Ansatz (s. Kurs) $$x(t) = Ae^{-\alpha t}\cos(\omega t + \varphi)$$

Davon die 1. und 2. Ableitung (Annahme ϕ=0): $$\frac{dx}{dt} = Ae^{-\alpha t}\left(-\omega\sin(\omega t) - \alpha\cos(\omega t)\right)$$

$$\frac{d^2x}{dt^2} = \underbrace{\frac{d}{dt}\left[Ae^{-\alpha t}\right.}_{\left[-A\alpha e^{-\alpha t}\left(-\omega\sin(\omega t)-\alpha\cos(\omega t)\right)\right]}\underbrace{\left.\left(-\omega\sin(\omega t) - \alpha\cos(\omega t)\right)\right]}_{\left[Ae^{-\alpha t}\left(-\omega^2\cos(\omega t)+\alpha\omega\sin(\omega t)\right)\right]}$$

(fg)'=f'g+fg'

$$= Ae^{-\alpha t}\left[2\alpha\omega\sin(\omega t) + \left(\alpha^2 - \omega^2\right)\cos(\omega t)\right]$$

In die Bewegungsgleichung einsetzen, nach sin und cos gruppieren

$$\frac{d^2x}{dt^2} + \frac{k}{m}x(t) + \frac{b}{m}\frac{dx}{dt} = 0$$

$$Ae^{-\alpha t}$$

C_1:
$$[2\alpha\omega]\sin(\omega t) + \frac{b}{m}[-\omega]\sin(\omega t)$$

C_2:
$$\left[\alpha^2-\omega^2\right]\cos(\omega t) + \frac{b}{m}[-\alpha]\cos(\omega t) + \frac{k}{m}\cos(\omega t)$$

Anhang A (Folge) Welches sind die Konstanten des gedämpften Pendels ?

$$Ae^{-\alpha t}\left[C_1 \sin(\omega t) + C_2 \cos(\omega t)\right] = 0$$

$$Ae^{-\alpha t}\begin{array}{lll} & & \\ C_1 & [2\alpha\omega] & \sin(\omega t) \\ & \frac{b}{m}[-\omega] & \sin(\omega t) \\ & [\alpha^2-\omega^2] & \cos(\omega t) \\ C_2 & \frac{b}{m}[-\alpha] & \cos(\omega t) \\ & \frac{k}{m} & \cos(\omega t) \end{array}$$

$$C_1 = 2\alpha\omega - \frac{b}{m}\omega$$

$$C_2 = \alpha^2 - \omega^2 - \frac{b}{m}\alpha + \frac{k}{m}$$

Für irgendeine Zeit t gültig :

$C_1 = C_2 = 0$

NB. $Ae^{-\alpha t} \neq 0$

$$C_1 = \left(2\alpha - \frac{b}{m}\right)\omega = 0 \quad \Rightarrow \quad \alpha = \frac{b}{2m}$$

$$C_2 = \left(\frac{b^2}{4m^2} - \omega^2\right) + \frac{k}{m} - \frac{b^2}{2m^2} = 0 \quad \Rightarrow \quad \omega^2 = \frac{k}{m} - \frac{b^2}{4m^2}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}\sqrt{1 - \frac{b^2}{4mk}}}$$

Eigenkreisfrequenz des harmonischen Oszillators (=ω_0)

Anhang B Wie bestimmt man die Bewegung in Anwesenheit einer periodischen Kraft?

Situation: Eine Masse ist an einer Feder aufgehängt. Eine sinusoidale Kraft wirkt auf die Masse. **Frage:** Welche Amplitude ergibt sich für die Bewegung der Masse ?

$$\frac{d^2x}{dt^2} + \omega_0^2 x(t) + 1/\tau \frac{dx}{dt} = \frac{F_0}{m}\cos(\omega t) \qquad \tau = \frac{m}{b} \qquad \omega_0^2 = \frac{k}{m}$$

Mit dem Ansatz $x(t) = R_0 \cos(\omega t - \varphi) = R_0 \cos\varphi \cos\omega t + R_0 \sin\varphi \sin\omega t$
$= A\cos\omega t + B\sin\omega t$

Seiner 1. und 2. Ableitung : $\frac{dx}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$ $\quad \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$

1) In die Bewegungsgleichung einsetzen

2) Nach sin und cos gruppieren $C_1 \sin\omega t + C_2\cos\omega t = 0$

Gültig für alle Zeiten t:

$C_1 = C_2 = 0$

C$_1$

$\left[\omega_0^2 - \omega^2\right] B\sin(\omega t)$

$-\frac{\omega}{\tau} \quad A\sin(\omega t)$

C$_1$

$\left[\omega_0^2 - \omega^2\right] A\cos(\omega t)$

$\frac{\omega}{\tau} \quad B\cos(\omega t)$

$-\frac{F_0}{m} \quad \cos(\omega t)$

Zwei Unbekannte (A, B)

Zwei Gleichungen (C_1=0, C_2=0)

Eine Lösung für A und B !

*8–9 Potential Energy Diagrams; Stable and Unstable Equilibrium

If only conservative forces do work on an object, we can learn a great deal about its motion simply by examining a potential energy diagram—the graph of $U(x)$ versus x. An example of a potential energy diagram is shown in Fig. 8–23. The rather complex curve represents some complicated potential energy $U(x)$. The total energy $E = K + U$ is constant and can be represented as a horizontal line on this graph. Four different possible values for E are shown, labeled E_0, E_1, E_2, and E_3. What the actual value of E will be for a given system depends on the initial conditions. (For example, the total energy E of a mass oscillating on the end of a spring depends on the amount the spring is initially compressed or stretched.) Kinetic energy $K = \frac{1}{2}mv^2$ cannot be less than zero (v would be imaginary), and because $E = U + K =$ constant, then $U(x)$ must be less than or equal to E for all situations: $U(x) \leq E$. Thus the minimum value which the total energy can take for the potential energy shown in Fig. 8–23 is that labeled E_0. For this value of E, the mass can only be at rest at $x = x_0$. The system has potential energy but no kinetic energy at this position.

If the system's total energy E is greater than E_0, say it is E_1 on our plot, the system can have both kinetic and potential energy. Because energy is conserved,

$$K = E - U(x).$$

Since the curve represents $U(x)$ at each x, the kinetic energy at any value of x is represented by the distance between the E line and the curve $U(x)$ at that value of x. In the diagram, the kinetic energy for an object at x_1, when its total energy is E_1, is indicated by the notation K_1.

An object with energy E_1 can oscillate only between the points x_2 and x_3. This is because if $x > x_2$ or $x < x_3$, the potential energy would be greater than E, meaning $K = \frac{1}{2}mv^2 < 0$ and v would be imaginary, and so impossible. At x_2 and x_3 the velocity is zero, since $E = U$ at these points. Hence x_2 and x_3 are called the **turning points** of the motion. If the object is at x_0, say, moving to the right, its kinetic energy (and speed) decreases until it reaches zero at $x = x_2$. The object then reverses direction, proceeding to the left and increasing in speed until it passes x_0 again. It continues to move, decreasing in speed until it reaches $x = x_3$, where again $v = 0$, and the object again reverses direction.

If the object has energy $E = E_2$ in Fig. 8–23, there are four turning points. The object can move in only one of the two potential energy "valleys," depending on where it is initially. It cannot get from one valley to the other because of the barrier between them—for example at a point such as x_4, $U > E_2$, which means v would be imaginary.† For energy E_3, there is only one turning point since $U(x) < E_3$ for all $x > x_5$. Thus our object, if moving initially to the left, varies in speed as it passes the potential valleys but eventually stops and turns around at $x = x_5$. It then proceeds to the right indefinitely, never to return.

How do we know the object reverses direction at the turning points? Because of the force exerted on it. The force F is related to the potential energy U by Eq. 8–7, $F = -dU/dx$. The force F is equal to the negative of the slope of the U-versus-x curve at any point x. At $x = x_2$, for example, the slope is positive so the force is negative, which means it acts to the left (toward decreasing values of x).

At $x = x_0$ the slope is zero, so $F = 0$. At such a point the particle is said to be in **equilibrium**. This term means simply that the net force on the object is zero. Hence, its acceleration is zero, and so if it is initially at rest, it remains at rest. If the object at rest at $x = x_0$ were moved slightly to the left or right, a nonzero force would act on it in the direction to move it back toward x_0. An object that returns toward its equilibrium point when displaced slightly is said to be at a point of **stable equilibrium**. Any *minimum* in the potential energy curve represents a point of stable equilibrium.

An object at $x = x_4$ would also be in equilibrium, since $F = -dU/dx = 0$. If the object were displaced a bit to either side of x_4, a force would act to pull the object *away* from the equilibrium point. Points like x_4, where the potential energy curve has a maximum, are points of **unstable equilibrium**. The object will *not* return to equilibrium if displaced slightly, but instead will move farther away.

Many objects vibrate or oscillate—an object on the end of a spring, a tuning fork, the balance wheel of an old watch, a pendulum, a plastic ruler held firmly over the edge of a table and gently struck, the strings of a guitar or piano. Spiders detect prey by the vibrations of their webs; cars oscillate up and down when they hit a bump; buildings and bridges vibrate when heavy trucks pass or the wind is fierce. Indeed, because most solids are elastic (see Chapter 12), they vibrate (at least briefly) when given an impulse. Electrical oscillations are necessary in radio and television sets. At the atomic level, atoms vibrate within a molecule, and the atoms of a solid vibrate about their relatively fixed positions. Because it is so common in everyday life and occurs in so many areas of physics, oscillatory motion is of great importance. Mechanical oscillations are fully described on the basis of Newtonian mechanics.

14–1 Oscillations of a Spring

When an object **vibrates** or **oscillates** back and forth, over the same path, each oscillation taking the same amount of time, the motion is **periodic**. The simplest form of periodic motion is represented by an object oscillating on the end of a uniform coil spring. Because many other types of oscillatory motion closely resemble this system, we will look at it in detail. We assume that the mass of the spring can be ignored, and that the spring is mounted horizontally, as shown in Fig. 14–1a, so that the object of mass m slides without friction on the horizontal surface. Any spring has a natural length at which it exerts no force on the mass m. The position of the mass at this point is called the **equilibrium position**. If the mass is moved either to the left, which compresses the spring, or to the right, which stretches it, the spring exerts a force on the mass that acts in the direction of returning the mass to the equilibrium position; hence it is called a *restoring force*. We consider the common situation where we can assume the restoring force F is directly proportional to the displacement x the spring has been stretched (Fig. 14–1b) or compressed (Fig. 14–1c) from the equilibrium position:

$$F = -kx. \qquad \text{[force exerted by spring]} \qquad \textbf{(14–1)}$$

Note that the equilibrium position has been chosen at $x = 0$ and the minus sign in Eq. 14–1 indicates that the restoring force is always in the direction opposite to the displacement x. For example, if we choose the positive direction to the right in Fig. 14–1, x is positive when the spring is stretched (Fig. 14–1b), but the direction of the restoring force is to the left (negative direction). If the spring is compressed, x is negative (to the left) but the force F acts toward the right (Fig. 14–1c).

Note that the force F in Eq. 14–1 is *not* a constant, but varies with position. Therefore the acceleration of the mass m is not constant, so we *cannot* use the equations for constant acceleration developed in Chapter 2.

Let us examine what happens when our uniform spring is initially compressed a distance $x = -A$, as shown in Fig. 14–2a, and then released on the frictionless surface. The spring exerts a force on the mass that pushes it toward the equilibrium position. But because the mass has inertia, it passes the equilibrium position with considerable speed. Indeed, as the mass reaches the equilibrium position, the force on it decreases to zero, but its speed at this point is a maximum, v_{max} (Fig. 14–2b). As the mass moves farther to the right, the force on it acts to slow it down, and it stops for an instant at $x = A$ (Fig. 14–2c). It then begins moving back in the opposite direction, accelerating until it passes the equilibrium point (Fig. 14–2d), and then slows down until it reaches zero speed at the original starting point, $x = -A$ (Fig. 14–2e). It then repeats the motion, moving back and forth symmetrically between $x = A$ and $x = -A$.

14–2 Simple Harmonic Motion

Any oscillating system for which the net restoring force is directly proportional to the negative of the displacement (as in Eq. 14–1, $F = -kx$) is said to exhibit **simple harmonic motion** (SHM). Such a system is often called a **simple harmonic oscillator** (SHO). We saw in Chapter 12 (Section 12–4) that most solid materials stretch or compress according to Eq. 14–1 as long as the displacement is not too great. Because of this, many natural oscillations are simple harmonic or close to it.

14–7 Damped Harmonic Motion

The amplitude of any real oscillating spring or swinging pendulum slowly decreases in time until the oscillations stop altogether. Figure 14–19 shows a typical graph of the displacement as a function of time. This is called **damped harmonic motion**. The damping† is generally due to the resistance of air and to internal friction within the oscillating system. The energy that is dissipated to thermal energy is reflected in a decreased amplitude of oscillation.

Since natural oscillating systems are damped in general, why do we even talk about (undamped) simple harmonic motion? The answer is that SHM is much easier to deal with mathematically. And if the damping is not large, the oscillations can be thought of as simple harmonic motion on which the damping is superposed, as represented by the dashed curves in Fig. 14–19. Although damping does alter the frequency of vibration, the effect is usually small if the damping is small. Let us look at this in more detail.

14–8 Forced Oscillations; Resonance

When an oscillating system is set into motion, it oscillates at its natural frequency (Eqs. 14–7a and 14–12b). However, a system may have an external force applied to it that has its own particular frequency and then we have a **forced oscillation**.

For example, we might pull the mass on the spring of Fig. 14–1 back and forth at a frequency f. The mass then oscillates at the frequency f of the external force, even if this frequency is different from the **natural frequency** of the spring, which we will now denote by f_0 where (see Eqs. 14–5 and 14–7a)

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}.$$

In a forced oscillation the amplitude of oscillation, and hence the energy transferred to the oscillating system, is found to depend on the difference between f and f_0 as well as on the amount of damping, reaching a maximum when the frequency of the external force equals the natural frequency of the system—that is, when $f = f_0$. The amplitude is plotted in Fig. 14–23 as a function of the external frequency f. Curve A represents light damping and curve B heavy damping. The amplitude can become large when the driving frequency f is near the natural frequency, $f \approx f_0$, as long as the damping is not too large. When the damping is small, the increase in amplitude near $f = f_0$ is very large (and often dramatic). This effect is known as **resonance**. The natural frequency f_0 of a system is called its **resonant frequency**.

A simple illustration of resonance is pushing a child on a swing. A swing, like any pendulum, has a natural frequency of oscillation that depends on its length ℓ. If you push on the swing at a random frequency, the swing bounces around and reaches no great amplitude. But if you push with a frequency equal to the natural frequency of the swing, the amplitude increases greatly. At resonance, relatively little effort is required to obtain a large amplitude.

3. (I) The springs of a 1500-kg car compress 5.0 mm when its 68-kg driver gets into the driver's seat. If the car goes over a bump, what will be the frequency of oscillations? Ignore damping.

9. (II) A small fly of mass 0.25 g is caught in a spider's web. The web oscillates predominately with a frequency of 4.0 Hz. (*a*) What is the value of the effective spring stiffness constant k for the web? (*b*) At what frequency would you expect the web to oscillate if an insect of mass 0.50 g were trapped?

59. (II) A damped harmonic oscillator loses 6.0% of its mechanical energy per cycle. (*a*) By what percentage does its frequency differ from the natural frequency $f_0 = (1/2\pi)\sqrt{k/m}$? (*b*) After how many periods will the amplitude have decreased to $1/e$ of its original value?

65. (II) An 1150 kg automobile has springs with $k = 16{,}000$ N/m. One of the tires is not properly balanced; it has a little extra mass on one side compared to the other, causing the car to shake at certain speeds. If the tire radius is 42 cm, at what speed will the wheel shake most?

9: Der Impuls und Systeme mit veränderlicher Masse

I. Welche Grössen sind während einer Kollision zweier Objekte (Stoss) erhalten ?

Impulserhaltungssatz

II. Was geschieht während eines Stosses ?

Inelastischer Stoss

III. Wie bestimmt man den Schub einer Rakete ?

IV. Welche Grössen sind während einer Kreisbewegung erhalten ?

Rotationsenergie

Erhaltung des Drehimpulses

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(2 Seiten)*:

9-1 Momentum and its relation to force
10-10 Systems of variable mass; rocket propulsion
11-3 angular momentum of a particle
11-6 conservation of angular momentum

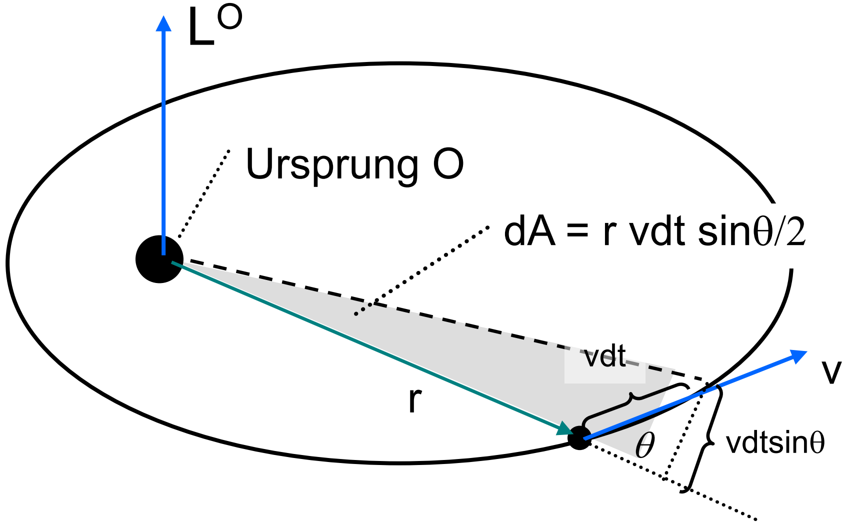
Vorbereitende Übungen *(4)* **vor der Übungssession** zu erledigen **:**

Giancoli 9-3, 9, 10, 24, 10-63; 11-1, 5, 7

Giancoli Kapitel 9-1; 9-2; 9-4 bis 9-7; 10-10; 11-1, 11-3, 11-4 (6)

Zugabe: Beweis des 2. Keplerschen Gesetzes

«Der Ortsvektor des Planeten überstreicht in gleichen Zeiten Δt gleich große Flächen»
(Lektion 6)



$$\vec{L} = \vec{r} \times \vec{p}$$

$$\underbrace{\frac{dA}{dt} = \frac{rv\sin\theta}{2} \qquad L = mrv\sin\theta}$$

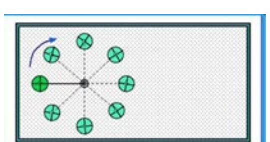
$$dA = \frac{L}{2m}dt$$

$$\int_{t_1}^{t_2} dA = \quad \Delta A = \frac{L}{2m}\Delta t \qquad \Delta t = \int_{t_1}^{t_2} dt$$

ΔA = konstant

(Manifestation der Drehimpulserhaltung)

Zentralkraft (Gravitation) :
$\tau^O = 0$



9–1 Momentum and Its Relation to Force

The **linear momentum** (or "momentum" for short) of an object is defined as the product of its mass and its velocity. Momentum (plural is *momenta*) is represented by the symbol $\vec{\mathbf{p}}$. If we let m represent the mass of an object and $\vec{\mathbf{v}}$ represent its velocity, then its momentum $\vec{\mathbf{p}}$ is defined as

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{9–1}$$

Velocity is a vector, so momentum too is a vector. The direction of the momentum is the direction of the velocity, and the magnitude of the momentum is $p = mv$. Because velocity depends on the reference frame, so does momentum; thus the reference frame must be specified. The unit of momentum is that of mass $\times$ velocity, which in SI units is $\mathrm{kg \cdot m/s}$. There is no special name for this unit.

Everyday usage of the term *momentum* is in accord with the definition above. According to Eq. 9–1, a fast-moving car has more momentum than a slow-moving car of the same mass; a heavy truck has more momentum than a small car moving with the same speed. The more momentum an object has, the harder it is to stop it, and the greater effect it will have on another object if it is brought to rest by striking that object. A football player is more likely to be stunned if tackled by a heavy opponent running at top speed than by a lighter or slower-moving tackler. A heavy, fast-moving truck can do more damage than a slow-moving motorcycle.

A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction. Newton originally stated his second law in terms of momentum (although he called the product mv the "quantity of motion"). Newton's statement of the **second law of motion**, translated into modern language, is as follows:

The rate of change of momentum of an object is equal to the net force applied to it.

We can write this as an equation,

$$\Sigma\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}, \tag{9–2}$$

where $\Sigma\vec{\mathbf{F}}$ is the net force applied to the object (the vector sum of all forces acting on it). We can readily derive the familiar form of the second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, from Eq. 9–2 for the case of constant mass. If $\vec{\mathbf{v}}$ is the velocity of an object at any moment, then Eq. 9–2 gives

$$\Sigma\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}} \qquad \text{[constant mass]}$$

because, by definition, $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$ and we assume m = constant. Newton's statement, Eq. 9–2, is actually more general than the more familiar one because it includes the situation in which the mass may change. This is important in certain circumstances, such as for rockets which lose mass as they burn fuel (Section 9–10) and in relativity theory (Chapter 36).

3. (II) The momentum of a particle, in SI units, is given by $\vec{\mathbf{p}} = 4.8\,t^2\hat{\mathbf{i}} - 8.0\hat{\mathbf{j}} - 8.9\,t\hat{\mathbf{k}}$. What is the force as a function of time?

9. (I) A 7700-kg boxcar traveling 18 m/s strikes a second car. The two stick together and move off with a speed of 5.0 m/s. What is the mass of the second car?

10. (I) A 9150-kg railroad car travels alone on a level frictionless track with a constant speed of 15.0 m/s. A 4350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?

24. (II) A 12-kg hammer strikes a nail at a velocity of 8.5 m/s and comes to rest in a time interval of 8.0 ms. (*a*) What is the impulse given to the nail? (*b*) What is the average force acting on the nail?

*9–10 Systems of Variable Mass; Rocket Propulsion

We now treat objects or systems whose mass varies. Such systems could be treated as a type of inelastic collision, but it is simpler to use Eq. 9–5, $d\vec{\mathbf{P}}/dt = \Sigma\vec{\mathbf{F}}_{\text{ext}}$, where $\vec{\mathbf{P}}$ is the total momentum of the system and $\Sigma\vec{\mathbf{F}}_{\text{ext}}$ is the net external force exerted on it. Great care must be taken to define the system, and to include all changes in momentum. An important application is to rockets, which propel themselves forward by the ejection of burned gases: the force exerted by the gases on the rocket accelerates the rocket. The mass M of the rocket decreases as it ejects gas, so for the rocket $dM/dt < 0$. Another application is the dropping of material (gravel, packaged goods) onto a conveyor belt. In this situation, the mass M of the loaded conveyor belt increases and $dM/dt > 0$.

To treat the general case of variable mass, let us consider the system shown in Fig. 9–33. At some time t, we have a system of mass M and momentum $M\vec{\mathbf{v}}$. We also have a tiny (infinitesimal) mass dM traveling with velocity $\vec{\mathbf{u}}$ which is about to enter our system. An infinitesimal time dt later, the mass dM combines with the system. For simplicity we will refer to this as a "collision." So our system has changed in mass from M to $M + dM$ in the time dt. Note that dM can be less than zero, as for a rocket propelled by ejected gases whose mass M thus decreases.

In order to apply Eq. 9–5, $d\vec{\mathbf{P}}/dt = \Sigma\vec{\mathbf{F}}_{\text{ext}}$, we must consider a definite fixed system of particles. That is, in considering the change in momentum, $d\vec{\mathbf{P}}$, we must consider the momentum of the same particles initially and finally. We will define our *total system* as including M plus dM. Then initially, at time t, the total momentum is $M\vec{\mathbf{v}} + \vec{\mathbf{u}}\,dM$ (Fig. 9–33). At time $t + dt$, after dM has combined with M, the velocity of the whole is now $\vec{\mathbf{v}} + d\vec{\mathbf{v}}$ and the total momentum is $(M + dM)(\vec{\mathbf{v}} + d\vec{\mathbf{v}})$. So the change in momentum $d\vec{\mathbf{P}}$ is

$$\begin{aligned} d\vec{\mathbf{P}} &= (M + dM)(\vec{\mathbf{v}} + d\vec{\mathbf{v}}) - (M\vec{\mathbf{v}} + \vec{\mathbf{u}}\,dM) \\ &= M\,d\vec{\mathbf{v}} + \vec{\mathbf{v}}\,dM + dM\,d\vec{\mathbf{v}} - \vec{\mathbf{u}}\,dM. \end{aligned}$$

The term $dM\,d\vec{\mathbf{v}}$ is the product of two differentials and is zero even after we "divide by dt," which we do, and apply Eq. 9–5 to obtain

$$\Sigma\vec{\mathbf{F}}_{\text{ext}} = \frac{d\vec{\mathbf{P}}}{dt} = \frac{M\,d\vec{\mathbf{v}} + \vec{\mathbf{v}}\,dM - \vec{\mathbf{u}}\,dM}{dt}.$$

Thus we get

$$\Sigma\vec{\mathbf{F}}_{\text{ext}} = M\frac{d\vec{\mathbf{v}}}{dt} - (\vec{\mathbf{u}} - \vec{\mathbf{v}})\frac{dM}{dt}. \tag{9–19a}$$

Note that the quantity $(\vec{\mathbf{u}} - \vec{\mathbf{v}})$ is the relative velocity, $\vec{\mathbf{v}}_{\text{rel}}$, of dM with respect to M. That is,

$$\vec{\mathbf{v}}_{\text{rel}} = \vec{\mathbf{u}} - \vec{\mathbf{v}}$$

is the velocity of the entering mass dM as seen by an observer on M. We can rearrange Eq. 9–19a:

$$M\frac{d\vec{\mathbf{v}}}{dt} = \Sigma\vec{\mathbf{F}}_{\text{ext}} + \vec{\mathbf{v}}_{\text{rel}}\frac{dM}{dt}. \tag{9–19b}$$

We can interpret this equation as follows. $M\,d\vec{\mathbf{v}}/dt$ is the mass times the acceleration of M. The first term on the right, $\Sigma\vec{\mathbf{F}}_{\text{ext}}$, refers to the external force on the mass M (for a rocket, it would include the force of gravity and air resistance). It does *not* include the force that dM exerts on M as a result of their collision. This is taken care of by the second term on the right, $\vec{\mathbf{v}}_{\text{rel}}(dM/dt)$, which represents the rate at which momentum is being transferred into (or out of) the mass M because of the mass that is added to (or leaves) it. It can thus be interpreted as the force exerted on the mass M due to the addition (or ejection) of mass. For a rocket this term is called the *thrust*, since it represents the force exerted on the rocket by the expelled gases. For a rocket ejecting burned fuel, $dM/dt < 0$, but so is $\vec{\mathbf{v}}_{\text{rel}}$ (gases are forced out the back), so the second term in Eq. 9–19b acts to increase $\vec{\mathbf{v}}$.

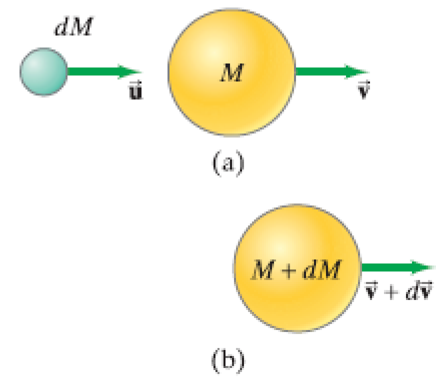


FIGURE 9–33 (a) At time t, a mass dM is about to be added to our system M. (b) At time $t + dt$, the mass dM has been added to our system.

11–3 Angular Momentum of a Particle

The most general way of writing Newton's second law for the translational motion of a particle (or system of particles) is in terms of the linear momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ as given by Eq. 9–2 (or 9–5):

$$\Sigma\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$$

The rotational analog of linear momentum is *angular momentum*. Just as the rate of change of $\vec{\mathbf{p}}$ is related to the net force $\Sigma\vec{\mathbf{F}}$, so we might expect the rate of change of angular momentum to be related to the net torque. Indeed, we saw this was true in Section 11–1 for the special case of a rigid object rotating about a fixed axis. Now we will see it is true in general. We first treat a single particle.

Suppose a particle of mass m has momentum $\vec{\mathbf{p}}$ and position vector $\vec{\mathbf{r}}$ with respect to the origin O in some chosen inertial reference frame. Then the general definition of the **angular momentum**, $\vec{\mathbf{L}}$, of the particle about point O is the vector cross product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \qquad \text{[particle]} \qquad \mathbf{(11–6)}$$

Angular momentum is a vector.† Its direction is perpendicular to both $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ as given by the right-hand rule (Fig. 11–12). Its magnitude is given by

$$L = rp\sin\theta$$

or

$$L = rp_\perp = r_\perp p$$

where θ is the angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ and $p_\perp (= p\sin\theta)$ and $r_\perp (= r\sin\theta)$ are the components of $\vec{\mathbf{p}}$ and $\vec{\mathbf{r}}$ perpendicular to $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$, respectively.

Now let us find the relation between angular momentum and torque for a particle. If we take the derivative of $\vec{\mathbf{L}}$ with respect to time we have

$$\frac{d\vec{\mathbf{L}}}{dt} = \frac{d}{dt}(\vec{\mathbf{r}} \times \vec{\mathbf{p}}) = \frac{d\vec{\mathbf{r}}}{dt} \times \vec{\mathbf{p}} + \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt}.$$

But

$$\frac{d\vec{\mathbf{r}}}{dt} \times \vec{\mathbf{p}} = \vec{\mathbf{v}} \times m\vec{\mathbf{v}} = m(\vec{\mathbf{v}} \times \vec{\mathbf{v}}) = 0,$$

since $\sin\theta = 0$ for this case. Thus

$$\frac{d\vec{\mathbf{L}}}{dt} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt}.$$

If we let $\Sigma\vec{\mathbf{F}}$ represent the resultant force on the particle, then in an inertial reference frame, $\Sigma\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$ and

$$\vec{\mathbf{r}} \times \Sigma\vec{\mathbf{F}} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt} = \frac{d\vec{\mathbf{L}}}{dt}.$$

But $\vec{\mathbf{r}} \times \Sigma\vec{\mathbf{F}} = \Sigma\vec{\boldsymbol{\tau}}$ is the net torque on our particle. Hence

$$\Sigma\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}. \qquad \text{[particle, inertial frame]} \qquad \mathbf{(11–7)}$$

The time rate of change of angular momentum of a particle is equal to the net torque applied to it. Equation 11–7 is the rotational equivalent of Newton's second law for a particle, written in its most general form. Equation 11–7 is valid only in an inertial frame since only then is it true that $\Sigma\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$, which was used in the proof.

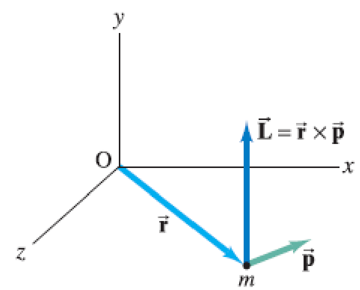


FIGURE 11–12 The angular momentum of a particle of mass m is given by $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$.

11–6 Conservation of Angular Momentum

In Chapter 9 we saw that the most general form of Newton's second law for the translational motion of a particle or system of particles is

$$\sum \vec{\mathbf{F}}_{\text{ext}} = \frac{d\vec{\mathbf{P}}}{dt},$$

where $\vec{\mathbf{P}}$ is the (linear) momentum, defined as $m\vec{\mathbf{v}}$ for a particle, or $M\vec{\mathbf{v}}_{\text{CM}}$ for a system of particles of total mass M whose CM moves with velocity $\vec{\mathbf{v}}_{\text{CM}}$, and $\Sigma\vec{\mathbf{F}}_{\text{ext}}$ is the net external force acting on the particle or system. This relation is valid only in an inertial reference frame.

In this Chapter, we have found a similar relation to describe the general rotation of a system of particles (including rigid objects):

$$\sum \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt},$$

where $\Sigma\vec{\boldsymbol{\tau}}$ is the net external torque acting on the system, and $\vec{\mathbf{L}}$ is the total angular momentum. This relation is valid when $\Sigma\vec{\boldsymbol{\tau}}$ and $\vec{\mathbf{L}}$ are calculated about a point fixed in an inertial reference frame, or about the CM of the system.

For translational motion, if the net force on the system is zero, $d\vec{\mathbf{P}}/dt = 0$, so the total linear momentum of the system remains constant. This is the law of conservation of linear momentum. For rotational motion, if the net torque on the system is zero, then

$$\frac{d\vec{\mathbf{L}}}{dt} = 0 \quad \text{and} \quad \vec{\mathbf{L}} = \text{constant}. \qquad [\Sigma\vec{\boldsymbol{\tau}} = 0] \quad \textbf{(11–12)}$$

In words:

The total angular momentum of a system remains constant if the net external torque acting on the system is zero.

This is the **law of conservation of angular momentum** in full vector form. It ranks with the laws of conservation of energy and linear momentum (and others to be discussed later) as one of the great laws of physics. In Section 11–1 we saw some Examples of this important law applied to the special case of a rigid object rotating about a fixed axis. Here we have it in general form. We use it now in interesting Examples.

63. (I) A centrifuge rotor has a moment of inertia of $4.25 \times 10^{-2}\,\text{kg}\cdot\text{m}^2$. How much energy is required to bring it from rest to 9750 rpm?

1. (I) What is the angular momentum of a 0.210-kg ball rotating on the end of a thin string in a circle of radius 1.35 m at an angular speed of 10.4 rad/s?

5. (II) A diver (such as the one shown in Fig. 11–2) can reduce her moment of inertia by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes 2.0 rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?

7. (II) Determine the angular momentum of the Earth (*a*) about its rotation axis (assume the Earth is a uniform sphere), and (*b*) in its orbit around the Sun (treat the Earth as a particle orbiting the Sun). The Earth has mass $= 6.0 \times 10^{24}\,\text{kg}$ and radius $= 6.4 \times 10^{6}\,\text{m}$, and is $1.5 \times 10^{8}\,\text{km}$ from the Sun.

10: Mehrkörpersysteme

I. Zusammenfassung der Erhaltungssätze

II. Wie beschreibt man die lineare Bewegung eines Mehrkörpersystems ?

- Massenschwerpunkt/Massenmittelpunkt (centre de masse, CM)
- Dynamik und mechanische Energie eines Systems mit N Körpern

III. Wann befindet sich ein starrer Körper im Gleichgewicht ?

- Drehmoment

IV. Was versetzt einen starren Körper in eine Drehbewegung ?

- z. Erinnerung: Dynamik der Drehbewegung des Massenpunktes

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(3.5 Seiten)*:

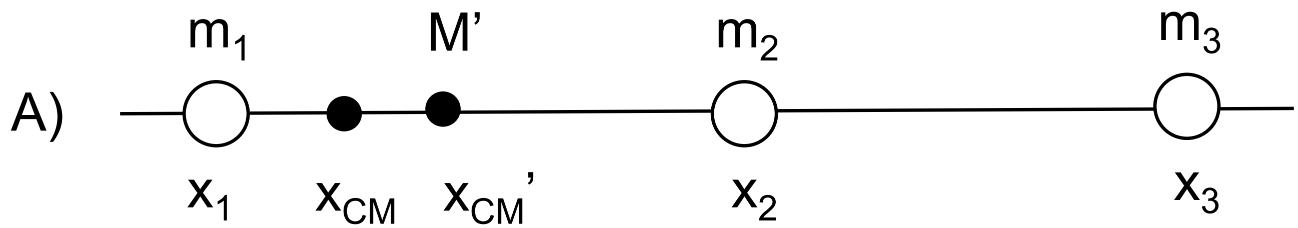
- 9-8 Center of Mass
- 11-4 Torque
- 12-1 The condition for equilibrium

Vorbereitende Übungen *(5)* **vor der Übungssession** zu erledigen **:**

- Giancoli 9-62, 63
- 10-25, 29, 30

Giancoli Kapitel 9-8, 9-9; 11-3, 11-4, 11-2, 12-1 bis 3

Annex: Massenschwerpunkt von 3 Objekten



Für 3 Objekte befindet sich der Massenschwerpunkt x_{CM}' bei

$$x_{CM}' \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M'} = \frac{x_{CM} M + m_3 x_3}{M + m_3}$$

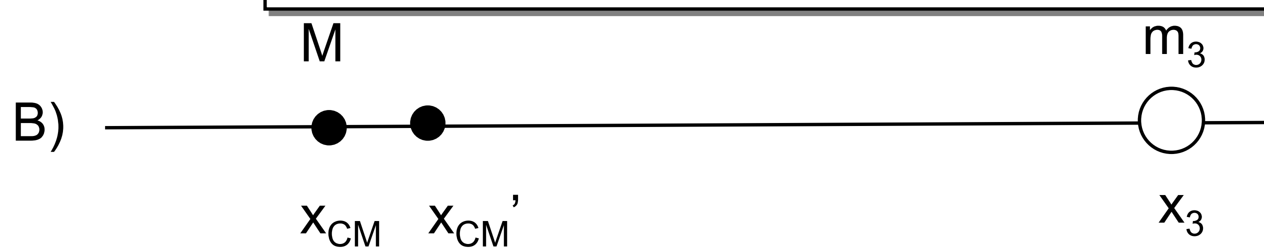
Gesamte Masse : $M' = m_1 + m_2 + m_3$

Mit der Definition der totalen Masse des CM von m_1 und m_2 (s. vorher)

$$\boxed{x_{CM}' \equiv \frac{M x_{CM} + m_3 x_3}{M + m_3}} \Rightarrow$$

Regel 2: Um den Massenschwerpunkt eines komplexen Objektes zu bestimmen, kann man den CM von den individuellen Objekten berechnen, und dann letzere wie ein Massenpunkt am respektiven Massenschwerpunkt behandeln.

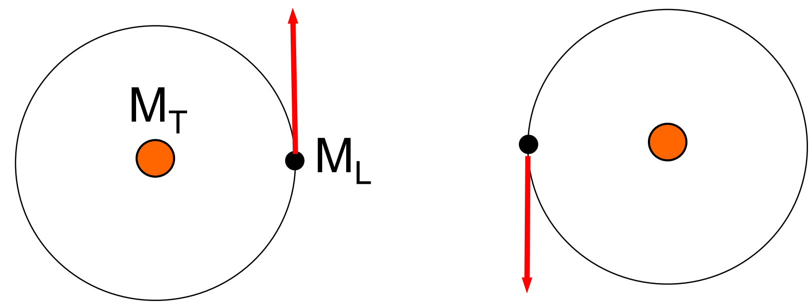
$\Rightarrow$ A) und B) sind äquivalent :



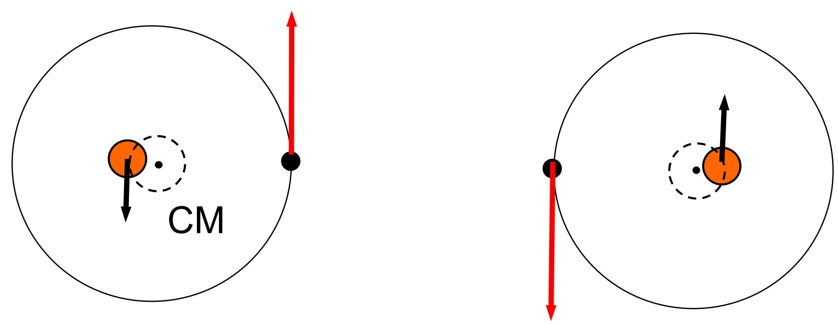
Beispiel eines isolierten Systems: Erde-Mond

Vorher | Nach einer halben Periode

Eher RICHTIG oder FALSCH ?

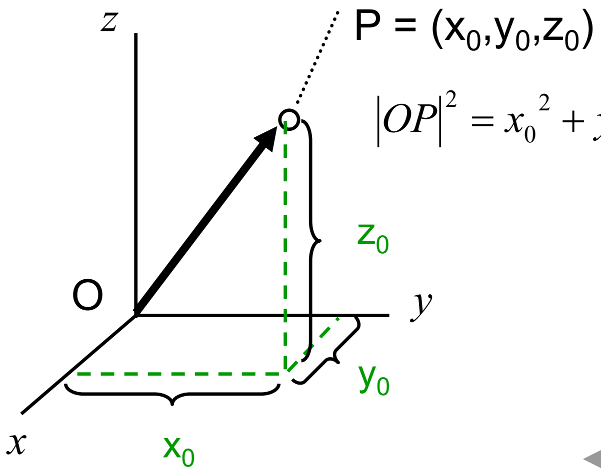


Eher RICHTIG oder FALSCH ?



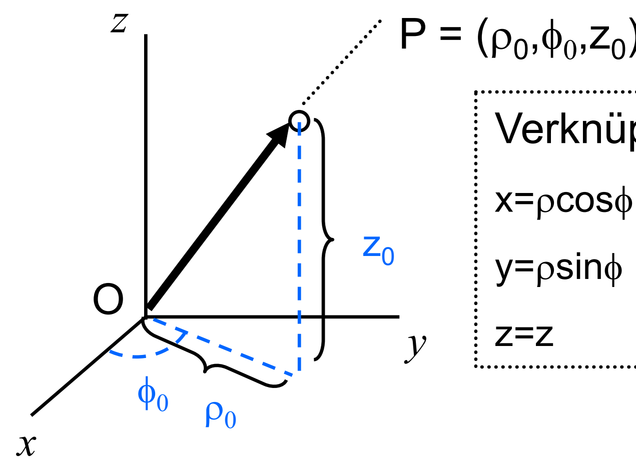
z. Erinnerung: Die zylindrischen Koordinaten

Kartesisch



$$|OP|^2 = x_0^{\ 2} + y_0^{\ 2} + z_0^{\ 2}$$

Zylindrisch



Verknüpfung:

$x=\rho\cos\phi$

$y=\rho\sin\phi$

$\left.\begin{matrix} x=\rho\cos\phi \\ y=\rho\sin\phi \end{matrix}\right\} \rho^2=x^2+y^2$

$z=z$ $\quad |OP|^2 = \rho_0^{\ 2} + z_0^{\ 2}$

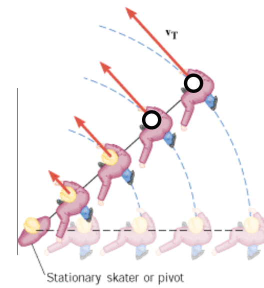
z.Erinnerung: Kinematik der Kreisbewegung

Definitionen Winkelgeschwindigkeit und -position (s. auch Lektion 3 und 5,6)

Winkelposition: θ

Winkelgeschwindigkeit: $$\boxed{\vec{\omega}(t) \equiv \frac{d\vec{\theta}(t)}{dt}}$$

NB. Die Winkelgeschwindigkeit (ω=v/r) ist dieseble für alle Punkte eines starren Körpers



Momentane **Winkelbeschleunigung** (ganz analog zur Kinematik definiert)

$$\boxed{\vec{\alpha}(t) \equiv \frac{d\vec{\omega}(t)}{dt} = \frac{d^2\vec{\theta}(t)}{dt^2}}$$

Welche Richtung für ω ?

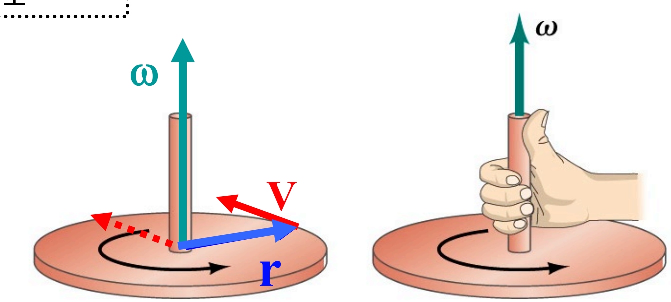
Wenn die Finger der rechten Hand der Kreisbewegung folgen, gilt ω_z als positiv

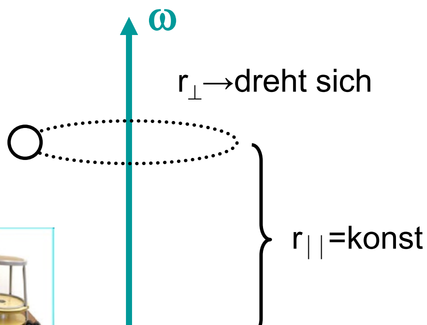
(« **Regel der rechten Hand** »)

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

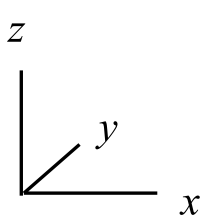
v=ωr = $|\Delta \mathbf{r}_\perp / \Delta t|$

?





$$\boxed{\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}}$$



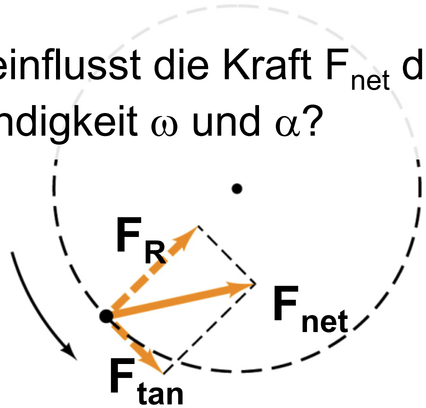
Beschreibt eine Drehung von **r** um den Vektor $\boldsymbol{\omega}$ mit Frequenz f=$\omega/2\pi$. Der Betrag von **r** bleibt dabei erhalten

$\Rightarrow$ Gilt für jede vektorielle Grösse **f** anstelle von **r**

z. Erinnerung: Kinematik und Dynamik der Kreibewegung
Drehmoment, Winkelbeschleunigung (Lektion 3 und 5)

Situation: Objekt befindet sich auf einem Kreis, und ist einer resultierenden Kraft F_{net} ausgesetzt.

Frage: Wie beeinflusst die Kraft F_{net} die Winkelgeschwindigkeit ω und α?



Kinematik der Kreisbewegung (Lektion 3, 5):

Tangentielle Beschleunigung a_{tan}

$$\vec{r} \times \vec{a}_{\tan} = r^2 \vec{\alpha}$$

} Ändert den Betrag und Richtung der Winkelgeschwindigkeit → Winkelbeschleunigung

2. Axiom:

Zerlegung in radiale und tangentielle Komponenten

$$\vec{F}_{net} = \sum \vec{F}_k = \vec{F}_R + \vec{F}_{\tan}$$

$$\vec{F}_{net} = m\vec{a} = m(\vec{a}_R + \vec{a}_{\tan})$$

NB. Zentripetalbedingung :

$$\vec{F}_R = m\vec{\omega}(t) \times (\vec{\omega}(t) \times \vec{r}) = -mr\omega(t)^2 \cdot \frac{\vec{r}}{r}$$

2. Gesetz der Rotation :

$$\boxed{\underbrace{\sum \vec{r} \times \vec{F}_k}_{\tau} = \underbrace{mr_{\perp}^{\ 2}}_{I} \vec{\alpha}}$$

τ: Drehmoment der Kraft F
I: Trägheitsmoment des Objektes
$r_{\perp}$: Distanz rel. zur Drehachse

NB. Das Drehmoment hängt von der Wahl des Koordinatenursprungs ab !
(Die Drehachse als Ursprung benutzen: Dies ist anders als für m und F)

z. B. Kann man damit die Türe öffnen ?

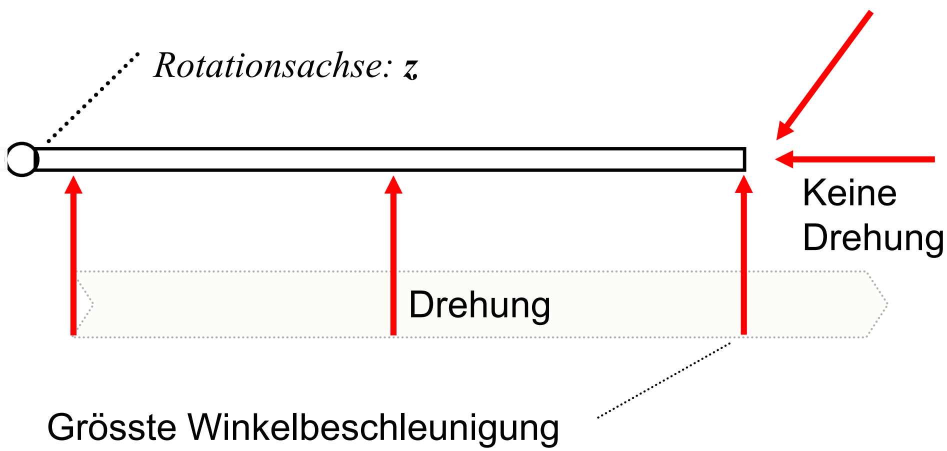
Die Proportionalität zwischen α und τ

Situation: Sie stossen eine Türe mit einem Finger an verschiedenen Stellen, immer mit derselben Kraft.

Frage: Welche Kraft ($F_1 = F_2$ etc) kann die grösste Winkelbeschleunigung α_z bewirken, i.e. erlaubt es, die Türe am schnellsten zu öffnen ?

$$\vec{\tau}^{\,O}_{net} = \sum \left(\vec{r} \times \vec{F} \right)$$

$$\tau_Z = \underset{\text{(Hebelarm)}}{R_\perp} F = RF_\perp = RF \sin\theta$$



9–8 Center of Mass (CM)

Momentum is a powerful concept not only for analyzing collisions but also for analyzing the translational motion of real extended objects. Until now, whenever we have dealt with the motion of an extended object (that is, an object that has size), we have assumed that it could be approximated as a point particle or that it undergoes only translational motion. Real extended objects, however, can undergo rotational and other types of motion as well. For example, the diver in Fig. 9–21a undergoes only translational motion (all parts of the object follow the same path), whereas the diver in Fig. 9–21b undergoes both translational and rotational motion. We will refer to motion that is not pure translation as *general motion*.

Observations indicate that even if an object rotates, or several parts of a system of objects move relative to one another, there is one point that moves in the same path that a particle would move if subjected to the same net force. This point is called the **center of mass** (abbreviated CM). The general motion of an extended object (or system of objects) can be considered as *the sum of the translational motion of the CM, plus rotational, vibrational, or other types of motion about the CM.*

As an example, consider the motion of the center of mass of the diver in Fig. 9–21; the CM follows a parabolic path even when the diver rotates, as shown in Fig. 9–21b. This is the same parabolic path that a projected particle follows when acted on only by the force of gravity (projectile motion, Section 3–7). Other points in the rotating diver's body, such as her feet or head, follow more complicated paths.

Figure 9–22 shows a wrench acted on by zero net force, translating and rotating along a horizontal surface. Note that its CM, marked by a red cross, moves in a straight line, as shown by the dashed white line.

We will show in Section 9–9 that the important properties of the CM follow from Newton's laws if the CM is defined in the following way. We can consider any extended object as being made up of many tiny particles. But first we consider a system made up of only two particles (or small objects), of masses m_A and m_B. We choose a coordinate system so that both particles lie on the x axis at positions x_A and x_B, Fig. 9–23. The center of mass of this system is defined to be at the position x_{CM}, given by

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

where $M = m_A + m_B$ is the total mass of the system. The center of mass lies on the line joining m_A and m_B. If the two masses are equal $(m_A = m_B = m)$, then x_{CM} is midway between them, since in this case

$$x_{CM} = \frac{m(x_A + x_B)}{2m} = \frac{(x_A + x_B)}{2}.$$

If one mass is greater than the other, say, $m_A > m_B$, then the CM is closer to the larger mass. If all the mass is concentrated at x_B, so $m_A = 0$, then $x_{CM} = (0x_A + m_B x_B)/(0 + m_B) = x_B$, as we would expect.

Now let us consider a system consisting of n particles, where n could be very large. This system could be an extended object which we consider as being made up of n tiny particles. If these n particles are all along a straight line (call it the x axis), we define the CM of the system to be located at

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_{i=1}^{n} m_i x_i}{M}, \qquad (9\text{–}10)$$

where $m_1, m_2, \ldots m_n$ are the masses of each particle and $x_1, x_2, \ldots x_n$ are their positions. The symbol $\sum_{i=1}^{n}$ is the summation sign meaning to sum over all the particles, where i takes on integer values from 1 to n. (Often we simply write $\Sigma m_i x_i$, leaving out the $i = 1$ to n.) The total mass of the system is $M = \Sigma m_i$.

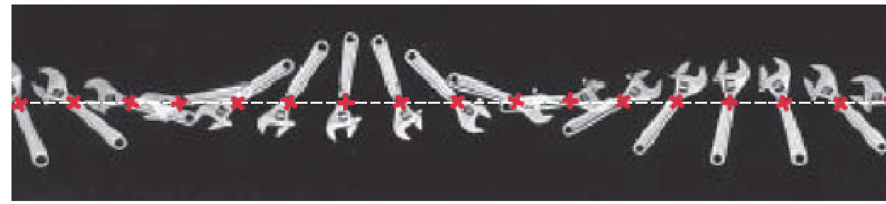


FIGURE 9–22 Translation plus rotation: a wrench moving over a horizontal surface. The CM, marked with a red cross, moves in a straight line.

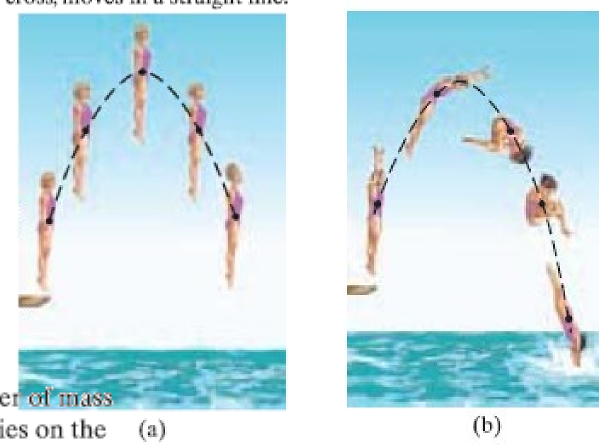
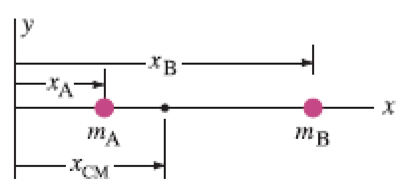


FIGURE 9–21 The motion of the diver is pure translation in (a), but is translation plus rotation in (b). The black dot represents the diver's CM at each moment.

FIGURE 9–23 The center of mass of a two-particle system lies on the line joining the two masses. Here $m_A > m_B$, so the CM is closer to m_A than to m_B.



It is often convenient to think of an extended object as made up of a continuous distribution of matter. In other words, we consider the object to be made up of n particles, each of mass Δm_i in a tiny volume around a point x_i, y_i, z_i, and we take the limit of n approaching infinity (Fig. 9–26). Then Δm_i becomes the infinitesimal mass dm at points x, y, z. The summations in Eqs. 9–11 and 9–12 become integrals:

$$x_{\mathrm{CM}} = \frac{1}{M}\int x\,dm, \quad y_{\mathrm{CM}} = \frac{1}{M}\int y\,dm, \quad z_{\mathrm{CM}} = \frac{1}{M}\int z\,dm, \qquad \textbf{(9–13)}$$

where the sum over all the mass elements is $\int dm = M$, the total mass of the object. In vector notation, this becomes

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\,dm. \qquad \textbf{(9–14)}$$

A concept similar to *center of mass* is **center of gravity** (CG). The CG of an object is that point at which the force of gravity can be considered to act. The force of gravity actually acts on *all* the different parts or particles of an object, but for purposes of determining the translational motion of an object as a whole, we can assume that the entire weight of the object (which is the sum of the weights of all its parts) acts at the CG. There is a conceptual difference between the center of gravity and the center of mass, but for nearly all practical purposes, they are at the same point.†

10–4 Torque

We have so far discussed rotational kinematics—the description of rotational motion in terms of angular position, angular velocity, and angular acceleration. Now we discuss the dynamics, or causes, of rotational motion. Just as we found analogies between linear and rotational motion for the description of motion, so rotational equivalents for dynamics exist as well.

To make an object start rotating about an axis clearly requires a force. But the direction of this force, and where it is applied, are also important. Take, for example, an ordinary situation such as the overhead view of the door in Fig. 10–11. If you apply a force $\vec{\mathbf{F}}_A$ to the door as shown, you will find that the greater the magnitude, F_A, the more quickly the door opens. But now if you apply the same magnitude force at a point closer to the hinge—say, $\vec{\mathbf{F}}_B$ in Fig. 10–11—the door will not open so quickly. The effect of the force is less: *where* the force acts, as well as its magnitude and direction, affects how quickly the door opens. Indeed, if only this one force acts, the angular acceleration of the door is proportional not only to the magnitude of the force, but is also directly proportional to *the perpendicular distance from the axis of rotation to the line along which the force acts*. This distance is called the **lever arm**, or **moment arm**, of the force, and is labeled R_A and R_B for the two forces in Fig. 10–11. Thus, if R_A in Fig. 10–11 is three times larger than R_B, then the angular acceleration of the door will be three times as great, assuming that the magnitudes of the forces are the same. To say it another way, if $R_A = 3R_B$, then F_B must be three times as large as F_A to give the same angular acceleration. (Figure 10–12 shows two examples of tools whose long lever arms are very effective.)

The angular acceleration, then, is proportional to the product of the *force times the lever arm*. This product is called the *moment of the force* about the axis, or, more commonly, it is called the **torque**, and is represented by τ (Greek lowercase letter tau). Thus, the angular acceleration α of an object is directly proportional to the net applied torque τ:

$$\alpha \propto \tau,$$

and we see that it is torque that gives rise to angular acceleration. This is the rotational analog of Newton's second law for linear motion, $a \propto F$.

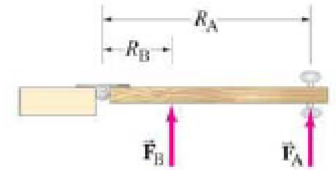


FIGURE 10–11 Top view of a door. Applying the same force with different lever arms, R_A and R_B. If $R_A = 3R_B$, then to create the same effect (angular acceleration), F_B needs to be three times F_A, or $F_A = \frac{1}{3}F_B$.

FIGURE 10–12 (a) A tire iron too can have a long lever arm. (b) A plumber can exert greater torque using a wrench with a long lever arm.

We defined the lever arm as the *perpendicular* distance from the axis of rotation to the line of action of the force—that is, the distance which is perpendicular both to the axis of rotation and to an imaginary line drawn along the direction of the force. We do this to take into account the effect of forces acting at an angle. It is clear that a force applied at an angle, such as $\vec{\mathbf{F}}_C$ in Fig. 10–13, will be less effective than the same magnitude force applied perpendicular to the door, such as $\vec{\mathbf{F}}_A$ (Fig. 10–13a). And if you push on the end of the door so that the force is directed at the hinge (the axis of rotation), as indicated by $\vec{\mathbf{F}}_D$, the door will not rotate at all.

The lever arm for a force such as $\vec{\mathbf{F}}_C$ is found by drawing a line along the direction of $\vec{\mathbf{F}}_C$ (this is the "line of action" of $\vec{\mathbf{F}}_C$). Then we draw another line, perpendicular to this line of action, that goes to the axis of rotation and is perpendicular also to it. The length of this second line is the lever arm for $\vec{\mathbf{F}}_C$ and is labeled R_C in Fig. 10–13b. The lever arm for $\vec{\mathbf{F}}_A$ is the full distance from the hinge to the door knob, R_A; thus R_C is much smaller than R_A.

The magnitude of the torque associated with $\vec{\mathbf{F}}_C$ is then $R_C F_C$. This short lever arm R_C and the corresponding smaller torque associated with $\vec{\mathbf{F}}_C$ is consistent with the observation that $\vec{\mathbf{F}}_C$ is less effective in accelerating the door than is $\vec{\mathbf{F}}_A$. When the lever arm is defined in this way, experiment shows that the relation $\alpha \propto \tau$ is valid in general. Notice in Fig. 10–13 that the line of action of the force $\vec{\mathbf{F}}_D$ passes through the hinge, and hence its lever arm is zero. Consequently, zero torque is associated with $\vec{\mathbf{F}}_D$ and it gives rise to no angular acceleration, in accord with everyday experience.

In general, then, we can write the magnitude of the torque about a given axis as

$$\tau = R_{\perp} F, \tag{10–10a}$$

where $R_{\perp}$ is the lever arm, and the perpendicular symbol ($\perp$) reminds us that we must use the distance from the axis of rotation that is perpendicular to the line of action of the force (Fig. 10–14a).

An equivalent way of determining the torque associated with a force is to resolve the force into components parallel and perpendicular to the line that connects the axis to the point of application of the force, as shown in Fig. 10–14b. The component $F_{\parallel}$ exerts no torque since it is directed at the rotation axis (its moment arm is zero). Hence the torque will be equal to $F_{\perp}$ times the distance R from the axis to the point of application of the force:

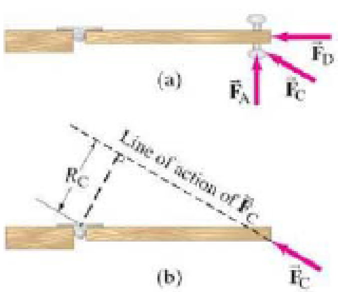
$$\tau = R F_{\perp}. \tag{10–10b}$$

This gives the same result as Eq. 10–10a because $F_{\perp} = F \sin\theta$ and $R_{\perp} = R \sin\theta$. So

$$\tau = RF \sin\theta \tag{10–10c}$$

in either case. [Note that θ is the angle between the directions of $\vec{\mathbf{F}}$ and R (radial line from the axis to the point where $\vec{\mathbf{F}}$ acts)]. We can use any of Eqs. 10–10 to calculate the torque, whichever is easiest.

FIGURE 10–13 (a) Forces acting at different angles at the doorknob. (b) The lever arm is defined as the perpendicular distance from the axis of rotation (the hinge) to the line of action of the force.



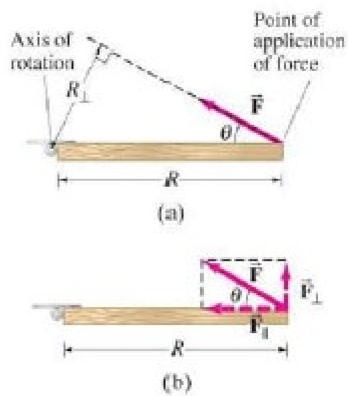


FIGURE 10–14 Torque $= R_{\perp}F = RF_{\perp}$.

Because torque is a distance times a force, it is measured in units of m·N in SI units,† cm·dyne in the cgs system, and ft·lb in the English system.

When more than one torque acts on an object, the angular acceleration α is found to be proportional to the *net* torque. If all the torques acting on an object tend to rotate it about a fixed axis of rotation in the same direction, the net torque is the sum of the torques. But if, say, one torque acts to rotate an object in one direction, and a second torque acts to rotate the object in the opposite direction (as in Fig. 10–15), the net torque is the difference of the two torques. We normally assign a positive sign to torques that act to rotate the object counterclockwise (just as θ is usually positive counterclockwise), and a negative sign to torques that act to rotate the object clockwise, when the rotation axis is fixed.

12–1 The Conditions for Equilibrium

Objects in daily life have at least one force acting on them (gravity). If they are at rest, then there must be other forces acting on them as well so that the net force is zero. A book at rest on a table, for example, has two forces acting on it, the downward force of gravity and the normal force the table exerts upward on it (Fig. 12–2). Because the book is at rest, Newton's second law tells us the net force on it is zero. Thus the upward force exerted by the table on the book must be equal in magnitude to the force of gravity acting downward on the book. Such an object is said to be in **equilibrium** (Latin for "equal forces" or "balance") under the action of these two forces.

Do not confuse the two forces in Fig. 12–2 with the equal and opposite forces of Newton's third law, which act on different objects. Here, both forces act on the same object; and they add up to zero.

The First Condition for Equilibrium

For an object to be at rest, Newton's second law tells us that the sum of the forces acting on it must add up to zero. Since force is a vector, the components of the net force must each be zero. Hence, a condition for equilibrium is that

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma F_z = 0. \tag{12–1}$$

We will mainly be dealing with forces that act in a plane, so we usually need only the x and y components. We must remember that if a particular force component points along the negative x or y axis, it must have a negative sign. Equations 12–1 are called the **first condition for equilibrium**.

The Second Condition for Equilibrium

Although Eqs. 12–1 are a necessary condition for an object to be in equilibrium, they are not always a sufficient condition. Figure 12–4 shows an object on which the net force is zero. Although the two forces labeled $\vec{\mathbf{F}}$ add up to give zero net force on the object, they do give rise to a net torque that will rotate the object. Referring to Eq. 10–14, $\Sigma\tau = I\alpha$, we see that if an object is to remain at rest, the net torque applied to it (calculated about *any* axis) must be zero. Thus we have the **second condition for equilibrium**: that the sum of the torques acting on an object, as calculated about any axis, must be zero:

$$\Sigma\tau = 0. \tag{12–2}$$

This condition will ensure that the angular acceleration, α, about any axis will be zero. If the object is not rotating initially $(\omega = 0)$, it will not start rotating. Equations 12–1 and 12–2 are the only requirements for an object to be in equilibrium.

We will mainly consider cases in which the forces all act in a plane (we call it the xy plane). In such cases the torque is calculated about an axis that is perpendicular to the xy plane. *The choice of this axis is arbitrary.* If the object is at rest, then $\Sigma\tau = 0$ about any axis whatever. Therefore we can choose any axis that makes our calculation easier. Once the axis is chosen, all torques must be calculated about that axis.

62. (I) The CM of an empty 1250-kg car is 2.50 m behind the front of the car. How far from the front of the car will the CM be when two people sit in the front seat 2.80 m from the front of the car, and three people sit in the back seat 3.90 m from the front? Assume that each person has a mass of 70.0 kg.

63. (I) The distance between a carbon atom ($m = 12\,\mathrm{u}$) and an oxygen atom ($m = 16\,\mathrm{u}$) in the CO molecule is 1.13×10^{-10} m. How far from the carbon atom is the center of mass of the molecule?

25. (I) Calculate the net torque about the axle of the wheel shown in Fig. 10–47. Assume that a friction torque of 0.40 m·N opposes the motion.

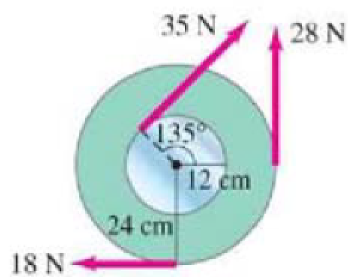


FIGURE 10–47 Problem 25.

29. (II) The bolts on the cylinder head of an engine require tightening to a torque of 75 m·N. If a wrench is 28 cm long, what force perpendicular to the wrench must the mechanic exert at its end? If the six-sided bolt head is 15 mm across (Fig. 10–49), estimate the force applied near each of the six points by a socket wrench.

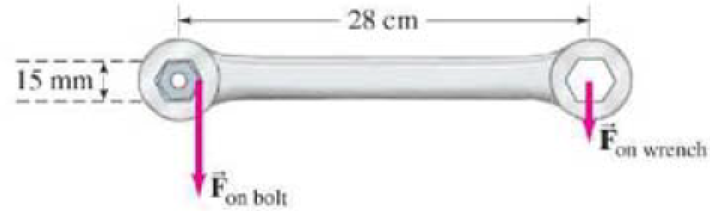


FIGURE 10–49 Problem 29.

30. (II) Determine the net torque on the 2.0-m-long uniform beam shown in Fig. 10–50. Calculate about (*a*) point C, the CM, and (*b*) point P at one end.

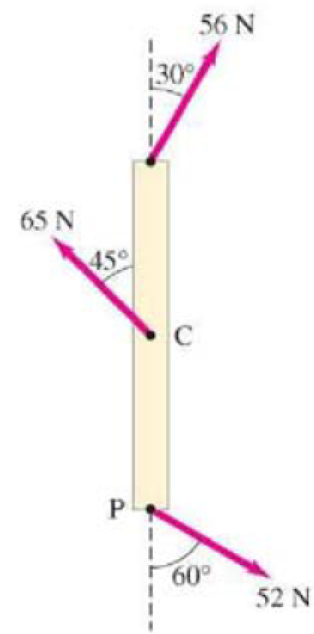


FIGURE 10–50 Problem 30.

14. (II) The force required to pull the cork out of the top of a wine bottle is in the range of 200 to 400 N. A common bottle opener is shown in Fig. 12–54. What range of forces F is required to open a wine bottle with this device?

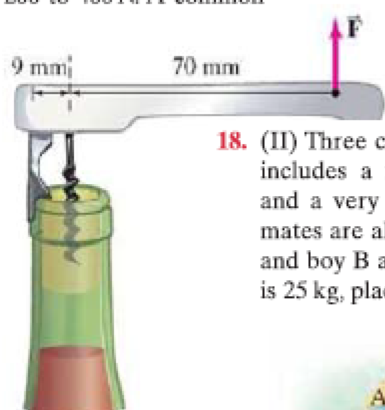


FIGURE 12–54 Problem 14.

18. (II) Three children are trying to balance on a seesaw, which includes a fulcrum rock acting as a pivot at the center, and a very light board 3.2 m long (Fig. 12–57). Two playmates are already on either end. Boy A has a mass of 45 kg, and boy B a mass of 35 kg. Where should girl C, whose mass is 25 kg, place herself so as to balance the seesaw?

11: Rotation – feste Achse

I. Wie beschreibt man die Dynamik des Rollens ohne Gleiten ?

z. Erinnerung: 2. Gesetz der Rotation

II. Wie bestimmt man das Trägheitsmoment eines starren Körpers ?

Parallelachsen-Theorem (Steinersche Satz)

III. Wie kann man die Rollbewegung als momentane Rotation um den Kontaktpunkt beschreiben ?

IV. Welches Trägheitsmoment für welche Körper ?

V. Welches ist die mechanische Energie eines rotierenden starren Körpers ?

z. Erinnerung: Dynamik

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(1.5 Seiten)*:

10-5 Torque and rotational inertia

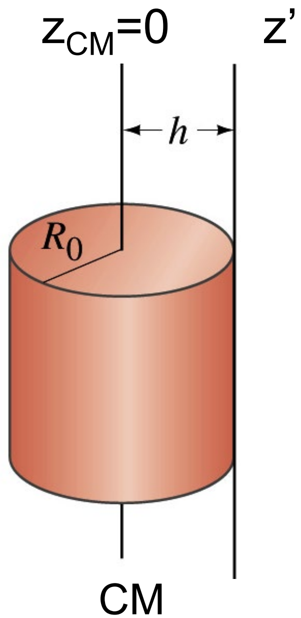
Vorbereitende Übungen *(4)* **vor der Übungssession** zu erledigen **:**

Giancoli 10-32, 41, 47a, 56, 10-65, 71

Giancoli Kapitel 10-5 bis 10-9

Beweis: Parallelachsen-Theorem

(Steinersches Theorem)



Situation: Nehmen wir an, dass Trägheitsmoment bezüglich eine Achse z durch den Massenschwerpunkt CM sei bekannt:

$I^{CM} \equiv \sum \Delta m_i r_i^2$.

Die Drehachse sei nun parallel um h verschoben und mit z' bezeichnet.

Frage: Welches ist die Beziehung zwischen den beiden Trägheitsmomenten ?

$$I' = \sum \Delta m_i (r_i + h)^2 = \overbrace{\sum \Delta m_i r_i^2}^{I^{CM}} + 2h \overbrace{\sum \Delta m_i r_i}^{M z_{CM}} + \overbrace{\sum \Delta m_i}^{M} h^2$$

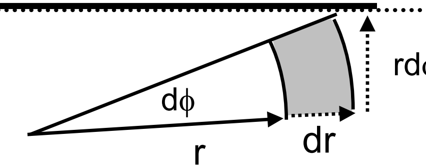
Rotation um eine Achse || zu derjenigen die durch den CM geht, wird durch ein Trägheitsmoment I' charakterisiert

$$\boxed{I' = I^{CM} + Mh^2}$$

Gut zu wissen: Sofern $h >> R_0 \rightarrow$ $I' \cong Mh^2$

$\Rightarrow$Bei grossen Distanzen kann man I^{CM} vernachlässigen

Anhang: Herleitung des Trägheitsmoments eines Zylinders

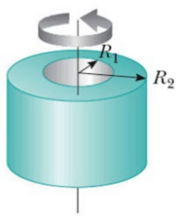


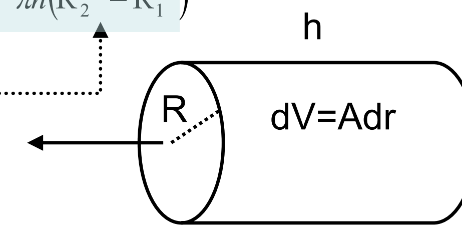
Hohlzylinder mit Radius r_i: $I_i = m_i r_i^2$ es folgt: $I = \sum m_i r_i^2$

Die Masse des Hohlzylinders: $V_i = 2\pi h r_i dr \rightarrow m_i = 2\pi\rho h r_i dr$

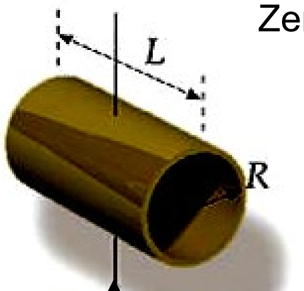
$$I = 2\pi\rho h \underbrace{\int_{R_1}^{R_2} r^3 dr}_{\frac{\left(R_2^4 - R_1^4\right)}{4}} = 2\pi\rho h \frac{1}{4}\left(R_2^2 - R_1^2\right)\left(R_1^2 + R_2^2\right) \rightarrow I = \frac{\pi h}{2}\frac{\left(R_2^2 - R_1^2\right)\left(R_1^2 + R_2^2\right)}{\pi h\left(R_2^2 - R_1^2\right)} M$$

$$V = \pi h\left(R_2^2 - R_1^2\right)$$





Zerlegung in dünne Stäbe:



$m_i h^2$ (Steinersches Theorem)

$$I_i = \frac{m_i L^2}{12} + \underbrace{m_i R^2 \sin^2\theta}_{\rho L R^3 \sin^2\theta d\theta}$$

$$m_i = \rho L R dR d\theta \rightarrow \rho L R^3 \sin^2\theta d\theta$$

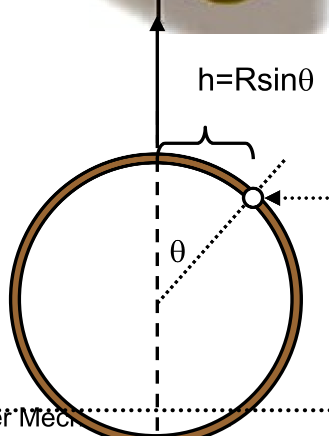
$$\sum I_i = \frac{L^2 \overbrace{\sum m_i}^{M}}{12} + \underbrace{R^2 \rho L R dR \overbrace{\int_0^{2\pi} \sin^2\theta d\theta}^{\pi}}_{\frac{R^2 M}{2}}$$

$$M = 2\pi\rho L R dR \rightarrow \frac{R^2 M}{2}$$

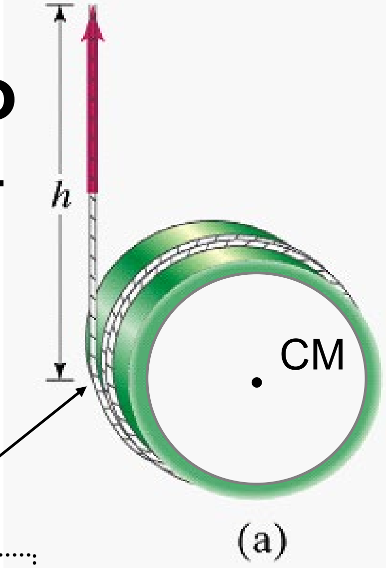
$$\underbrace{\int_0^{2\pi}\left(\sin^2 x + \cos^2 x\right)dx = 2\pi}$$

$$\int_0^{2\pi} \sin^2 x dx + \underbrace{\int_0^{2\pi} \cos^2 x dx}_{\int_0^{2\pi} \sin^2 x dx}$$

$$\int \sin^2 x dx = \frac{\sin x \cos x + x}{2}$$



Beweis: Zugspannung F_T eines Yoyo



Entspricht derselben Situation wie ein rollendes Rad (s. vorher)

das durch eine Kraft Mg an seinem CM gezogen wird. Die Zugspannung F_T entspricht also der Reibungskraft des Rades F_f, für die gilt, dass

$$F_f = -\frac{I_z^{CM}}{R^2} a_{CM} = F_T$$

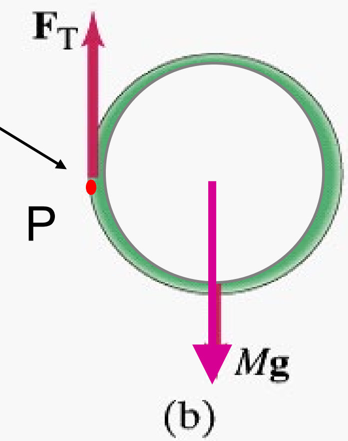
$$a_{CM} = \frac{F}{M}\left(\frac{1}{1+I_z^{CM}/MR^2}\right)$$

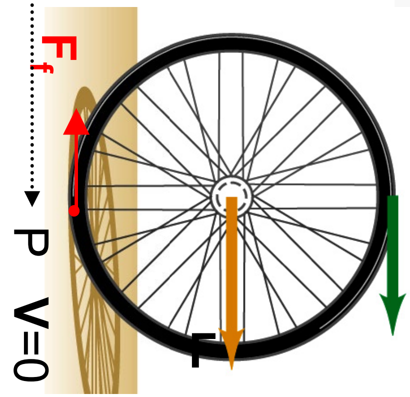
am Kontaktpunkt P gilt v=0

$$F_T = -F I_z^{CM}\left(\frac{1}{MR^2 + MR^2 I_z^{CM}/MR^2}\right)$$

$$F = -Mg$$

$$F_T = Mg\left(\frac{1}{MR^2/I_z^{CM}+1}\right)$$





Mit $I_z^{CM}=MR^2$ folgt

$$F_T = Mg\left(\frac{1}{MR^2/MR^2+1}\right)$$

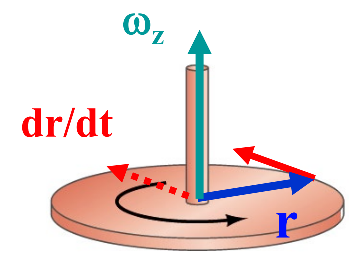
$F_T = F/2 = Mg/2$

z. Erinnerung: Kinematik und Dynamik der Rotation

Kinematik der Drehbewegung

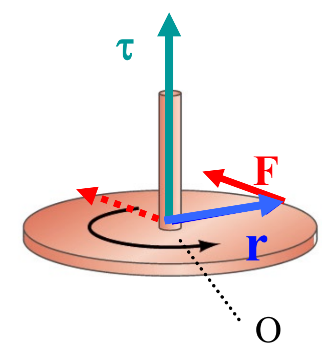
Momentane Winkelgeschwindigkeit :

$$\boxed{\Rightarrow \quad \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}}$$



Momentane Winkelbeschleunigung

$$\vec{\alpha}(t) \equiv \frac{d\vec{\omega}(t)}{dt}$$



Dynamik der Rotation

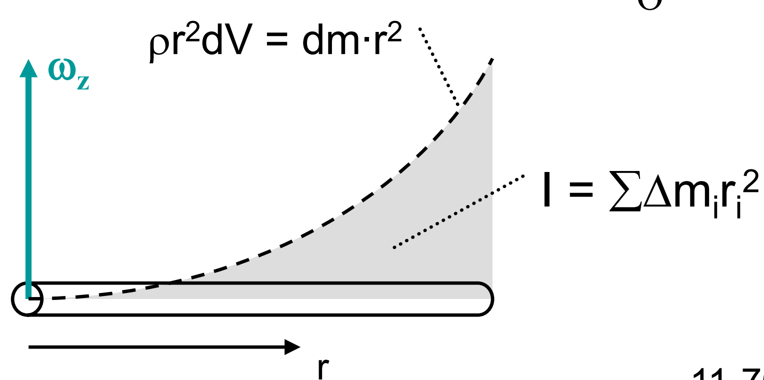
Drehmoment einer Kraft bezüglich eines Punktes O

$$\Rightarrow \quad \vec{\tau}_{\vec{r}_0} \equiv \Delta\vec{r} \times \vec{F} = \left(\vec{r} - \vec{r}_0\right) \times \vec{F}$$

Trägheitsmoment bezüglich einer Drehachse

$$I \equiv \int\limits_{\substack{\text{bezüglich} \\ \text{Drehachse}}} r^2 dm$$

r = Distanz zur Drehachse



Zugabe: Spule und Schnur - Analyse bezüglich dem CM

2. Gesetz Rotation:

$$\tau_z^{CM} = RF_f - rF = \frac{d\omega_z}{dt} I_z^{CM}$$

$$\frac{d\omega_z}{dt} \underset{\omega = -\frac{v_{CM}}{R}}{=} \frac{a_{CM}}{R} \quad \text{(Rollen)}$$

$$RF_f - rF = -\frac{a_{CM}}{R} I_z^{CM}$$

2. Axiom:

$$F\cos\theta + F_f = Ma_{CM}$$

$$-RF_f - RF\cos\theta = -MRa_{CM}$$

$$\underbrace{\left(RF_f - rF = -\frac{a_{CM}}{R} I_z^{CM}\right) \qquad \left(-RF_f - RF\cos\theta = -MRa_{CM}\right)}_{\text{Addition}}$$

$$-RF\cos\theta - rF = -MRa_{CM} - \frac{a_{CM}}{R} I_z^{CM}$$

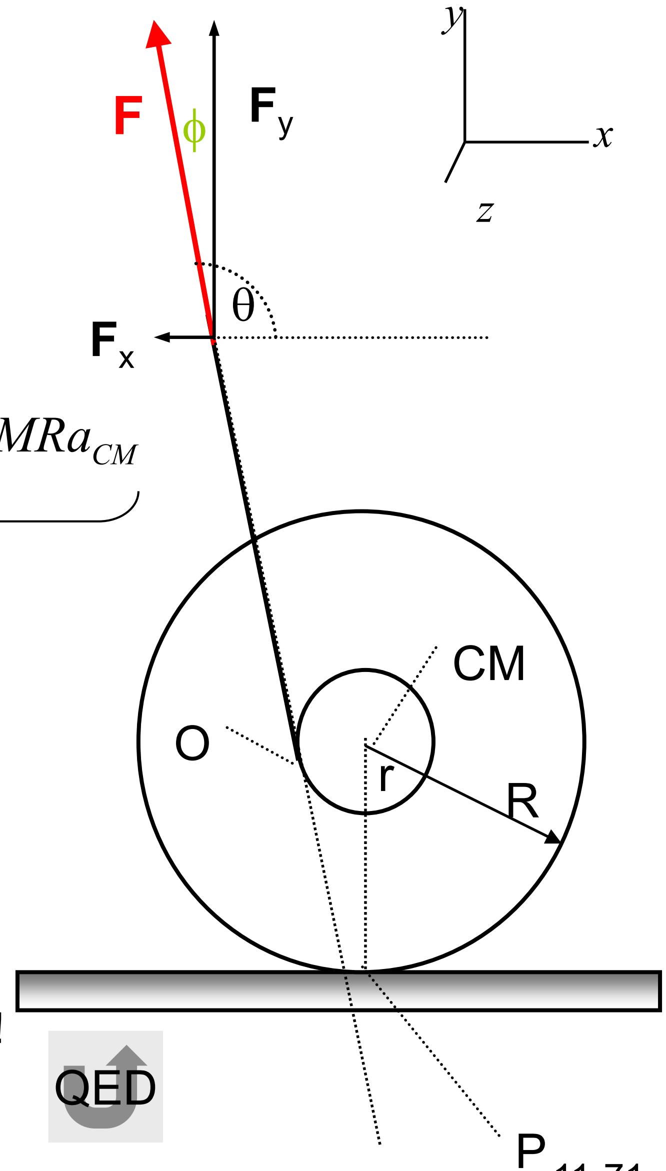
$$FR(\cos\theta + r/R) = M\left(R + \frac{I_z^{CM}}{MR}\right)a_{CM}$$

$$a_{CM} = \frac{F}{M}\frac{(r/R + \cos\theta)}{\left(1 + \frac{I_z^{CM}}{MR^2}\right)}$$

Je nach θ ist a_{CM} positiv oder negativ:
Wenn $a_{CM} = 0$: $R\cos\theta = -r \rightarrow R\sin\phi = r$

$\Rightarrow$ F ist entlang OP!

$\cos\theta = -\sin(\theta - 90) = -\sin\phi$



10–5 Rotational Dynamics; Torque and Rotational Inertia

We discussed in Section 10–4 that the angular acceleration α of a rotating object is proportional to the net torque τ applied to it:

$$\alpha \propto \Sigma\tau,$$

where we write $\Sigma\tau$ to remind us that it is the *net* torque (sum of all torques acting on the object) that is proportional to α. This corresponds to Newton's second law for translational motion, $a \propto \Sigma F$, but here torque has taken the place of force, and, correspondingly, the angular acceleration α takes the place of the linear acceleration a. In the linear case, the acceleration is not only proportional to the net force, but it is also inversely proportional to the inertia of the object, which we call its mass, m. Thus we could write $a = \Sigma F/m$. But what plays the role of mass for the rotational case? That is what we now set out to determine. At the same time, we will see that the relation $\alpha \propto \Sigma\tau$ follows directly from Newton's second law, $\Sigma F = ma$.

We first consider a very simple case: a particle of mass m rotating in a circle of radius R at the end of a string or rod whose mass we can ignore compared to m (Fig. 10–17), and we assume that a single force F acts on m tangent to the circle as shown. The torque that gives rise to the angular acceleration is $\tau = RF$. If we use Newton's second law for linear quantities, $\Sigma F = ma$, and Eq. 10–5 relating the angular acceleration to the tangential linear acceleration, $a_{\tan} = R\alpha$, then we have

$$\begin{aligned} F &= ma \\ &= mR\alpha, \end{aligned}$$

where α is given in rad/s^2. When we multiply both sides of this equation by R, we find that the torque $\tau = RF = R(mR\alpha)$, or

$$\tau = mR^2\alpha. \qquad \text{[single particle]} \qquad \textbf{(10–11)}$$

Here at last we have a direct relation between the angular acceleration and the applied torque τ. The quantity mR^2 represents the *rotational inertia* of the particle and is called its *moment of inertia*.

Now let us consider a rotating rigid object, such as a wheel rotating about a fixed axis through its center, such as an axle. We can think of the wheel as consisting of many particles located at various distances from the axis of rotation. We can apply Eq. 10–11 to each particle of the object; that is, we write $\tau_i = m_i R_i^2 \alpha$ for the i^{th} particle of the object. Then we sum over all the particles. The sum of the various torques is just the total torque, $\Sigma\tau$, so we obtain:

$$\Sigma\tau_i = \left(\Sigma m_i R_i^2\right)\alpha \qquad \text{[axis fixed]} \qquad \textbf{(10–12)}$$

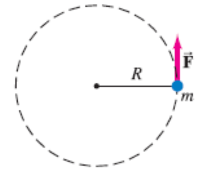


FIGURE 10–17 A mass m rotating in a circle of radius R about a fixed point.

NEWTON'S SECOND LAW FOR ROTATION

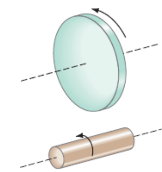


FIGURE 10–18 A large-diameter cylinder has greater rotational inertia than one of equal mass but smaller diameter.

where we factored out the α since it is the same for all the particles of a rigid object. The resultant torque, $\Sigma\tau$, represents the sum of all internal torques that each particle exerts on another, plus all external torques applied from the outside: $\Sigma\tau = \Sigma\tau_{\text{ext}} + \Sigma\tau_{\text{int}}$. The sum of the internal torques is zero from Newton's third law. Hence $\Sigma\tau$ represents the resultant *external* torque.

The sum $\Sigma m_i R_i^2$ in Eq. 10–12 represents the sum of the masses of each particle in the object multiplied by the square of the distance of that particle from the axis of rotation. If we give each particle a number $(1, 2, 3, \ldots)$, then

$$\Sigma m_i R_i^2 = m_1 R_1^2 + m_2 R_2^2 + m_3 R_3^2 + \cdots.$$

This summation is called the **moment of inertia** (or *rotational inertia*) I of the object:

$$I = \Sigma m_i R_i^2 = m_1 R_1^2 + m_2 R_2^2 + \cdots. \qquad \textbf{(10–13)}$$

Combining Eqs. 10–12 and 10–13, we can write

$$\Sigma\tau = I\alpha. \qquad \begin{bmatrix}\text{axis fixed in} \\ \text{inertial reference frame}\end{bmatrix} \qquad \textbf{(10–14)}$$

This is the rotational equivalent of Newton's second law. It is valid for the rotation of a rigid object about a fixed axis.[†] It can be shown (see Chapter 11) that Eq. 10–14 is valid even when the object is translating with acceleration, as long as I and α are calculated about the center of mass of the object, and the rotation axis through the CM doesn't change direction. (A ball rolling down a ramp is an example.) Then

$$(\Sigma\tau)_{\text{CM}} = I_{\text{CM}}\alpha_{\text{CM}}, \qquad \begin{bmatrix}\text{axis fixed in direction,} \\ \text{but may accelerate}\end{bmatrix} \qquad \textbf{(10–15)}$$

where the subscript CM means "calculated about the center of mass."

We see that the moment of inertia, I, which is a measure of the rotational inertia of an object, plays the same role for rotational motion that mass does for translational motion. As can be seen from Eq. 10–13, the rotational inertia of an object depends not only on its mass, but also on how that mass is distributed with respect to the axis. For example, a large-diameter cylinder will have greater rotational inertia than one of equal mass but smaller diameter (and therefore greater length), Fig. 10–18. The former will be harder to start rotating, and harder to stop. When the mass is concentrated farther from the axis of rotation, the rotational inertia is greater. For rotational motion, the mass of an object *cannot* be considered as concentrated at its center of mass.

For most ordinary objects, the mass is distributed continuously, and the calculation of the moment of inertia, ΣmR^2, can be difficult. Expressions can, however, be worked out (using calculus) for the moments of inertia of regularly shaped objects in terms of the dimensions of the objects, as we will discuss in Section 10–7. Figure 10–20 gives these expressions for a number of solids rotated about the axes specified. The only one for which the result is obvious is that for the thin hoop or ring rotated about an axis passing through its center perpendicular to the plane of the hoop (Fig. 10–20a). For this hoop, all the mass is concentrated at the same distance from the axis, R_0. Thus $\Sigma mR^2 = (\Sigma m)R_0^2 = MR_0^2$, where M is the total mass of the hoop.

When calculation is difficult, I can be determined experimentally by measuring the angular acceleration α about a fixed axis due to a known net torque, $\Sigma\tau$, and applying Newton's second law, $I = \Sigma\tau/\alpha$, Eq. 10–14.

	Object	Location of axis	Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center	MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter	$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center	$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center	$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center	$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end	$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center	$\frac{1}{12}M(\ell^2 + w^2)$

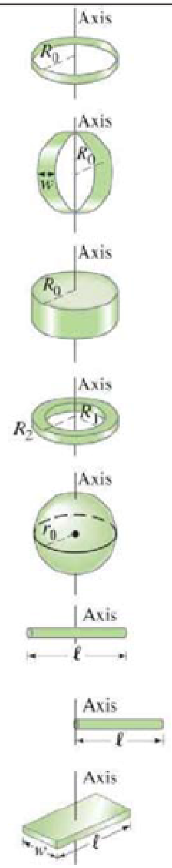


FIGURE 10–20 Moments of inertia for various objects of uniform composition. [We use R for radial distance from an axis, and r for distance from a point (only in e, the sphere), as discussed in Fig. 10–2.]

32. (I) Estimate the moment of inertia of a bicycle wheel 67 cm in diameter. The rim and tire have a combined mass of 1.1 kg. The mass of the hub can be ignored (why?).

41. (II) A merry-go-round accelerates from rest to 0.68 rad/s in 24 s. Assuming the merry-go-round is a uniform disk of radius 7.0 m and mass 31,000 kg, calculate the net torque required to accelerate it.

47. (II) A helicopter rotor blade can be considered a long thin rod, as shown in Fig. 10–55. (*a*) If each of the three rotor helicopter blades is 3.75 m long and has a mass of 135 kg, calculate the moment of inertia of the three rotor blades about the axis of rotation. (*b*) How much torque must the motor apply to bring the blades from rest up to a speed of 5.0 rev/s in 8.0 s?

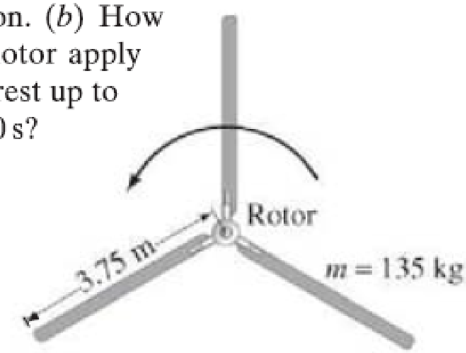


FIGURE 10–55 Problem 47.

56. (II) Determine the moment of inertia of a 19-kg door that is 2.5 m high and 1.0 m wide and is hinged along one side. Ignore the thickness of the door.

65. (II) A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

71. (I) A bowling ball of mass 7.3 kg and radius 9.0 cm rolls without slipping down a lane at 3.7 m/s. Calculate its total kinetic energy.

12: Der Drehimpuls

I. Wie bestimmt man den Drehimpuls eines starren Körpers ?
Drehungen im Ungleichgewicht - Zentrifuge

II. Wozu dient die Drehimpulserhaltung ?

III. Wie beschreibt man die Bewegung eines Kreisels ?
Gyroskop – die gleichmässige Drehbewegung der Rotation

Vorbereitung auf die Vorlesung und Übungen

Kapitel im Giancoli **vor dem Kurs zu lesen** *(2.5 Seiten)*:

11-1 Angular Momentum – objects rotating about a fixed axis

11-7 The spinning top and gyroscope

Vorbereitende Übungen *(12)* **vor der Übungssession** zu erledigen**:**

11-2, 4, 6, 15, 55, 57

Giancoli Kapitel 11-4 bis 11-7 (wenig Übereinstimmung)

Anhang: Rollbewegung ohne fixierte Achse

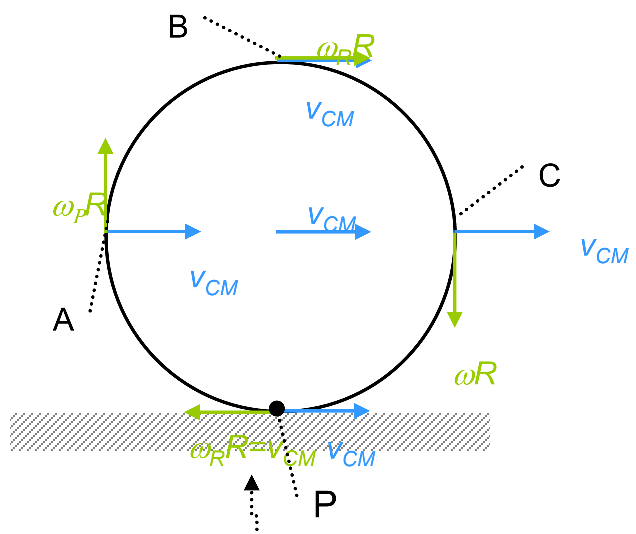
ohne Gleiten

Situation: Ein Geldstück mit Radius R rollt ohne zu gleiten. Rekapitulation:

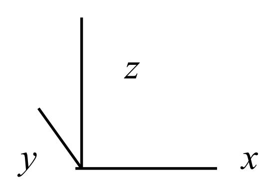
Translation des v_{CM}

+

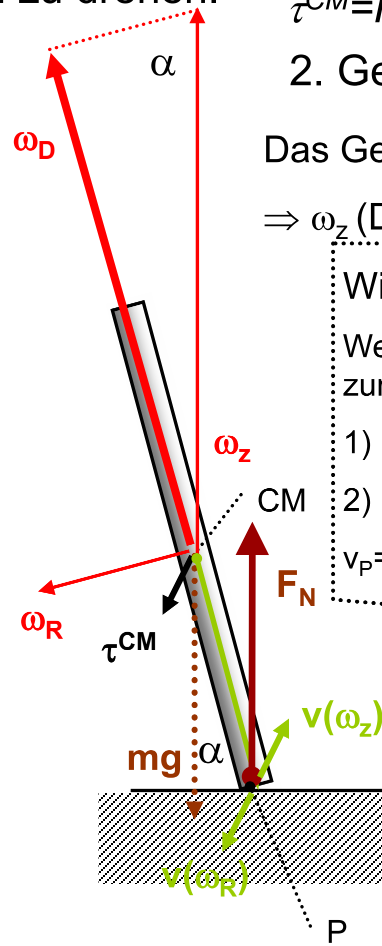
Rotation um CM mit ω_R=-v/R



Rollen ohne gleiten: $v_P=0$



Beobachtung: Nach einer kurzen Zeit, befindet sich aufgründ von Störungen die Achse des Geldstücks nicht mehr ⊥ zur Vertikalen ($\alpha<90^0$). Das Stück ist abgelenkt und beginnt sich mit ω_z um sein CM zu drehen.



$\tau^{CM}=R\cos\alpha F_N$

2. Gesetz rotation(τ=Iα): $\frac{d\vec{\omega}}{dt}=\frac{\vec{\tau}^{CM}}{I^{CM}}$

Das Geldstück ist abgelenkt

$\Rightarrow \omega_z$ (Drehung um den CM entlang z)

Wie verhalten sich ω_R und ω_z ?

Wenn $\mathbf{v}_{CM}$=0, ergeben sich zwei Beiträge zur Geschwindigkeit am Kontaktpunkt P:

1) Durch ω_z: $v_y(\omega_z)=\omega_z(R\cos\alpha)$

2) Durch ω_R: $v_y(\omega_R)= -\omega_R R$

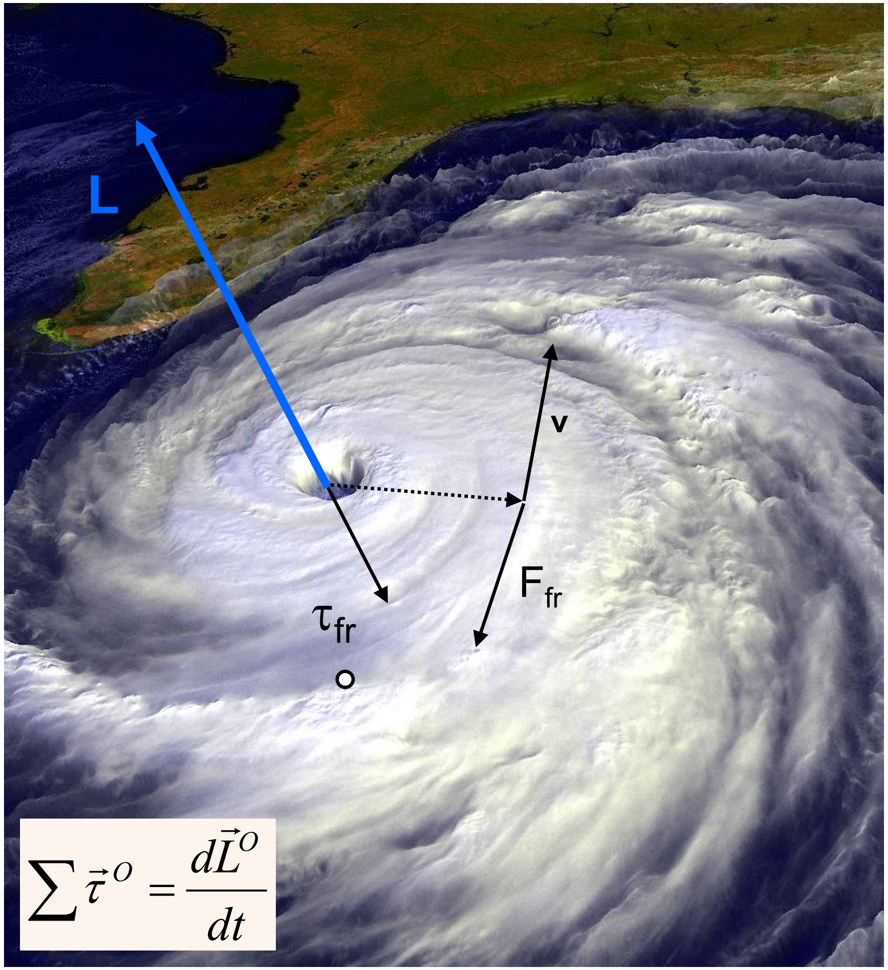
$v_P=v_y(\omega_z)+v_y(\omega_R)=0$ (sans glisser)

$\omega_z R\cos\alpha = \omega_R R \quad \rightarrow \omega_R = \omega_z\cos\alpha$

$\Rightarrow \boxed{\vec{\omega}_D=\vec{\omega}_z+\vec{\omega}_R}$

Die resultierende Drehung ist ω_D, durch CM und P definiert

Zugabe: Wirbelstürme



$$\sum \vec{\tau}^{O} = \frac{d\vec{L}^{O}}{dt}$$

Beobachtung: Wirbelstürme entstehen über warmem Wasser und Ihre Rotation mit starken Winden von 200-300km/h ist über Tage hinaus erhalten.

Die sich drehende Luft erfährt Reibungskräfte mit langsmer fliessender Luft sowie mit dem Wasser.

Das resultiernde Drehmoment reduziert den Drehimpules …

Frage I: Welche Kraft/Energie erhält den Wirbelstrum aufrecht ?

Frage II: Welches sind gewissse Folgen der Klimaerwärmung ?

http://webphysics.iupui.edu/webscience/physics_archive/hurricanes.html

http://www.physics.ubc.ca/outreach/phys420/p420_04/sean/

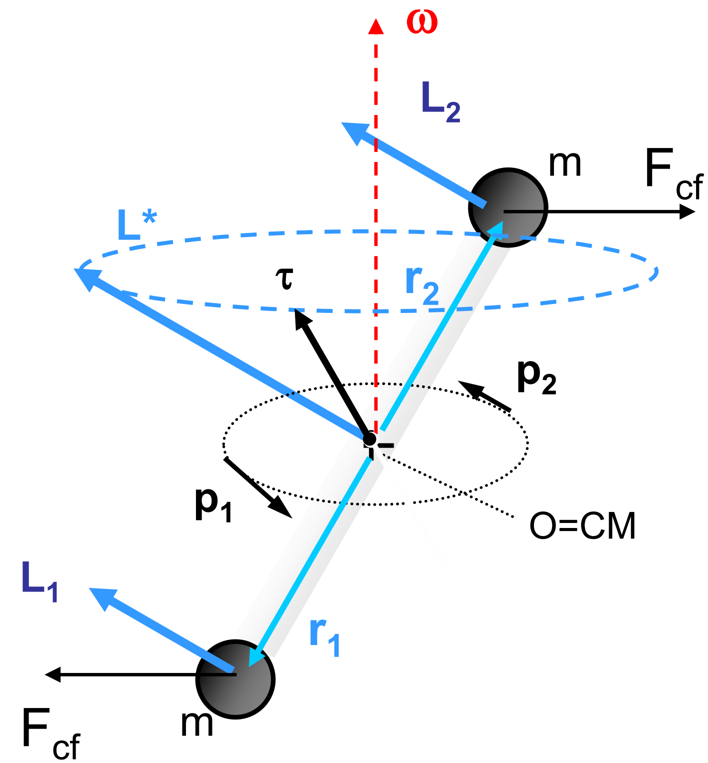
Zugabe: Das Drehmoment einer freien Rotation

fast entlang einer Symmetrieachse

Situation: Objekt das sich fast entlang seiner Symmetrieachse dreht. Der Grossteil des Objektes ist immer noch symmetrisch zur Drehachse von L, mit Ausnahme des Ungleichgewichts, hier durch zwei Massen modelliert, die sich um ω drehen, beide mit einem Drehimpuls L* assoziiert, der um ω präzessiert.

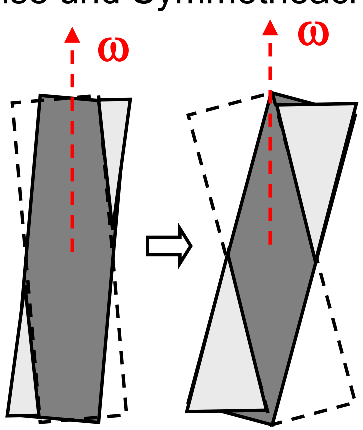
Um die Drehachse stabil zu halten, braucht es ein externes Drehmoment (siehe Zentrifuge und Kugellager).

Analyse *(im Koordinatensystem des Objektes)*: Die Zentrifugalkräfte der zwei Massen resultieren in einem Drehmoment von $2m\omega^2 r\sin\phi$.



$$\vec{p}\cdot\vec{F}_{cf}=0\rightarrow\vec{\tau}\cdot\vec{L}=0 \cdots\cdots\rightarrow L^2\text{=konstant}$$

Das Drehmoment erhöht das Ungleichgewicht zwischen Drehachse und Symmetrieachse.



11–1 Angular Momentum—Objects Rotating About a Fixed Axis

In Chapter 10 we saw that if we use the appropriate angular variables, the kinematic and dynamic equations for rotational motion are analogous to those for ordinary linear motion. In like manner, the linear momentum, $p = mv$, has a rotational analog. It is called **angular momentum**, L, and for an object rotating about a fixed axis with angular velocity ω, it is defined as

$$L = I\omega, \tag{11–1}$$

where I is the moment of inertia. The SI units for L are $\mathrm{kg \cdot m^2/s}$; there is no special name for this unit.

We saw in Chapter 9 (Section 9–1) that Newton's second law can be written not only as $\Sigma F = ma$, but also more generally in terms of momentum (Eq. 9–2), $\Sigma F = dp/dt$. In a similar way, the rotational equivalent of Newton's second law, which we saw in Eqs. 10–14 and 10–15 can be written as $\Sigma\tau = I\alpha$, can also be written in terms of angular momentum: since the angular acceleration $\alpha = d\omega/dt$ (Eq. 10–3), then $I\alpha = I(d\omega/dt) = d(I\omega)/dt = dL/dt$, so

$$\Sigma\tau = \frac{dL}{dt}. \tag{11–2}$$

This derivation assumes that the moment of inertia, I, remains constant. However, Eq. 11–2 is valid even if the moment of inertia changes, and applies also to a system of objects rotating about a fixed axis where $\Sigma\tau$ is the net external torque (discussed in Section 11–4). Equation 11–2 is Newton's second law for rotational motion about a fixed axis, and is also valid for a moving object if its rotation is about an axis passing through its center of mass (as for Eq. 10–15).

Conservation of Angular Momentum

Angular momentum is an important concept in physics because, under certain conditions, it is a conserved quantity. What are the conditions for which it is conserved? From Eq. 11–2 we see immediately that if the net external torque $\Sigma\tau$ on an object (or system of objects) is zero, then

$$\frac{dL}{dt} = 0 \quad \text{and} \quad L = I\omega = \text{constant}. \qquad [\Sigma\tau = 0]$$

This, then, is the **law of conservation of angular momentum** for a rotating object:

> **The total angular momentum of a rotating object remains constant if the net external torque acting on it is zero.**

The law of conservation of angular momentum is one of the great conservation laws of physics, along with those for energy and linear momentum.

When there is zero net torque acting on an object, and the object is rotating about a fixed axis or about an axis through its center of mass whose direction doesn't change, we can write

$$I\omega = I_0\omega_0 = \text{constant}.$$

I_0 and ω_0 are the moment of inertia and angular velocity, respectively, about the axis at some initial time $(t = 0)$, and I and ω are their values at some other time. The parts of the object may alter their positions relative to one another, so that I changes. But then ω changes as well and the product $I\omega$ remains constant.

Many interesting phenomena can be understood on the basis of conservation of angular momentum. Consider a skater doing a spin on the tips of her skates, Fig. 11–1. She rotates at a relatively low speed when her arms are outstretched, but when she brings her arms in close to her body, she suddenly spins much faster. From the definition of moment of inertia, $I = \Sigma mR^2$, it is clear that when she pulls her arms in closer to the axis of rotation, R is reduced for the arms so her moment of inertia is reduced. Since the angular momentum $I\omega$ remains constant (we ignore the small torque due to friction), if I decreases, then the angular velocity ω must increase. If the skater reduces her moment of inertia by a factor of 2, she will then rotate with twice the angular velocity.

A similar example is the diver shown in Fig. 11–2. The push as she leaves the board gives her an initial angular momentum about her center of mass. When she curls herself into the tuck position, she rotates quickly one or more times. She then stretches out again, increasing her moment of inertia which reduces the angular velocity to a small value, and then she enters the water. The change in moment of inertia from the straight position to the tuck position can be a factor of as much as $3\frac{1}{2}$.

Note that for angular momentum to be conserved, the net torque must be zero, but the net force does not necessarily have to be zero. The net force on the diver in Fig. 11–2, for example, is not zero (gravity is acting), but the net torque about her CM is zero because the force of gravity acts at her center of mass.

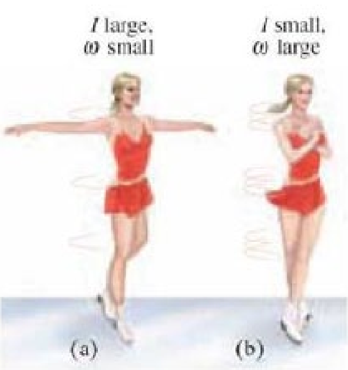


FIGURE 11–1 A skater doing a spin on ice, illustrating conservation of angular momentum: (a) I is large and ω is small; (b) I is smaller so ω is larger.

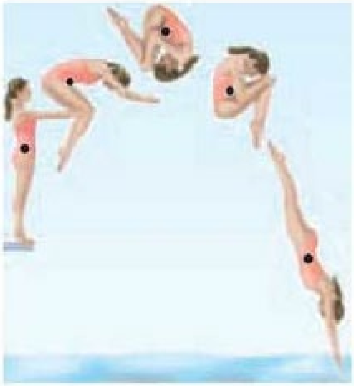


FIGURE 11–2 A diver rotates faster when arms and legs are tucked in than when they are outstretched. Angular momentum is conserved.

Directional Nature of Angular Momentum

Angular momentum is a vector, as we shall discuss later in this Chapter. For now we consider the simple case of an object rotating about a fixed axis, and the direction of $\vec{\mathbf{L}}$ is specified by a plus or minus sign, just as we did for one-dimensional linear motion in Chapter 2.

For a symmetrical object rotating about a symmetry axis (such as a cylinder or wheel), the direction of the angular momentum[†] can be taken as the direction of the angular velocity $\vec{\omega}$. That is,

$$\vec{\mathbf{L}} = I\vec{\omega}.$$

As a simple example, consider a person standing at rest on a circular platform capable of rotating friction-free about an axis through its center (that is, a simplified merry-go-round). If the person now starts to walk along the edge of the platform, Fig. 11–5a, the platform starts rotating in the opposite direction. Why? One explanation is that the person's foot exerts a force on the platform. Another explanation (and this is the most useful analysis here) is as an example of the conservation of angular momentum. If the person starts walking counterclockwise, the person's angular momentum will be pointed upward along the axis of rotation (remember how we defined the direction of $\vec{\omega}$ using the right-hand rule in Section 10–2). The magnitude of the person's angular momentum will be $L = I\omega = (mR^2)(v/R)$, where v is the person's speed (relative to the Earth, not the platform), R is his distance from the rotation axis, m is his mass, and mR^2 is his moment of inertia if we consider him a particle (mass concentrated at one point). The platform rotates in the opposite direction, so its angular momentum points downward. If the initial total angular momentum was zero (person and platform at rest), it will remain zero after the person starts walking. That is, the upward angular momentum of the person just balances the oppositely directed downward angular momentum of the platform (Fig. 11–5b), so the total vector angular momentum remains zero. Even though the person exerts a force (and torque) on the platform, the platform exerts an equal and opposite torque on the person. So the net torque on the *system* of person plus platform is zero (ignoring friction) and the total angular momentum remains constant.

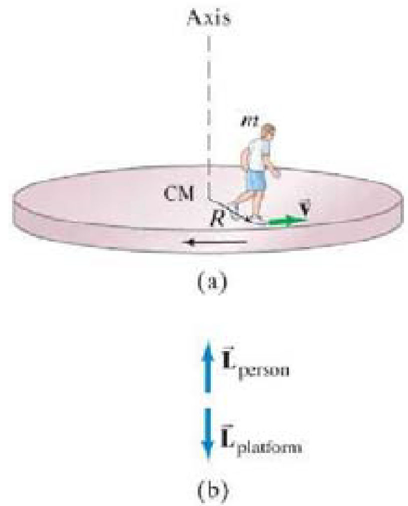


FIGURE 11–5 (a) A person on a circular platform, both initially at rest, begins walking along the edge at speed v. The platform, assumed to be mounted on friction-free bearings, begins rotating in the opposite direction, so that the total angular momentum remains zero, as shown in (b).

* 11–7 The Spinning Top and Gyroscope

The motion of a rapidly spinning top, or a gyroscope, is an interesting example of rotational motion and of the use of the vector equation

$$\sum \vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt}.$$

Consider a symmetrical top of mass M spinning rapidly about its symmetry axis, as in Fig. 11–23. The top is balanced on its tip at point O in an inertial reference frame. If the axis of the top makes an angle ϕ to the vertical (z axis), when the top is carefully released its axis will move, sweeping out a cone about the vertical as shown by the dashed lines in Fig. 11–23. This type of motion, in which a torque produces a change in the direction of the rotation axis, is called **precession**. The rate at which the rotation axis moves about the vertical (z) axis is called the angular velocity of precession, Ω (capital Greek omega). Let us now try to understand the reasons for this motion, and calculate Ω.

If the top were not spinning, it would immediately fall to the ground when released due to the pull of gravity. The apparent mystery of a top is that when it is spinning, it does not immediately fall to the ground but instead precesses—it moves slowly sideways. But this is not really so mysterious if we examine it from the point of view of angular momentum and torque, which we calculate about the point O. When the top is spinning with angular velocity ω about its symmetry axis, it has an angular momentum $\vec{\mathbf{L}}$ directed along its axis, as shown in Fig. 11–23. (There is also angular momentum due to the precessional motion, so that the total $\vec{\mathbf{L}}$ is not exactly along the axis of the top; but if $\Omega \ll \omega$, which is usually the case, we can ignore this.) To change the angular momentum, a torque is required. If no torque were applied to the top, $\vec{\mathbf{L}}$ would remain constant in magnitude and direction; the top would neither fall nor precess. But the slightest tip to the side results in a net torque about O, equal to $\vec{\tau}_{\text{net}} = \vec{\mathbf{r}} \times M\vec{\mathbf{g}}$, where $\vec{\mathbf{r}}$ is the position vector of the top's center of mass with respect to O, and M is the mass of the top. The direction of $\vec{\tau}_{\text{net}}$ is perpendicular to both $\vec{\mathbf{r}}$ and $M\vec{\mathbf{g}}$ and by the right-hand rule is, as shown in Fig. 11–23, in the horizontal (xy) plane. The change in $\vec{\mathbf{L}}$ in a time dt is

$$d\vec{\mathbf{L}} = \vec{\tau}_{\text{net}}\, dt,$$

which is perpendicular to $\vec{\mathbf{L}}$ and horizontal (parallel to $\vec{\tau}_{\text{net}}$), as shown in Fig. 11–23. Since $d\vec{\mathbf{L}}$ is perpendicular to $\vec{\mathbf{L}}$, the magnitude of $\vec{\mathbf{L}}$ does not change. Only the direction of $\vec{\mathbf{L}}$ changes. Since $\vec{\mathbf{L}}$ points along the axis of the top, we see that this axis moves to the right in Fig. 11–23. That is, the upper end of the top's axis moves in a horizontal direction perpendicular to $\vec{\mathbf{L}}$. This explains why the top precesses rather than falls. The vector $\vec{\mathbf{L}}$ and the top's axis move together in a horizontal circle. As they do so, $\vec{\tau}_{\text{net}}$ and $d\vec{\mathbf{L}}$ rotate as well so as to be horizontal and perpendicular to $\vec{\mathbf{L}}$.

To determine Ω, we see from Fig. 11–23 that the angle $d\theta$ (which is in a horizontal plane) is related to dL by

$$dL = L \sin\phi\, d\theta,$$

since $\vec{\mathbf{L}}$ makes an angle ϕ to the z axis. The angular velocity of precession is $\Omega = d\theta/dt$, which becomes (since $d\theta = dL/L\sin\phi$)

$$\Omega = \frac{1}{L\sin\phi}\frac{dL}{dt} = \frac{\tau}{L\sin\phi}. \qquad \text{[spinning top]} \quad \textbf{(11–13a)}$$

But $\tau_{\text{net}} = |\vec{\mathbf{r}} \times M\vec{\mathbf{g}}| = rMg\sin\phi$ [because $\sin(\pi - \phi) = \sin\phi$] so we can also write

$$\Omega = \frac{Mgr}{L}. \qquad \text{[spinning top]} \quad \textbf{(11–13b)}$$

Thus the rate of precession does not depend on the angle ϕ; but it is inversely proportional to the top's angular momentum. The faster the top spins, the greater L is and the slower the top precesses.

From Eq. 11–1 (or Eq. 11–11) we can write $L = I\omega$, where I and ω are the moment of inertia and angular velocity of the spinning top about its spin axis. Then Eq. 11–13b for the top's precession angular velocity becomes

$$\Omega = \frac{Mgr}{I\omega}. \qquad \textbf{(11–13c)}$$

Equations 11–13 apply also to a toy gyroscope, which consists of a rapidly spinning wheel mounted on an axle (Fig. 11–24). One end of the axle rests on a support. The other end of the axle is free and will precess like a top if its "spin" angular velocity ω is large compared to the precession rate ($\omega \gg \Omega$). As ω decreases due to friction and air resistance, the gyroscope will begin to fall, just as does a top.

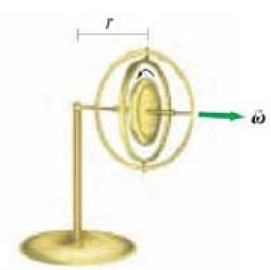


FIGURE 11–24 A toy gyroscope.

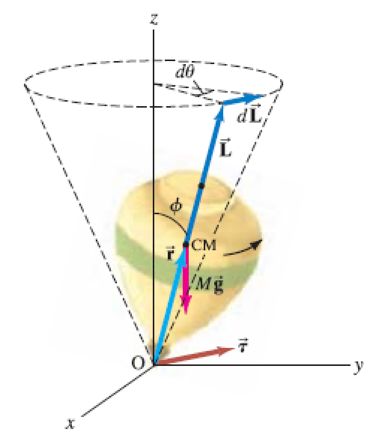


FIGURE 11–23 Spinning top.

65. (II) A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

71. (I) A bowling ball of mass 7.3 kg and radius 9.0 cm rolls without slipping down a lane at 3.7 m/s. Calculate its total kinetic energy.

2. (I) (*a*) What is the angular momentum of a 2.8-kg uniform cylindrical grinding wheel of radius 18 cm when rotating at 1300 rpm? (*b*) How much torque is required to stop it in 6.0 s?

4. (II) A figure skater can increase her spin rotation rate from an initial rate of 1.0 rev every 1.5 s to a final rate of 2.5 rev/s. If her initial moment of inertia was $4.6\ \mathrm{kg \cdot m^2}$, what is her final moment of inertia? How does she physically accomplish this change?

6. (II) A uniform horizontal rod of mass M and length ℓ rotates with angular velocity ω about a vertical axis through its center. Attached to each end of the rod is a small mass m. Determine the angular momentum of the system about the axis.

15. (II) A nonrotating cylindrical disk of moment of inertia I is dropped onto an identical disk rotating at angular speed ω. Assuming no external torques, what is the final common angular speed of the two disks?

how long would it take to precess once?

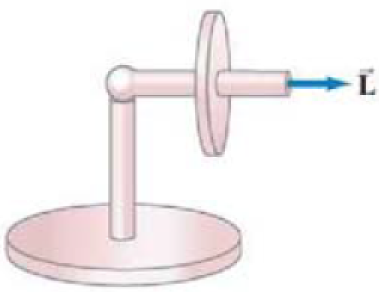


FIGURE 11–41 A wheel, rotating about a horizontal axle supported at one end, precesses. Problems 54, 55, and 56.

***55.** (II) Suppose the solid wheel of Fig. 11–41 has a mass of 300 g and rotates at 85 rad/s; it has radius 6.0 cm and is mounted at the center of a horizontal thin axle 25 cm long. At what rate does the axle precess?

***57.** (II) A bicycle wheel of diameter 65 cm and mass m rotates on its axle; two 20-cm-long wooden handles, one on each side of the wheel, act as the axle. You tie a rope to a small hook on the end of one of the handles, and then spin the bicycle wheel with a flick of the hand. When you release the spinning wheel, it precesses about the vertical axis defined by the rope, instead of falling to the ground (as it would if it were not spinning). Estimate the rate and direction of precession if the wheel rotates counterclockwise at 2.0 rev/s and its axle remains horizontal.